Modular arithmetic

Integers mod 2 $\mathbb{Z}/2\mathbb{Z}$

$\bar{0} = \bar{2}, \bar{1}$

$\frac{27}{2} = 13 + \frac{1}{2}$

$\overline{27} = \bar{1}$

$2\overline{)27}$ 13 r 1

$\mathbb{Z}/3\mathbb{Z}$

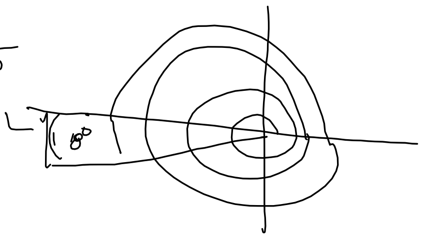
$\bar{0}, \bar{1}, \bar{2}$

$3\overline{)19}$ 6 r 1

$\overline{19} = \bar{1}$

1278° to find where it lives divide by 360° & take the remainder.

$\frac{1278}{360} = 3.55$




```
1278/360
             3.55
3*360
             1080
Ans-1278
             -198
```

$+198^\circ$

18° is the reference angle for this purposes.

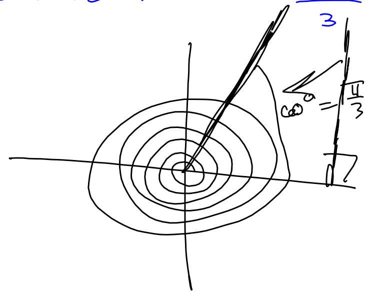
So, $1278^\circ = 198^\circ \mod 360^\circ$

$19 = 1 \mod 3$

Same with radians. It's probably easier to see if $\frac{37\pi}{3}$ you convert to degrees.

2 tricks!

$\frac{37\pi}{3}$



$60^\circ = \frac{\pi}{3}$

$\frac{36\pi}{3} + \frac{\pi}{3} = 12\pi + \frac{\pi}{3}$

6 times around the circle

$\frac{\pi}{3}$

$$\left(\frac{37\pi}{3}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{37 \cdot 180}{3} = 37 \cdot 60 = 2220^\circ$$

4 376

6

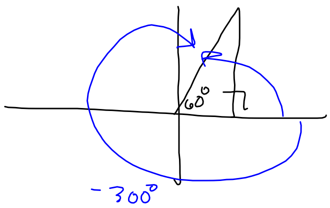
22 20

$\frac{2220^\circ}{360^\circ} = 6.1\bar{6}$ → 60°

6 times around

```
                 -198
222/36
          6.166666667
6*360
                 2160
Ans-2220
                  -60
```

Find 2 angles between -360 and 360 that are coterminal with 2220 degrees.



60° & -300°

-300°

Test question was $\frac{35\pi}{6}$ & $-2\pi < \theta < 2\pi$

Nearest multiple of 6 to 35:

