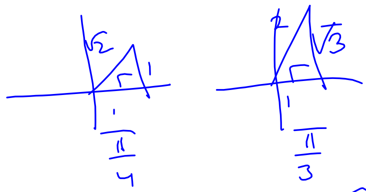
$\cos\left(\frac{7\pi}{12}\right)$ in 2 ways

Today's discussion from Test 2, Spring, 2018

$$\frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$$



$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2} - \sqrt{3}\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{2\cdot 2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

Write much. Think little.

$$\cos(2u) = 2\cos^2 u - 1 \implies$$

$$\frac{\cos(2u)+1}{2} = \cos^2(u)$$

$$\cos(u) = \pm\sqrt{\frac{1+\cos(2u)}{2}}$$

with the $\pm$ depending on what quadrant you're in.

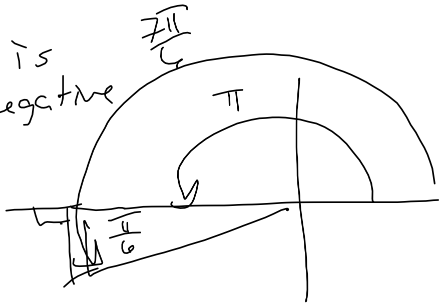
$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\cos\left(\frac{7\pi}{12}\right) \implies \frac{u}{2} = \frac{7\pi}{12} \implies u = \frac{7\pi}{6}$$

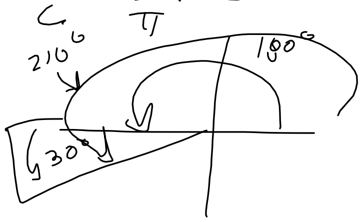
$$\cos\left(\frac{7\pi}{12}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{7\pi}{6}\right)}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \ast$$

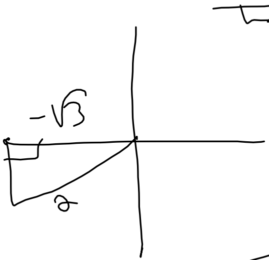
$$\left(\frac{7\pi}{12}\right)\left(\frac{180}{\pi}\right) = \frac{7\cdot 30}{2} = 7\cdot 15$$
$$= 105^\circ$$

cosine is negative



$$\frac{7\pi}{6}\cdot\frac{180}{\pi} = 7\cdot 30 = 210^\circ$$





$$* = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

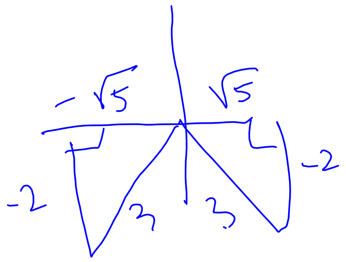
$$= -\sqrt{\left(\frac{2-\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)} = -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}} = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2}}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4} \quad ?!$$

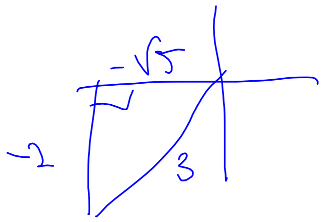
what?!

Yes, if you had the faith of a mustard seed, you could move mountains!

$\sin(u) = -\frac{2}{3}$, $\tan(u) > 0$ Find $\sin(2u)$, $\cos(2u)$, $\tan(2u)$



& $\tan(u) > 0$



$9 - 4 = 5$

$$\sin(2u) = 2\sin u \cos u = 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = \boxed{\frac{4\sqrt{5}}{9} = \sin(2u)}$$

$$\cos(2u) = 2\cos^2(u) - 1 = 2\left(-\frac{\sqrt{5}}{3}\right)^2 - 1 = 2 \cdot \frac{5}{9} - 1$$

$$= \frac{10}{9} - \frac{9}{9} = \boxed{\frac{1}{9} = \cos(2u)}$$

$\tan(2u) =$

$$\frac{\sin(2u)}{\cos(2u)} = \frac{\frac{4\sqrt{5}}{9}}{\frac{1}{9}} = \frac{4\sqrt{5}}{9} \cdot \frac{9}{1} = \boxed{4\sqrt{5}}$$

