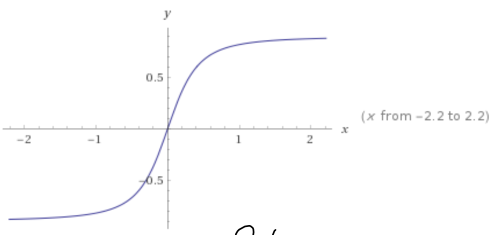
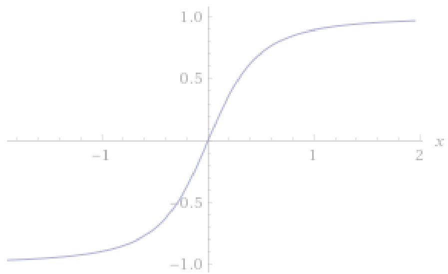
Graphical Analysis In Exercises 75 and 76, use a graphing utility to graph f and g in the same viewing window to verify that the two functions are equal. Explain why they are equal. Identify any asymptotes of the graphs.

#75 NA

75. $f(x) = \sin(\arctan 2x)$, $\quad g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

76. $f(x) = \tan\left(\arccos \dfrac{x}{2}\right)$, $\quad g(x) = \dfrac{\sqrt{4 - x^2}}{x}$



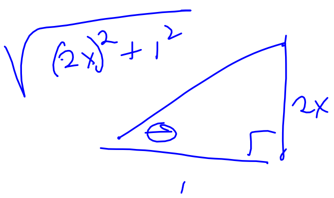


$g(x) = \dfrac{2x}{\sqrt{4x^2+1}}$

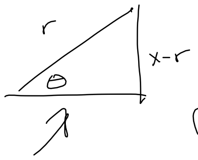
They're the same! See?

$\arctan(2x)$

$\sin(\arctan(2x)) = \sin\theta = \dfrac{2x}{\sqrt{4x^2+1}}$ See?



$$\cos\left(\arcsin\left(\frac{x-r}{r}\right)\right) = \cos\theta = \frac{\sqrt{2xr-x^2}}{r}$$



Pythagorus.

$$\sqrt{r^2-(x-r)^2} =$$

$$\sqrt{r^2-(x^2-2xr+r^2)}$$

$$= \sqrt{-x^2+2xr}$$

$$\left(x-\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right) = x^2-\frac{3}{4} = 0 \Rightarrow$$

$$4x^2-3=0$$

$$4\sin^2\theta-3=0$$

$$\left(x-\frac{1}{2}\right)\left(4x^2-3\right)$$

$$4x^3-3x-2x^2+\frac{3}{2}=0$$

$$\Rightarrow 8x^3-6x-4x^2+3=0$$

Put a $\sin\theta$ in for x.

$$\left(x-\frac{1}{\sqrt{2}}\right)\left(x+\frac{1}{\sqrt{2}}\right) = x^2-\frac{1}{2} = 0$$

$$\Rightarrow 2x^2-1=0$$

$$2\cos^2\theta-1=0$$

Now $4\sin^2\theta-3=0$

$$4u^2-3=0$$

$$(2u-\sqrt{3})(2u+\sqrt{3})=0$$

$$2u=\sqrt{3} \qquad 2u=-\sqrt{3}$$

$$u=\frac{\sqrt{3}}{2} \qquad u=-\frac{\sqrt{3}}{2}$$

$$4u^2=3$$

$$u^2=\frac{3}{4}$$

$$u=\pm\sqrt{\frac{3}{4}}=\pm\frac{\sqrt{3}}{2}$$

$$=4u^2+0u-3=0$$

$$a=4,\ b=0,\ c=-3$$

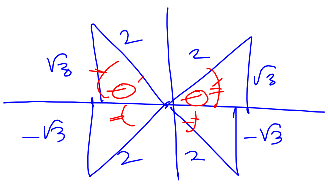
$$b^2-4ac = 0^2-4(4)(-3)$$

$$= 48$$

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array} \Rightarrow \sqrt{48}=4\sqrt{3}$$

