What to do if there isn't a nice assignment sheet?

Work the exercises in the notes that accompany the videos!

§2.1 Neat Stuff Potpourri.

Identities

Trig Polynomials

Odd & Even

$\boxed{\text{Trig Substitution}}$

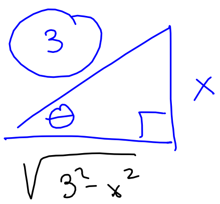
$$\int \sqrt{9-x^2}\,dx$$

Rewrite $\sqrt{9-x^2}$ as a trig expression.

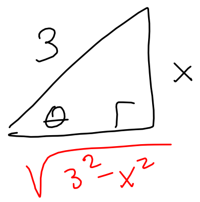
$= \sqrt{3^2-x^2}$

Book GIVES:

use $x = 3\sin\theta$




$\frac{x}{3} = \sin\theta$




$a^2+b^2=c^2$

$c^2-a^2=b^2$

$x = 3\sin\theta$

$\sqrt{3^2-(3\sin\theta)^2}$

$= \sqrt{9-9\sin^2\theta}$

$\sqrt{9(1-\sin^2\theta)}$

$3\sqrt{\cos^2\theta} = 3|\cos\theta|$

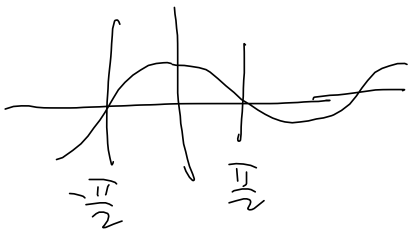
$\sqrt{x^2} = |x|$

$x=3 \quad \sqrt{3^2} = 3$

$x=-3 \quad \sqrt{(-3)^2} = 3$

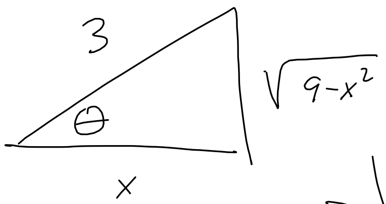
Problem specifies $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

THAT makes $|\cos\theta| = \cos\theta$




$\cos\theta > 0$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Book should be ASHAMED for telling you what substitution to make. You should draw the picture and figure it out. For this one there are two different possibilities, depending on how you drew the triangle:




$\frac{x}{3} = \cos\theta$

$x = 3\cos\theta$

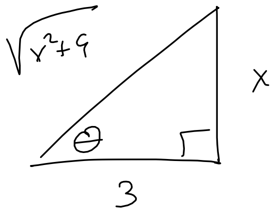
$\sqrt{9 - 9\cos^2\theta}$

$= 3\sqrt{\sin^2\theta} = 3|\sin\theta|$

Now, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ doesn't get rid of the absolute value.

In calculus, you might prefer one picture over the other, depending on the domain (where $\theta$ lives).

Same for $\sqrt{x^2+9}$




$\frac{x}{3} = \tan\theta$

$x = 3\tan\theta$

$$\sqrt{x^2+9}$$
$$= \sqrt{9\tan^2\theta + 9}$$
$$= 3\sqrt{\tan^2\theta + 1}$$
$$= 3\sqrt{\sec^2\theta}$$
$$= 3|\sec\theta|$$

and getting rid of absolute value depends on the domain

Book: $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

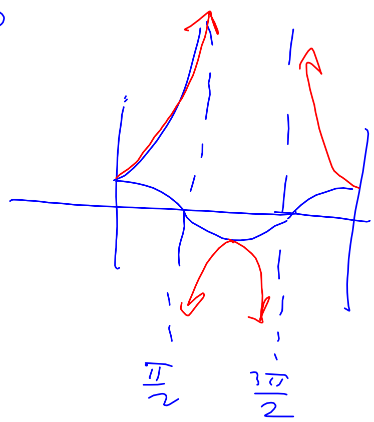
$\cos\theta = \frac{1}{\sec\theta} > 0$, so

$3\sec\theta$.

what if $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ ?

Then $3|\sec\theta| =$

$= -3\sec\theta$



A SLEW of identities.

Cofunction Identies

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$, etc.

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

These like never come up. Good cheat-sheet material, but don't get used very often.