$$\cos(\arctan(x) - \arcsin(x))$$

$$= \cos(u-v) = \cos(u + (-v))$$

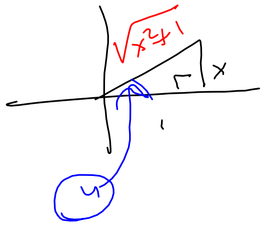
$$= \cos(u)\cos(-v) - \sin(u)\sin(-v)$$

$$= \cos(u)\cos(v) + \sin(u)\sin(v)$$

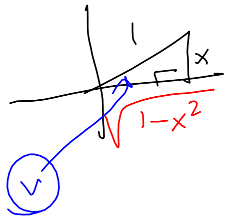
$$= \left(\frac{1}{\sqrt{x^2+1}}\right)\left(\frac{\sqrt{1-x^2}}{1}\right) + \left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{x}{1}\right)$$

$$= \boxed{\frac{\sqrt{1-x^2} + x^2}{\sqrt{x^2+1}}}$$

$\arctan(x) = u$



$\arcsin(x) = v$



Not simplified radical form, but the question said don't sweat that additional piece, and that's... OK.

$$3\tan^3 x - 3\tan^2 x - \tan x + 1 = 0$$

$$\Rightarrow 3u^3 - 3u^2 - u + 1$$

$$= 3u^2(u-1) - 1(u-1)$$

$$= (u-1)[3u^2 - 1] = 0$$

1

$u - 1 = 0$

$u = 1$

$3u^2 - 1 = 0$

$3u^2 = 1$

$u^2 = \frac{1}{3}$

$u = \pm \frac{1}{\sqrt{3}}$

Square Root Property

$u = \pm \frac{1}{\sqrt{3}} = \tan(x)$

quadratic eq'n

$a = 3,\ b = 0,\ c = -1$

$au^2 + bu + c = 0$

$b^2 - 4ac = 0^2 - 4(3)(-1) = 12$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{12}}{2(3)}$$

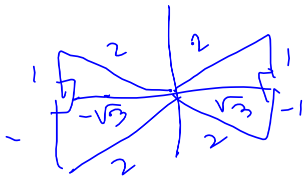
$$= \frac{\pm\sqrt{12}}{6} = \frac{\pm 2\sqrt{3}}{6}$$

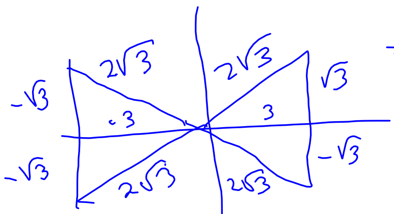
$$= \pm \frac{\sqrt{3}}{3}$$

$2\,|\,12$, $2\,|\,6$, 3

$\arctan\left(\frac{\sqrt{3}}{3}\right) = 30^\circ$

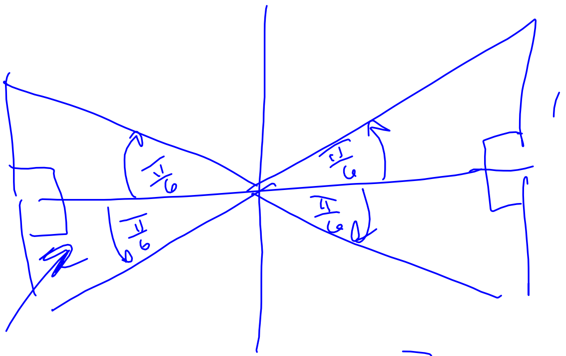
$= \frac{\pi}{6}$





$$\sqrt{\sqrt{3}^2 + 3^2} = \sqrt{3+9}$$

$$= \sqrt{12} = 2\sqrt{3}$$



$\frac{\pi}{6},\ \pi - \frac{\pi}{6},\ \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6} \leadsto \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

$30^\circ,\ 180^\circ + 30^\circ,\ 180^\circ - 30^\circ,\ 360^\circ - 30^\circ \leadsto \{30^\circ, 150^\circ, 210^\circ, 330^\circ\}$