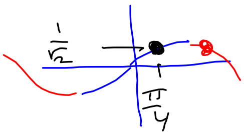
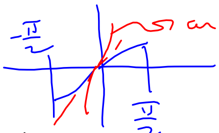
Vocabulary: Fill in the blanks.

Function	Alternative Notation	Domain How?	Range
1. $y = \arcsin(x)$	$\sin^{-1}(x)$	________	$\{x \mid -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$
2. ________	$y = \cos^{-1} x$	$-1 \le x \le 1$	________
3. $y = \arctan x$	________	________	________

4. Without restrictions, no trigonometric function has an __________ function.





$\arcsin(x)$

$\sin\left(\frac{2\pi}{3}\right)$ POIFECK!

Evaluating an Inverse Trigonometric Function

In Exercises 5–18, evaluate the expression without using a calculator. #11 NA DRAW PICS!

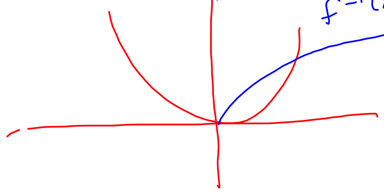
5. $\arcsin \frac{1}{2}$
6. $\arcsin 0$
7. $\arccos \frac{1}{2}$
8. $\arccos 0$
9. $\arctan \frac{\sqrt{3}}{3}$
11. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
14. $\arctan \sqrt{3}$
15. $\arccos\left(-\frac{1}{2}\right)$

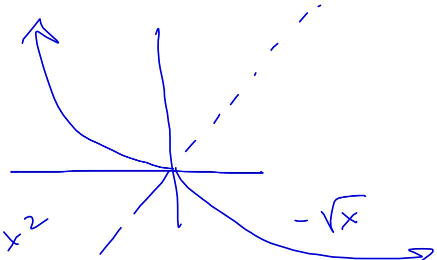
$\sin 2\pi/3$

Disambiguate.

$f(x) = x^2, x \ge 0$

$f^{-1}(x) = \sqrt{x}$





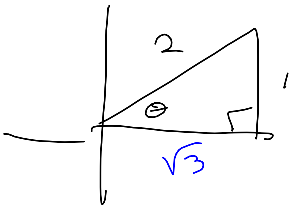
x^2

$x \le 0$

$-\sqrt{x}$

#5 $\arcsin\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}$

$\sin(x) = \frac{1}{2}$



$\arcsin(\sin(x)) = \arcsin\left(\frac{1}{2}\right)$

$\theta = 30^\circ$ or $\frac{\pi}{6}$

Calculator - Degrees mode.

$\sin^{-1}$ - KEY

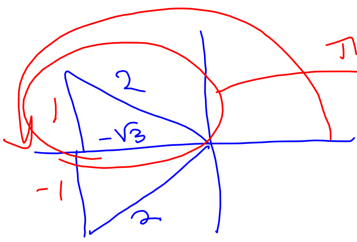
arcsine only sees angles between (& including) $\boxed{-\frac{\pi}{2} \text{ \& } \frac{\pi}{2}}$

$\frac{\pi}{6}$ = .5... ugh!

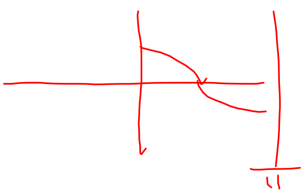
Degrees mode to SEE.

$\arccos\left(-\frac{\sqrt{3}}{2}\right) =$

2 pics for $\cos(\theta) = -\frac{\sqrt{3}}{2}$



This one, fool! Which one does your calculator see?



WORD PROBLEMS:

Write two of them up, nicely to turn in, but remember that this is the knowledge I want you to OWN.

S 1.8 has a ton of 'em.

Graphing an Inverse Trigonometric Function In Exercises 19 and 20, use a graphing utility to graph f, g, and $y = x$ in the same viewing window to verify geometrically that g is the inverse function of f. (Be sure to restrict the domain of f properly.)

19. $f(x) = \cos x, \quad g(x) = \arccos x$

20. $f(x) = \tan x, \quad g(x) = \arctan x$

We did this last Tuesday.

Calculators and Inverse Trigonometric Functions In Exercises 21–38, use a calculator to evaluate the expression. Round your result to two decimal places.