$\frac{35\pi}{6} = \frac{30\pi}{6} + \frac{5\pi}{6} = 5\pi + \frac{5\pi}{6}$

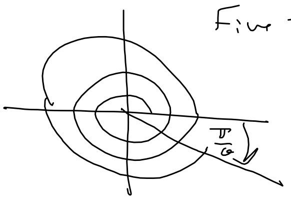
want an even multiple

$\frac{24\pi}{6} + \frac{11\pi}{6} = 4\pi + \frac{11\pi}{6}$

Twice around + this

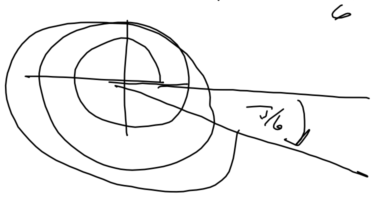
$5\pi + \frac{5\pi}{6}$

Five πs



$4\pi + \frac{11\pi}{6}$

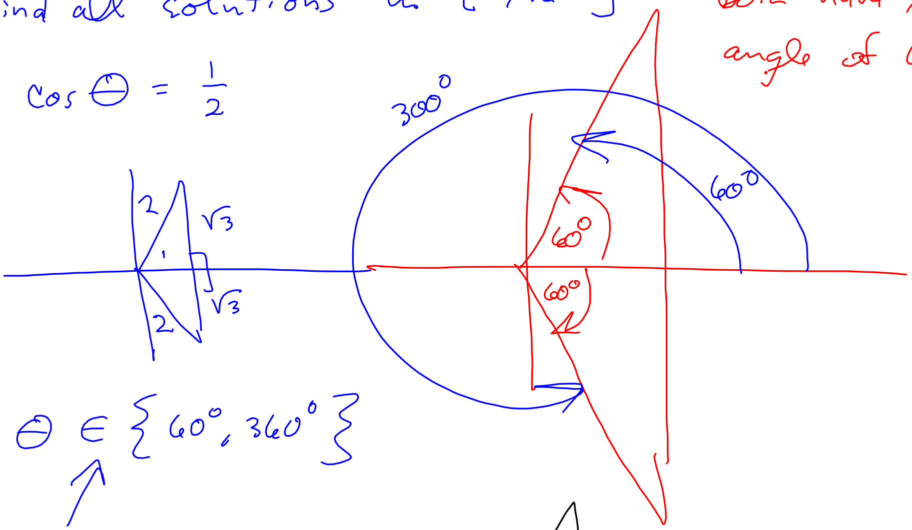
2 2πs plus $\frac{11\pi}{6}$



Find all solutions in $[0°, 360°]$

Both have reference angle of $60°$

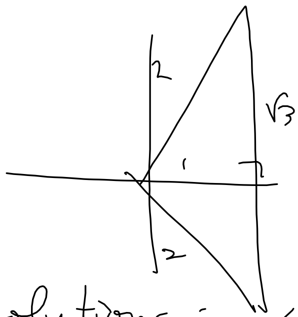
$\cos\theta = \frac{1}{2}$



$\theta \in \{60°, 360°\}$

is in

$\cos\theta = \frac{1}{2}$



$\frac{\pi}{3}$, $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\theta \in \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

Solve. Find all solutions in $(-2\pi, 2\pi)$

$4\sin^2\theta - 3 = 0$

$4u^2 - 3 = 0$

$\sin^2\theta = (\sin\theta)^2$

$\sin^3\theta = (\sin\theta)^3$

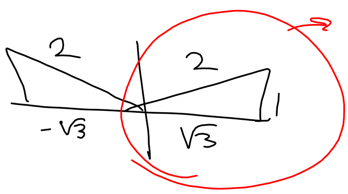
$\sin^{-1}\theta$ = Depends on context?! Dadgum mathematicians!

Depends whether it's arithmetic "-1" power, or "function inverse" as in

$\sin\theta = \frac{1}{2} \Rightarrow$

$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

$\sin^{-1}$ Key is, for this



calculator only sees this

Solve. Find all solutions in $(-2\pi, 2\pi)$

$4\sin^2\theta - 3 = 0$

$4u^2 - 3 = 0 \Longrightarrow (2u)^2 - (\sqrt{3})^2 = a^2 - b^2 = (a-b)(a+b)$

Twisted way:

$(2u - \sqrt{3})(2u + \sqrt{3}) = 0$

1 $4u^2 - 3 = 0$

$a = 4,\ b = 0,\ c = -3$

$\boxed{b^2 - 4ac} = 0^2 - 4(4)(-3) = 48$

$ax^2 + bx + c = 0$

DO DISCRIMINANT FIRST, SEPARATELY

$\Longrightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-0 \pm \sqrt{48}}{2(4)}$

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

$\sqrt{48} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$

$x = \dfrac{\pm 4\sqrt{3}}{8} = \boxed{\pm \dfrac{\sqrt{3}}{2} = u}$

Twisted way:

2 $(2u - \sqrt{3})(2u + \sqrt{3}) = 0$

$2u - \sqrt{3} = 0$

$2u = \sqrt{3}$

$\boxed{u = \dfrac{\sqrt{3}}{2} \qquad u = -\dfrac{\sqrt{3}}{2}}$

3 $4u^2 - 3 = 0$

$4u^2 = 3$

$u^2 = \dfrac{3}{4}$

$u = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$

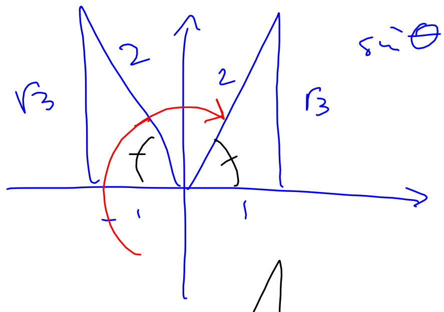
So, $u = \pm \frac{\sqrt{3}}{2} = \sin\theta$

