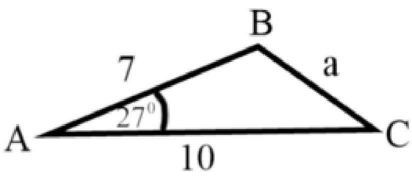
1. Consider the triangle in the figure. Assume lengths are in centimeters.
   a. (10 pts) Use the Law of Cosines to find the length of side a, to 4 decimal places.
   b. (10 pts) Use the Law of Sines to find angle C to 4 decimal places.



(a)

$a^2 = b^2 + c^2 - 2bc\cos A$

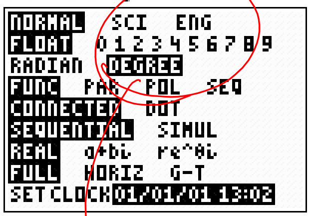
$= 100 + 49 - 2(10)(7)\cos 27^\circ$

$= 149 - 140\cos 27^\circ$

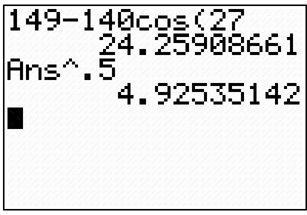
≈ 24.25908661

$\Rightarrow a \approx 4.92535142$

$\boxed{a \approx 4.9254}$



Always check mode.



(b)

$\frac{\sin C}{c} = \frac{\sin A}{a}$

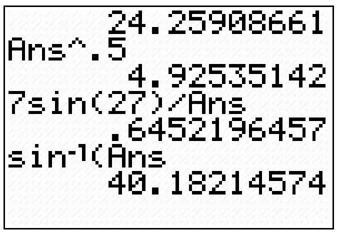
$\frac{\sin C}{7} = \frac{\sin 27^\circ}{a}$

$\sin C = \frac{7\sin 27^\circ}{4.92\ldots}$

$\approx .6452196457$

$\Rightarrow C \approx 40.18214574$

$\Rightarrow \boxed{C \approx 40.1821^\circ}$

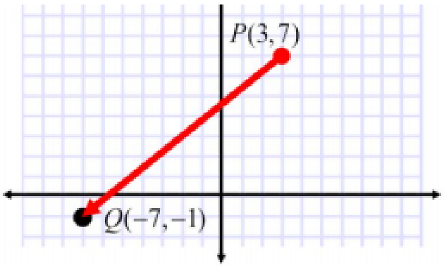


2. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\bar{u}$:

   a. (5 pts) Express the vector $\bar{u} = \overrightarrow{PQ}$ in component form.

   b. (5 pts) Compute the magnitude of $\bar{u}$. Leave your answer in simplified radical form.

   c. (10 pts) Find the direction angle of $\bar{u}$. Use degrees, rounded to 4 places.

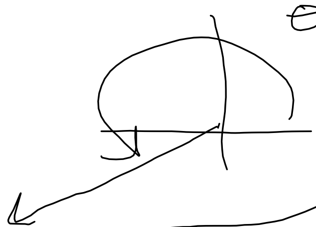


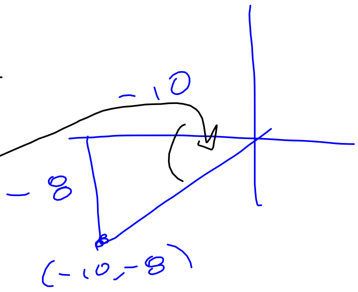
(a) $\bar{u} = \langle -7-3, -1-7 \rangle$

$= \langle -10, -8 \rangle$

(c) Direction angle is positive angle measured from the positive x-axis

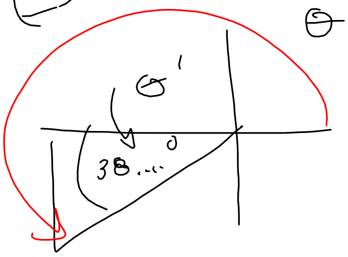
θ





$\theta' = \arctan\left(\frac{4}{5}\right)$

θ




```
tan⁻¹(4/5)
         38.65980825
```

$\Rightarrow \theta = 180^\circ + 38....^\circ \approx 218.6598^\circ \approx \theta$

$\theta \approx 180^\circ + 38.65980825^\circ$

Dot Product:

$\bar{u} = \langle u_1, u_2 \rangle$ & $\bar{v} = \langle v_1, v_2 \rangle$, then