$$u=\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-0\pm\sqrt{48}}{2(4)} = \frac{\pm4\sqrt{3}}{8} = \pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin\theta=\pm\frac{\sqrt{3}}{2}$$



$$\theta'=60^\circ$$

$$60^\circ = \frac{\pi}{3}$$

$$180^\circ-60^\circ = 120^\circ = \frac{2\pi}{3}$$

$$180^\circ+60^\circ = 240^\circ = \frac{4\pi}{3}$$

$$360^\circ-60^\circ = 300^\circ = \frac{5\pi}{3}$$

Find all solutions in

$[0, 2\pi) = [0°, 360°)$:

$\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\} = \{60°, 120°, 240°, 300°\}$

Find all sol'ns:

$\left\{ \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi \;\middle|\; n \in \mathbb{Z} \right\}$

$= \{ 60° + 360°n, 120° + 360°n, 240° + 360°n, 300° + 360°n \mid n \in \mathbb{Z} \}$

= Get clever =

$\left\{ \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi \;\middle|\; n \in \mathbb{Z} \right\}$

$= \{ 60° + 180°n, 120° + 180°n \mid n \in \mathbb{Z} \}$

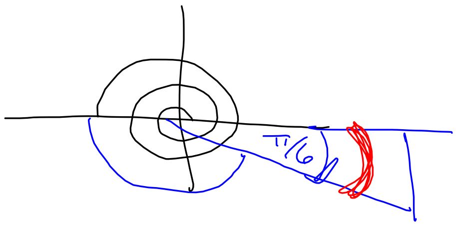
Graph sine and arcsine, together

cosine and arccosine

tangent and arctangent

1. (10 pts) Find two angles, between -2π and 2π (i.e., 0^0 and 360^0) that are coterminal with $\frac{35\pi}{6}$. Give exact answers in degrees and radians.

$$\frac{35\pi}{6} = \frac{30\pi}{6} + \frac{5\pi}{6} = 5\pi + \frac{5\pi}{6}$$



$$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$\pi + \frac{5\pi}{6} = \frac{11\pi}{6}$$ Duh.

Now for a Negative:

$$-\frac{\pi}{6}$$

Coterminals:

$$\boxed{\left\{ -\frac{\pi}{6}, \frac{11\pi}{6} \right\} \text{ Radians}}$$

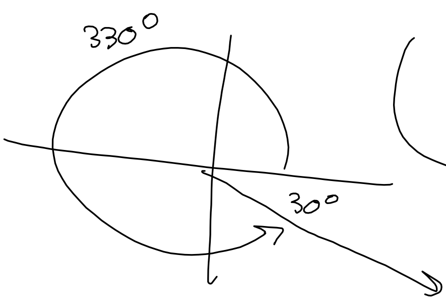
$$\left(\frac{35\pi}{6}\right)\left(\frac{180}{\pi}\right) = (35)(30^\circ) = 1050^0$$

360

720

1080^0

$$1050^0 - 720^0 = 330^0$$



$$\boxed{\{-30^0, 330^0\} \text{ Degrees}}$$

WORK BIG

WORK DARK

LEAVE PLENTY OF SPACE

SHOW THE FLOW OF THE PROCESS - THE NARRATIVE!!!!

WORK ONLY IN ONE COLUMN DON'T USE 2 COLUMNS OF PROBLEMS.

PUT YOUR DOGGONE NAME AT THE TOP, PRINTED.

I WILL PROVIDE PAPER.

DO NOT WORRY ABOUT THE "CONTEXT" LIKE IN HOMEWORK. YOU'RE PROVIDING IT WITH THE COVER SHEET AND YOU'RE UNDER A TIME CONTROL.