Pytagorean Identities

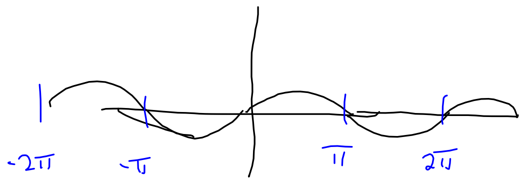
$\sin^2\theta + \cos^2\theta = 1 \longrightarrow 1 - \sin^2\theta = \cos^2\theta$

$\tan^2\theta + 1 = \sec^2\theta \qquad \cos^2\theta - 1 = -\sin^2\theta$

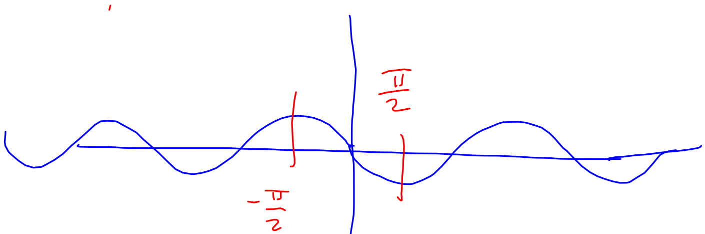
$\cot^2\theta + 1 = \csc^2\theta$

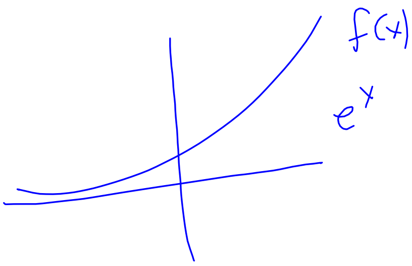
$\sin\left(\frac{\pi}{2}-\theta\right) = \sin\left(-\left(\theta - \frac{\pi}{2}\right)\right)$

$\sin(\theta)$




$\sin(-\theta)$ Flip Horizontal

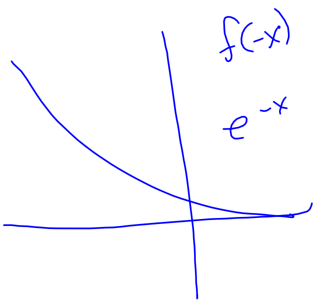


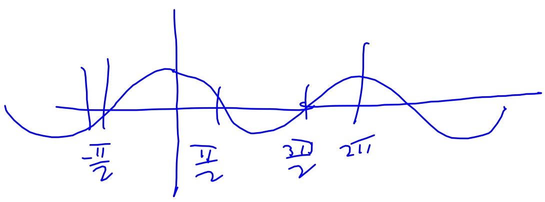

sine is odd, so $\sin(-\theta)$

$= -\sin\theta$ is also a

reflection about the x-axis




$\sin\left(-\left(\theta - \frac{\pi}{2}\right)\right) = \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

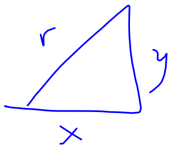


Simplifying

$$\tan\theta = \frac{y}{x} = \frac{y \cdot \frac{1}{r}}{x \cdot \frac{1}{r}} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{\sin\theta}{\cos\theta}$$

$\cot\theta \sec\theta$ !

$$= \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} = \frac{1}{\sin\theta} = \csc\theta$$

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$$\frac{\sin^2 x}{1-\cos x} = \left(\frac{\sin^2 x}{1-\cos x}\right)\left(\frac{1+\cos x}{1+\cos x}\right) = \frac{(\sin^2 x)(1+\cos x)}{(1-\cos^2 x)}$$

$$= \frac{(\sin^2 x)(1+\cos x)}{\sin^2 x} = 1+\cos x \ !$$

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# Trig Polynomials

$\cot^3 x + \cot^2 x + \cot x + 1$ FACTOR IT

$u^3 + u^2 + u + 1$ cyclotomic

$= u^2(u+1) + 1(u+1) = (u^2+1)(u+1)$

$(\cot^2 x + 1)(\cot x + 1)$

$= (\csc^2 x)(\cot x + 1)$

$(u+i)(u-i) = u^2 + 1$

$\tan^2 x + 1 = \dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x} = \sec^2 x$