$\bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2$

$\bar{u} = \langle 1, 5 \rangle$, $\bar{v} = \langle 2, -6 \rangle \implies$

$\bar{u} \cdot \bar{v} = 1 \cdot 2 + 5 \cdot (-6) = 2 - 30 = -28.$

Vector length: $\|\bar{u}\| = \sqrt{u_1^2 + u_2^2}$

$\|\langle 1, 5 \rangle\| = \|\bar{u}\| = \sqrt{1^2 + 5^2} = \sqrt{26}$

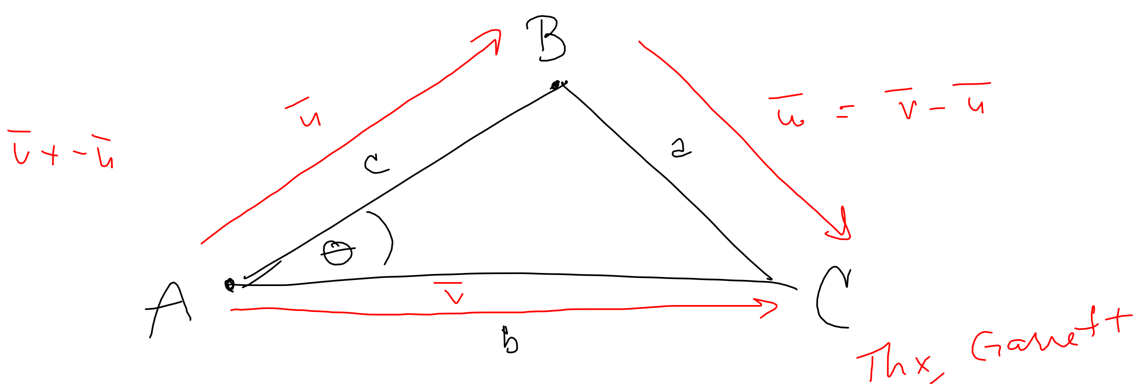
Note: $\bar{u} \cdot \bar{u} = 1^2 + 5^2 = 26$

$= \langle 1, 5 \rangle \cdot \langle 1, 5 \rangle =$

so $\|\bar{u}\| = \sqrt{\bar{u} \cdot \bar{u}}$

Dot Product's relationship to angles...

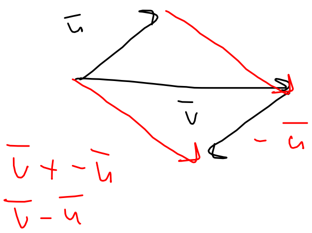
$\cos\theta$



$\overrightarrow{AB} = \bar{u}$ $\quad \overrightarrow{BC} = \bar{w}$ $\quad \overrightarrow{AC} = \bar{v}$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\|\bar{v} - \bar{u}\|^2 = \|\bar{v}\|^2 + \|\bar{u}\|^2 - 2\|\bar{v}\|\,\|\bar{u}\|\cos A$$



$$(\bar{v} - \bar{u}) \cdot (\bar{v} - \bar{u}) = \bar{v} \cdot \bar{v} + \bar{u} \cdot \bar{u} - 2\|\bar{u}\|\,\|\bar{v}\|\cos A$$

$$= (\langle v_1, v_2\rangle - \langle u_1, u_2\rangle) \cdot (\langle v_1, v_2\rangle - \langle u_1, u_2\rangle) = \ldots$$

$$= \langle v_1 - u_1, v_2 - u_2\rangle \cdot \langle v_1 - u_1, v_2 - u_2\rangle$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$= (v_1 - u_1)(v_1 - u_1) + (v_2 - u_2)(v_2 - u_2) = v_1^2 - 2v_1u_1 + u_1^2 + v_2^2 - 2v_2u_2 + u_2^2$$

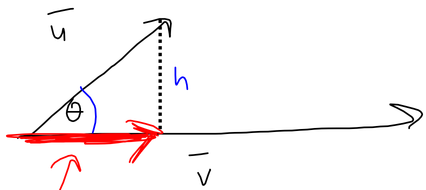
$$= v_1^2 - 2u_1v_1 + u_1^2 + v_2^2 - 2u_2v_2 + u_2^2 = v_1^2 + v_2^2 + u_1^2 + u_2^2 - 2\|\bar{u}\|\,\|\bar{v}\|\cos A$$

$$2\|\bar{u}\|\|\bar{v}\|\cos A = 2u_1v_1 + 2u_2v_2$$

$$\|\bar{u}\|\|\bar{v}\|\cos A = u_1v_1 + u_2v_2 = \bar{u}\cdot\bar{v}$$

$$\cos A = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \cos\theta,$$ where

θ is the angle between $\bar{u}$ & $\bar{v}$!

