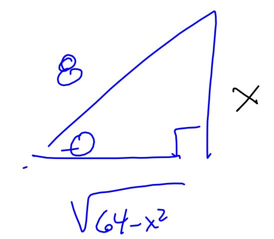
$\sqrt{64-x^2}$



$$\frac{x}{8} = \sin\theta$$

$$x = 8\sin\theta$$

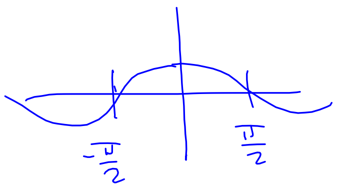
$$\Rightarrow \sqrt{64-x^2} = \sqrt{64-64\sin^2\theta}$$

$$= 8\sqrt{1-\sin^2\theta} = 8\sqrt{\cos^2\theta}$$

$$= 8|\cos\theta|$$

NOW, look at Domain:

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \cos\theta \geq 0$$



$$\Rightarrow 8|\cos\theta| = \boxed{8\cos\theta.}$$

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

$$\tan^2\theta + 1 = \left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{1}\cdot\frac{\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

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§2.2 Trig Identities

$9x + 72 = 9(x+8)$ So this is an IDENTITY.

$\Rightarrow 9x + 72 = 9x + 72$

$9x = 9x$

$0 = 0$ is true, regardless of the value of x.

§2.2 #35 (OLD BOOK)

$$\frac{\tan x \cot x}{\cos x} = \boxed{\sec x}$$

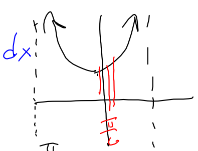
$$\frac{\tan x \cot x}{\cos x} = \frac{1}{\cos x} = \sec x \qquad \frac{\sin x}{\cos x}\cdot\frac{\cos x}{\sin x}$$

$\tan x = \dfrac{1}{\cot x}$

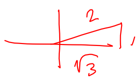
Find the area under the curve from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{4}$

$\underbrace{\cot 30° = 1/\tan(30°)}_{\text{Calculator for cotangent.}}$

$$\int_{\pi/6}^{\pi/4} \frac{\tan x \cot x}{\cos x}\,dx$$

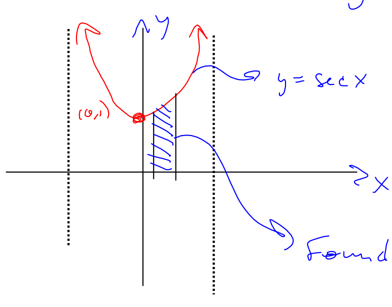


$$= \int_{\pi/6}^{\pi/4} \sec x\,dx = \ln|\sec x + \tan x|\Big]_{\pi/6}^{\pi/4} = \boxed{\ln|\sqrt{2}+1| - \ln\left|\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right|}$$



$$\boxed{\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)\sec x} = \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} = \frac{\frac{d}{dx}[\sec x + \cot x]}{\sec x + \tan x}$$

$= \dfrac{du}{u}$ & $\displaystyle\int \frac{du}{u}$ is known to be $\ln|u| + C$



This is maybe the last cofunction identity you'll see:

Show that

$\tan\left(\frac{\pi}{2}-\theta\right)\tan\theta = 1$

$\Rightarrow \cot\theta\tan\theta = 1$

$\sin^{-\frac{1}{2}}x\cos x - \sin^{\frac{5}{2}}x\cos x = \cos^3 x\sqrt{\sin x}$ sweet!

$\sin^{-\frac{1}{2}}x\cos x - \sin^{\frac{5}{2}}x\cos x$

$= \sin^{-\frac{1}{2}}x\cos x\left[1 - \sin^2 x\right]$

$\dfrac{\sin^{\frac{5}{2}}x\,\cancel{\cos x}}{\sin^{-\frac{1}{2}}x\,\cancel{\cos x}} = \sin^2 x$!