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$$\frac{5}{\tan x + \sec x} = \left(\frac{5}{\tan x + \sec x}\right)\left(\frac{\tan x - \sec x}{\tan x - \sec x}\right) = \frac{5(\tan x - \sec x)}{\tan^2 x - \sec^2 x}$$

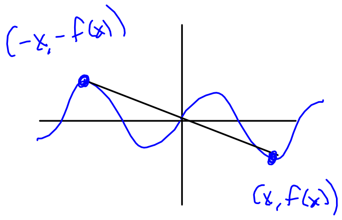
$$= \frac{5(\tan x - \sec x)}{\sec^2 x - 1 - \sec^2 x} = \frac{5(\tan x + \sec x)}{-1} = -5\tan x - 5\sec x$$

# Odd & Even Functions

$f(-x) = -f(x)$

ODD

Symmetric thru the origin



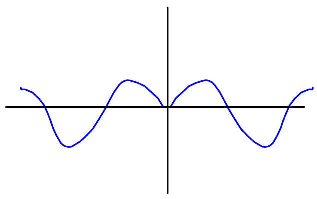

$\sin x,\ x,\ x^3,\ x^5,\ x^7, \ldots$

$\tan x,\ \cot x$

$f(-x) = f(x)$

EVEN

Symmetric about the y-axis.



$\cos x,\ 1,\ x^2,\ x^4,\ x^6, \ldots$

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\tan x$$

$$\frac{\sin x}{\cos x} = \frac{-}{+} = \frac{\text{ODD}}{\text{EVEN}} = - = \text{ODD}$$

ODD + EVEN = NEITHER

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$$\frac{\sin x \tan x}{(\cos x)(x^2+4)} = \frac{(-)(-)}{(+)(+)} = + \quad \text{is even}$$

ODD + ODD = ODD

EVEN + EVEN = EVEN

SOLVE $\forall x \in [0, 2\pi)$

$3\sin^2 x - 5\sin x - 2 = 0$

$= 3u^2 - 5u - 2$

$= 3u^2 - 6u + 1u - 2$

$= 3u(u-2) + 1(u-2)$

$= (u-2)(3u+1)$

$= (\sin x - 2)(3\sin x + 1)$

$\sin x = 2$ Nevah!

$3\sin x + 1 = 0$

$3\sin x = -1$

$\sin x = -\frac{1}{3}$

what if you suck at factoring? NONSENSE! EVERYONE CAN FACTOR!!!

$a = 3, b = -5, c = -2$

Discriminant $= D = b^2 - 4ac =$

$(-5)^2 - 4(3)(-2) = 25 + 24 = 49$

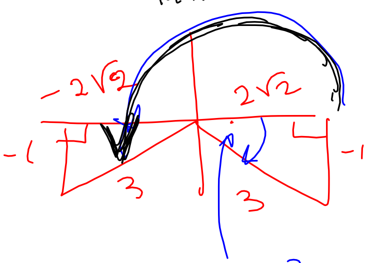
$x = \frac{5 \pm 7}{2(3)}$ $\nearrow \frac{12}{6} = 2$ $\searrow \frac{-2}{6} = -\frac{1}{3}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$3(x-2)(x+\frac{1}{3})$ FACTORED!

$= (x-2)(3x+1)$

199...$^\circ$




$-19...^\circ$

$180^\circ + 19...^\circ$

$\arcsin(-\frac{1}{3}) \approx$ -19.47122062

$\approx -19.47^\circ$

$\theta \approx$ +-199.4712206

$\theta \in \{-19.47^\circ, -199.47^\circ\}$

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$\left(x - \frac{5+\sqrt{97}}{24}\right)\left(x - \frac{5-\sqrt{97}}{24}\right)(12) = 12x^2 - 5x - 50$

EVERYTHING FACTORS

$34x^2 - 107x + 55$ $\qquad x = \frac{5}{2}, \frac{11}{17}$

$= 34\left(x - \frac{5}{2}\right)\left(x - \frac{11}{17}\right)$

$= 2\left(x - \frac{5}{2}\right)(17)\left(x - \frac{11}{17}\right)$

$= \boxed{(2x-5)(17x-11)}$

§2.1 Exercises in NOTES

Get ROLLING!

Take a bite outta §2.2.