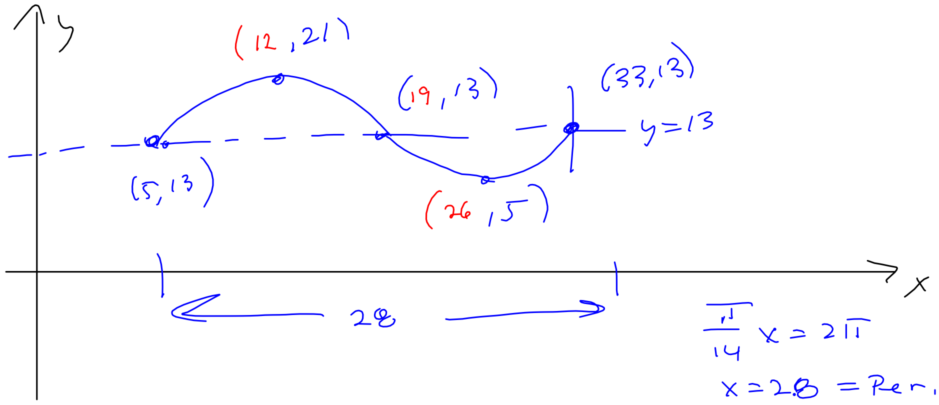
I'LL COME IN AND BE READY TO TEST AT 8 A.M. THAT GIVES YOU A FULL 1.5 HOURS.

6. (10 pts) Sketch the graph of $f(x) = 8\sin\left(\frac{\pi}{14}x - \frac{5\pi}{14}\right) + 13$. $= 8\sin\left(\frac{\pi}{14}(x-5)\right) + 13$

Amp

7. (10 pts) Write the cosine function that achieves its maximum height of $y = 7$ centimeters at time $t = 2$ seconds and its minimum height of $y = -5$ centimeters at $t = 30$ seconds.

A

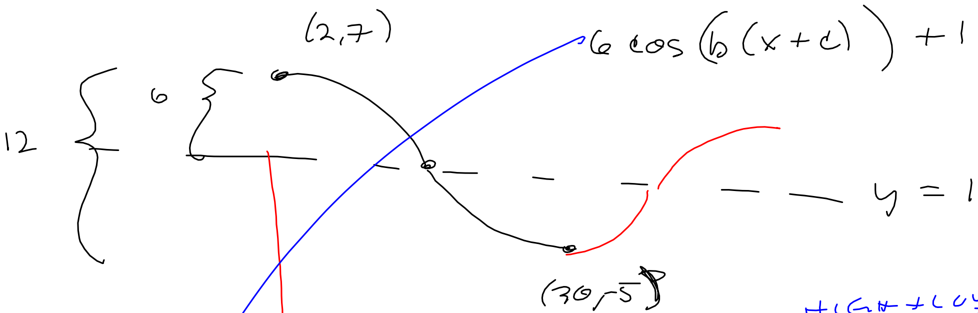


$\frac{\pi}{14}x = 2\pi$

$x = 28$ = Period

7. (10 pts) Write the cosine function that achieves its maximum height of $y = 7$ centimeters at time $t = 2$ seconds and its minimum height of $y = -5$ centimeters at $t = 30$ seconds.

A



$6\cos(b(x+c)) + 1$

$\frac{\text{HIGH} + \text{LOW}}{2} = \text{MID}$

$\frac{\text{HIGH} - \text{LOW}}{2} = \text{Amp.}$

$\frac{1}{2}$-Period is 28

$\Rightarrow T = 56$

$bx = 2\pi$ when $x = 56$

$56b = 2\pi$

$\boxed{a\cos(b(x-c)) + d}$

$b = \frac{2\pi}{56} = \frac{\pi}{28}$

$a = 6$

$b = \frac{\pi}{28}$, $c = 2$, $d = 1$

$\boxed{6\cos\left(\frac{\pi}{28}(x-2)\right) + 1}$