$4(\sin\theta)^2 - 3 = 0$

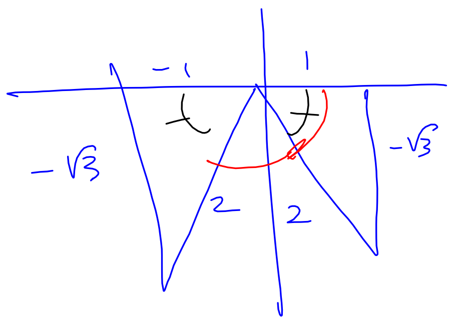
$4\sin^2\theta - 3 = 0$

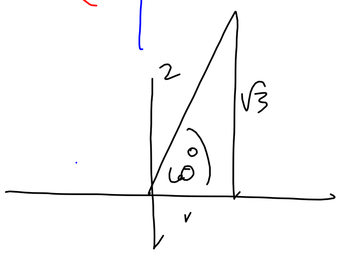
$\sin\theta = \pm \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$



$\sin\theta = -\frac{\sqrt{3}}{2}$





$\theta = 60°, 120°, 240°, 300°$

But... But... teacher wanted radians, (such a jerk) $\theta \in (-2\pi, 2\pi)$

I'll finish in degrees & do radians @ the end.

$\theta \in (-360°, 360°)$

$-60°, -120°, -240°, -300°$

Radians: $\theta \in \left\{ -\frac{5\pi}{3}, -\frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

$= \left\{ \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}, \pm\frac{5\pi}{3} \right\}$ is more efficient & still complete.

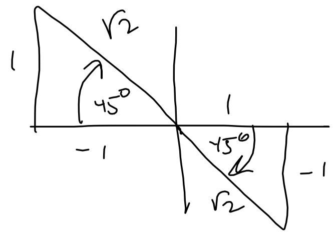
Jan 29-9:01 AM

$\cot \theta = -1$

$\tan \theta = \frac{1}{-1} = -1$

§1.4 #93b Want $\theta \in [0°, 360°)$

or $[0, 2\pi)$



$\Rightarrow \theta = 180° - 45° = 135° = \frac{3\pi}{4} = \theta$

or $360° - 45° = 315° = \frac{7\pi}{4} = \theta$

$\theta \in \left\{ \frac{3\pi}{4}, \frac{7\pi}{4} \right\}$

~~assuming~~ or

$\theta \in \{135°, 315°\}$

§1.4

#23 STANDARD TEST QUESTION

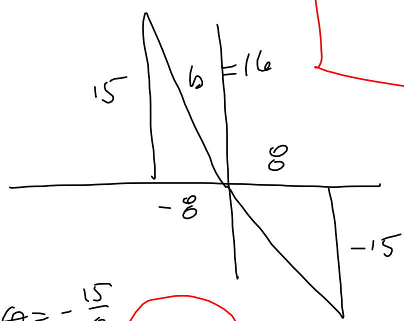
Find the 6 trigs, given $\tan \theta = -\frac{15}{8}$ & $\sin \theta > 0$

$\tan \theta = -\frac{15}{8}$

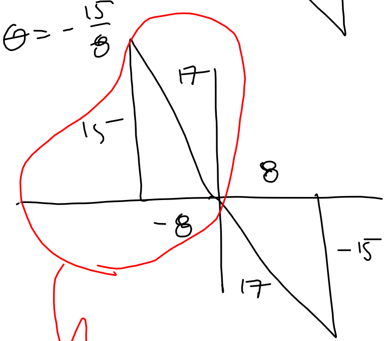
$15^2 - 8^2 = 225 - 64 = 161$

$\Rightarrow \sqrt{161} = b$

Did Pythagorus Wrong. Idiot.



$\tan \theta = -\frac{15}{8}$



$\sin \theta > 0$

$8^2 + 15^2 = 64 + 225 = 289 = 17^2$

$\sin \theta = \frac{15}{17}$ $\csc \theta = \frac{17}{15}$

$\cos \theta = -\frac{8}{17}$ $\sec \theta = -\frac{17}{8}$

$\tan \theta = -\frac{15}{8}$ $\cot \theta = -\frac{8}{15}$