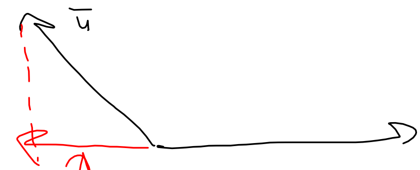


Let $\bar{w} = proj_{\bar{v}}\bar{u}$ = projection onto $\bar{v}$ of $\bar{u}$

$$\frac{\|\bar{w}\|}{\|\bar{u}\|} = \cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$

$$\|\bar{w}\| = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|} = comp_{\bar{v}}\bar{u}$$ = component of $\bar{u}$ the direction of $\bar{v}$.

One caveat: Notice the left hand side is positive. The right hand side might not be. It's all good, as long as the angle between the vectors is between 0,degrees and 90 degrees. So I should've said at the start that the angle was acute. Notice that if it IS greater than 90 degrees, it looks like this:



$proj_{\bar v}\bar u$ is in the opposite direction from $\bar u$.

To GET the projection, we need a VECTOR:

$$\frac{\bar u \cdot \bar v}{\|\bar v\|}$$

Right size.

want it parallel to $\bar v$!

Cheat sheet

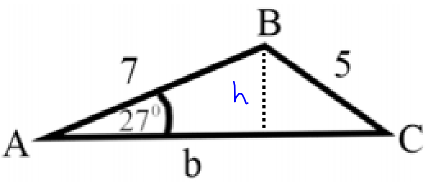
$$\left(\frac{\bar u \cdot \bar v}{\|\bar v\|}\right)\frac{\bar v}{\|\bar v\|} = \boxed{\frac{\bar u \cdot \bar v}{\|\bar v\|^2}\,\bar v}$$

unit vector in direction of $\bar v$.

5. Consider the triangle in the figure on the right.

   a. (10 pts) Prove there are 2 triangles that are possible from this ambiguous information.

   b. (10 pts) Find the two possible values for Angle C.




```
7sin(27)
        3.177933498
```

② $\frac{h}{7} = \sin 27°$

$h = 7\sin 27° \approx 3.177933498$

$5 > h \Rightarrow$ it's long enough to reach

$5 < 7 \Rightarrow$ it's short enough for there to be 2 possibilities

① C acute

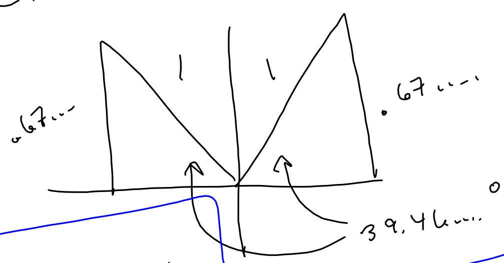
② C obtuse

$\frac{\sin C}{7} = \frac{\sin A}{5}$

$\sin C = \frac{7\sin A}{5} \approx .6355866996$

```
7sin(27)
        3.177933498
7sin(27)/5
        .6355866996
sin⁻¹(Ans
        39.46351248
```

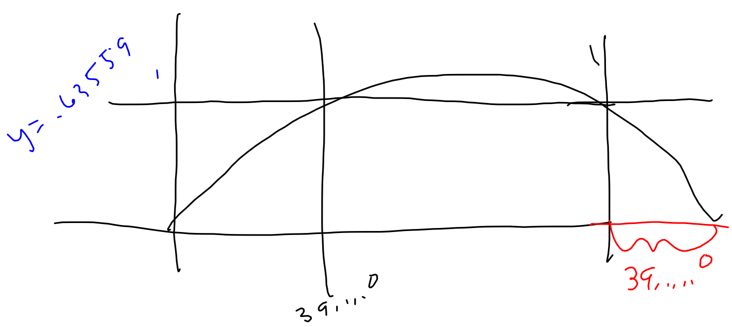
$\Rightarrow C \approx \sin^{-1}(.67\ldots) \approx 39.46351248°$



$\sin C = .67\ldots$, pics

Acute $C \approx 39.4635°$

OR $C = 180° - 39.46\ldots° \approx 140.5365° \approx C$ OBTUSE




```
        3.177933498
7sin(27)/5
        .6355866996
sin⁻¹(Ans
        39.46351248
Ans-180
        -140.5364875
```

6. (10 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = -\frac{7}{11}$ and $\sin(u) > 0$.

Preview

1. Let $f(x)=x^3-7x^2+25x-39$
   a. (10 pts) Use synthetic division to find $f(-3)$.
   b. (10 pts) Use synthetic division to show that $x=2+3i$ is a solution of the equation $f(x)=0$.
   c. (10 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.
2. Let $z=-3+3\sqrt{3}\,i$
   a. (10 pts) Find $z+\bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.
   b. (10 pts) Express z in trigonometric form.