$= \sin^{-\frac{1}{2}}x\cos x\left[\cos^2 x\right] = \cos^3 x\sqrt{\sin x}$

$5x+10 = 5\left(\boxed{\frac{5x}{5}+\frac{10}{5}}\right)$

$= 5(x+2)$

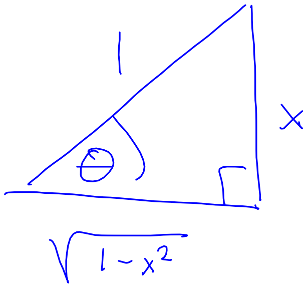
$\dfrac{a^{\frac{5}{2}}}{a^{\frac{1}{2}}} = a^{\frac{5}{2}-\frac{1}{2}} = a^{\frac{4}{2}} = a^2$

legerdemain

Show

$$\tan\theta = \underbrace{\tan(\sin^{-1}x)} = \frac{x}{\sqrt{1-x^2}}$$

Write this as an algebraic expression.

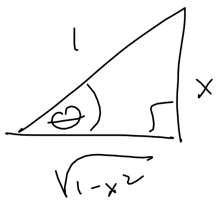


$$\Rightarrow \tan\theta = \frac{x}{\sqrt{1-x^2}}$$

Writeup style.

Show that:

$$\tan(\sin^{-1}x) = \tan\theta = \frac{x}{\sqrt{1-x^2}}$$



$$\Rightarrow \tan\theta = \frac{x}{\sqrt{1-x^2}} \quad \blacksquare$$

There are some questions that ask you to graph both sides of the equation to convince yourself that the statements are actually true, visually.

Do as many or as few as you like, as required to build your intuition (or your faith that this stuff actually works).

$\sin^2 25^\circ + \sin^2 65^\circ = 1$

$= \sin^2(90^\circ - 65^\circ) + \sin^2 65^\circ$

$= \cos^2 65^\circ + \sin^2 65^\circ = 1$

Cofunction questions are so CONTRIVED.

At most, a 5-point bonus on a test.

§2.3: I sneaked a lot of this in, already.

Trig Equations.

$\boxed{E}$ $4\cos^2 x - 1 = 0$

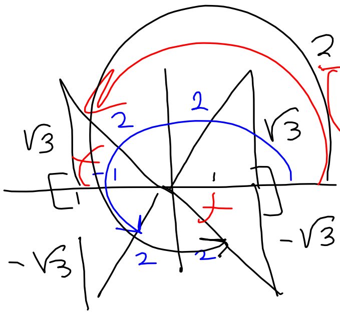
2 versions: for $\theta \in [0, 2\pi]$:

M1

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$



$\theta \in \{60^\circ, 120^\circ, 240^\circ, 300^\circ\}$

OR

$\theta = 60^\circ, 120^\circ, 240^\circ, 360^\circ$

$= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

M2

$4\cos^2 x - 1 = (2\cos x - 1)(2\cos x + 1) = 0$

$a^2 - b^2 = (a-b)(a+b) \Rightarrow 2\cos x - 1 = 0$ OR $2\cos x + 1 = 0$

etc.

M3

$4\cos^2 x + 0\cos x - 1 = 0$

$a = 4, b = 0, c = -1$

$b^2 - 4ac = 0^2 - 4(4)(-1) = 16$

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{16}}{2(4)} = \frac{\pm 4}{8} = \pm \frac{1}{2} = \cos x$, etc.

$\boxed{E}$ $4\cos^2 x - 1 = 0$

Trig Equations,

2 versions: ① for $\theta \in [0, 2\pi]$:

$\theta \in \{60^\circ, 120^\circ, 240^\circ, 300^\circ\}$

OR

$\theta = 60^\circ, 120^\circ, 240^\circ, 360^\circ$

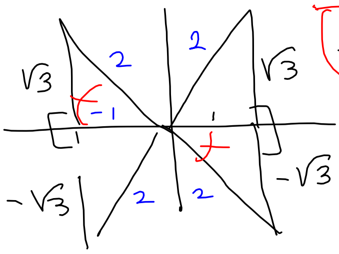
$= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

M1

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$



Version 2: FIND ALL θ.

$\theta \in \{60^\circ + 360^\circ n, 120^\circ + 360^\circ n, 240^\circ + 360^\circ n, 300^\circ + 360^\circ n \mid n \in \mathbb{Z}\}$

$= \{60^\circ + 180^\circ n, 120^\circ + 180^\circ n \mid n \in \mathbb{Z}\}$

OR, in Radians

$\theta \in \{\frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n \mid n \in \mathbb{Z}\}$

OR

$\theta \in \{\frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n \mid n \in \mathbb{Z}$

$\sin^{\frac{1}{2}} x \cos x \quad \{1 -$