22. arcsin 0.65

24. arccos(−0.7)

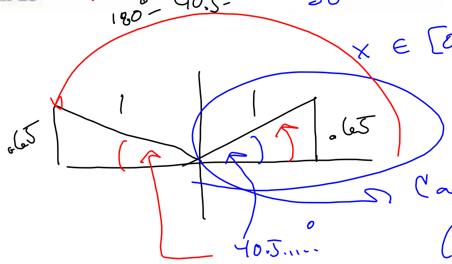
26. arctan 25

I want to show how arcsine is an assist, but you need the picture to find all solutions to

sin(x)=0.65

2.4... $180° - 40.5...°$

Solve $\sin(x) = .65$ Find all sol'ns $x \in [0, 2\pi)$



Calculator only "sees" this one (Restricted sine curve)

```
sin⁻¹(.65)
   40.54160187
Ans-180
  -139.4583981
```

so 139....°

positive, by pic.

$x \approx 40.54160187°, 139.4583981°$

$\Rightarrow \boxed{x \in \{40.54°, 139.46°\}}$

FIND ALL SOLUTIONS!

$x \in \{40.54° + 360°n \mid n \in \mathbb{Z}\} \cup \{139.46° + 360°n \mid n \in \mathbb{Z}\}$

$2.4... = \pi - \theta \Rightarrow$

$\theta = \pi - 2.4... \approx$

```
    40.54160187
Ans-180
   -139.4583981
Ans*π/180
   -2.434008217
Ans+180
    177.5659918
```

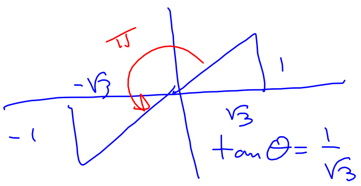
Idiot mixing degrees & radians, here.

```
   -2.434008217
Ans+180
    177.5659918
Ans-180
   -2.434008217
Ans+π
    .7075844367
```

THAT'S WHAT WE WANT!

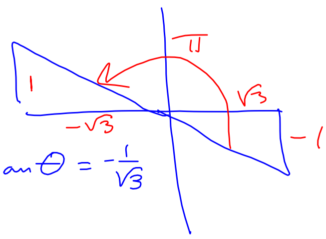
$\theta \in \{2.43 + 2n\pi \mid n \in \mathbb{Z}\} \cup \{.707 + 2n\pi \mid n \in \mathbb{Z}\}$

The "$n\pi$" thing is for THESE pictures



$\tan\theta = \frac{1}{\sqrt{3}}$

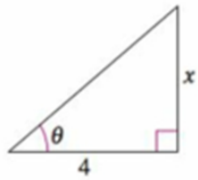
OR



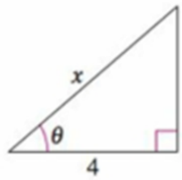
$\tan\theta = -\frac{1}{\sqrt{3}}$

Using an Inverse Trigonometric Function In Exercises 41–46, use an inverse trigonometric function to write θ as a function of x.

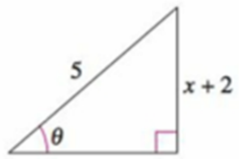
41.



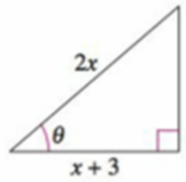
42.



43.



45.



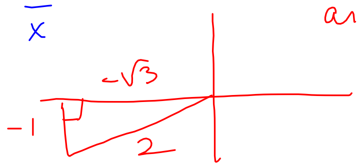
(41) $\tan\theta = \frac{x}{4} \Rightarrow$

$\arctan(\tan\theta) = \arctan\left(\frac{x}{4}\right)$

$\Rightarrow \theta = \arctan\left(\frac{x}{4}\right)$ is "generally" true, but not always. NOTE

(42) $\cos\theta = \frac{4}{x}$

$\arctan\left(\tan\left(\frac{7\pi}{6}\right)\right) = \arctan(\tan(210°))$



$= \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} = 30°$

So $\arctan\left(\tan\left(\frac{7\pi}{6}\right)\right) \neq \frac{7\pi}{6}$

i.e., $\arctan(\tan(\theta))$ isn't always θ!

It will have the same reference angle

(1st Quadrant is GOLD)

Using Inverse Properties In Exercises 47–52, use the properties of inverse trigonometric functions to evaluate the expression.

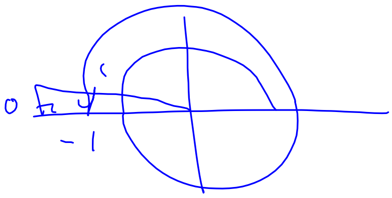
$\boxed{\#48 \text{ NA}}$

47. sin(arcsin 0.3) **48.** tan(arctan 45)

Using Inverse Properties In Exercises 47–52, use the properties of inverse trigonometric functions to evaluate the expression.

51. $\arcsin(\sin 3\pi) = \pi$ **52.** $\arccos\left(\cos \frac{7\pi}{2}\right)$

$= \arcsin(0) = 0$



Had cosine in my brain.

This was my deal:

$\arccos(\cos(3\pi)) = \arccos(-1)$

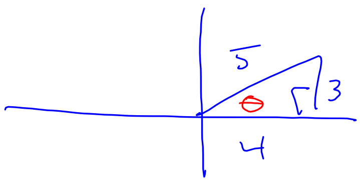
$= \pi$

Evaluating a Composition of Functions In Exercises 53–64, find the exact value of the expression. (*Hint:* Sketch a right triangle.) DRAW THE PICTURES!

53. $\sin\left(\arctan \frac{3}{4}\right)$ **54.** $\sec\left(\arcsin \frac{4}{5}\right)$ #56 NA

55. $\cos(\tan^{-1} 2)$ **56.** $\sin\left(\cos^{-1} \frac{\sqrt{5}}{5}\right)$ **58.** $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$

53 $\sin\left(\arctan\left(\frac{3}{4}\right)\right) = \sin(\theta) = \frac{3}{5}$



Writing an Expression In Exercises 65–74, write an algebraic expression that is equivalent to the given expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

66. $\sin(\arctan x)$ **68.** $\sec(\arctan 3x)$

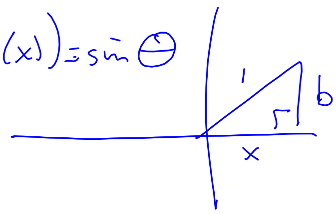
69. $\sin(\arccos x)$ **70.** $\sec[\arcsin(x-1)]$

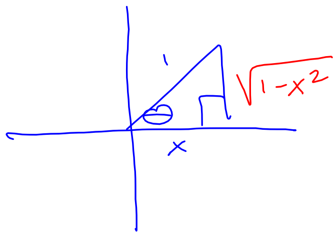
69 $\boxed{\sin}(\arccos(x)) = \sin\theta$

$\boxed{= \sqrt{1-x^2}}$

Transcendental!

Algebraic!





$x^2 + b^2 = 1^2$

$\sqrt{b^2} = \sqrt{1^2 - x^2}$

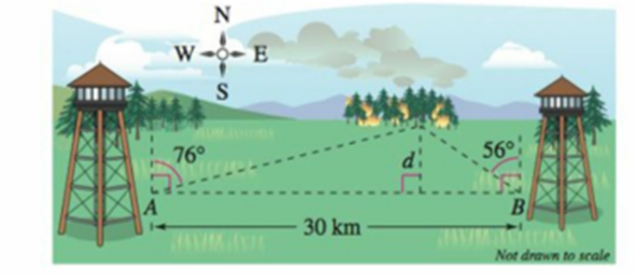
$|b| = \sqrt{1-x^2}$

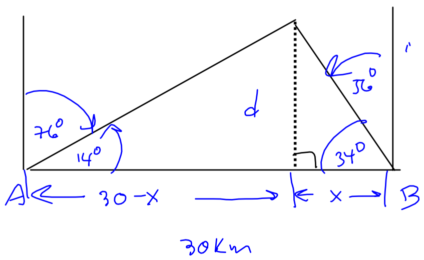
QI

$b = \sqrt{1-x^2}$

Section 1.8:

42. **Location of a Fire** Fire tower *A* is 30 kilometers due west of fire tower *B*. A fire is spotted from the towers, and the bearings from *A* and *B* are N 76° E and N 56° W, respectively (see figure). Find the distance *d* of the fire from the line segment *AB*.





$$\frac{d}{30-x} = \tan(14^\circ) \qquad \frac{d}{x} = \tan(34^\circ)$$

$$d = d$$

$$d = (30-x)\tan 14^\circ = x\tan(34^\circ)$$ Solve for x!

$$= -(x-30)a = x\,b$$

$$\Longrightarrow -ax + 30a = bx$$

$$-ax - bx = -30a$$

$$ax + bx = 30a$$

$$x(a+b) = 30a$$

$$x = \frac{30a}{a+b} = \frac{30\tan(14^\circ)}{\tan(14^\circ)+\tan(34^\circ)}$$

The rest be calculator work.

Get x, then $d = x\tan(34^\circ)$

& check with

$d = (3-x)\tan(14^\circ)$