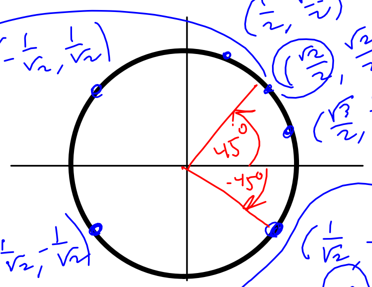
56. Using the Unit Circle Use the unit circle to verify that the cosine and secant functions are even and that the sine, cosecant, tangent, and cotangent functions are odd.



$\cos(45^\circ)$

$\sin\frac{\pi}{4}$

$\cos(-45^\circ) =$

cosine is even:

$\cos\left(\frac{\pi}{4}\right) = \cos(45^\circ) = \frac{1}{\sqrt{2}}$

$\cos\left(-\frac{\pi}{4}\right) = \cos(-45^\circ)$

$\sec\theta = \frac{1}{\cos\theta}$

$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)}$

$= \sec(\theta)$

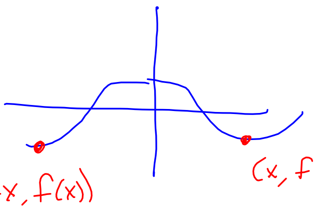
is definition of even function!

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

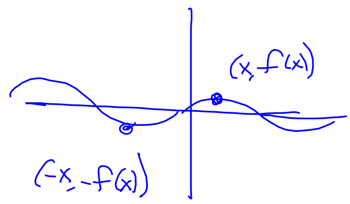
$\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

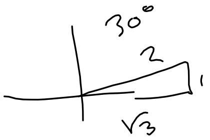
In general,

EVEN: $f(-x) = f(x)$

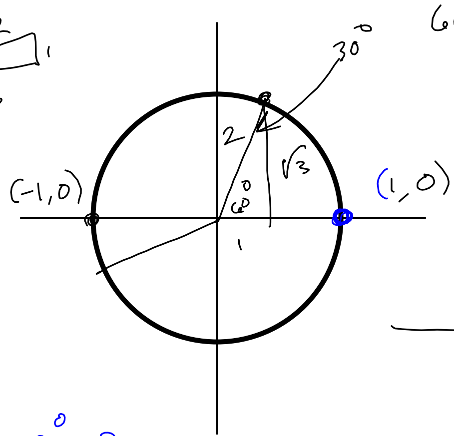


ODD: $f(-x) = -f(x)$





$60^\circ = \frac{\pi}{3}$



$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

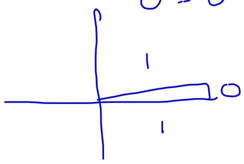
$\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}$

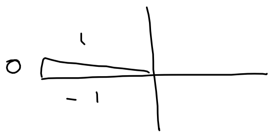
$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$0^\circ = 0$

sohcahtoa



$\nexists$



$\cos(0) = 1 = \frac{1}{1} \Rightarrow \sec(0) = 1$

$\sin(0) = 0 = \frac{0}{1} \Rightarrow \csc(0) = \frac{1}{0}\ \nexists$

$\tan(0) = \frac{0}{1} = 0 \Rightarrow \cot(0)\ \nexists$ OR DNE

$\cos\pi = \cos 180^\circ = \frac{-1}{1} = -1$

$\sin\pi = \frac{0}{1} = 0$

$\tan\pi = \frac{0}{-1} = 0$

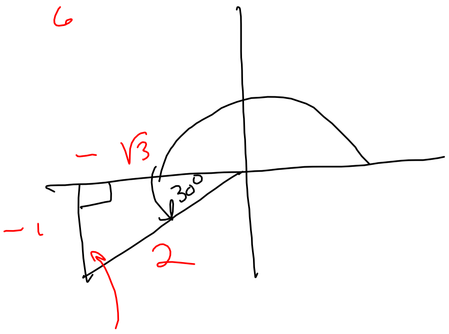
<u>WebAssign</u> Like MyMathLab.

Interested?

$(210^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{6}$

$210^\circ = \frac{7\pi}{6}$

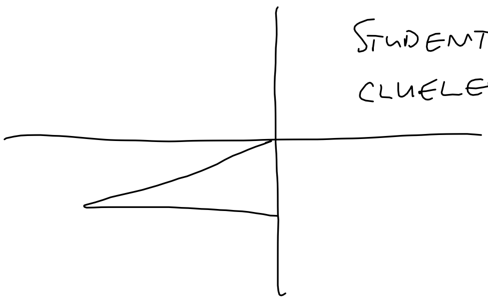
$210^\circ - 180 = 30^\circ$

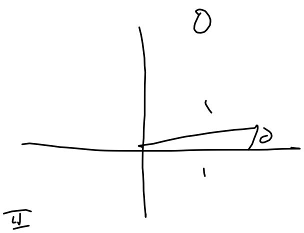


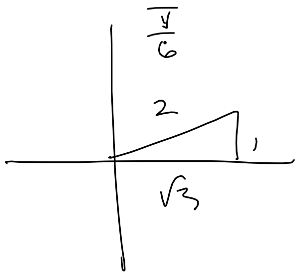
30° is the reference angle, formed by dropping a perpendicular to the x-axis

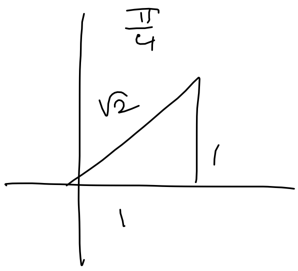
$\sin 210^\circ = -\frac{1}{2}$	$\csc 210^\circ = -2$
$\cos 210^\circ = -\frac{\sqrt{3}}{2}$	$\sec 210^\circ = -\frac{2}{\sqrt{3}}$
$\tan 210^\circ = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$	$\cot 210^\circ = \sqrt{3}$

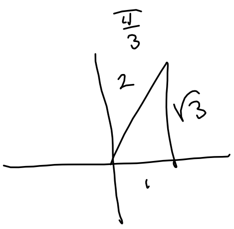
STUDENT'S CLUELESS.

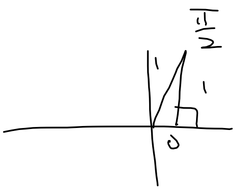












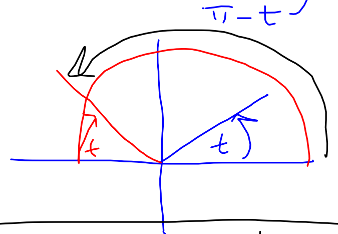
55. Conjecture Let (x_1, y_1) and (x_2, y_2) be points on the unit circle corresponding to $t = t_1$ and $t = \pi - t_1$, respectively.

(a) Identify the symmetry of the points (x_1, y_1) and (x_2, y_2).

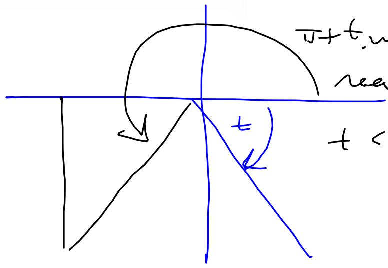
(b) Make a conjecture about any relationship between $\sin t_1$ and $\sin(\pi - t_1)$.

(c) Make a conjecture about any relationship between $\cos t_1$ and $\cos(\pi - t_1)$.

Assume t is an angle.



$\pi - t$ looks like $\pi + t$, until you realize that $t < 0$.



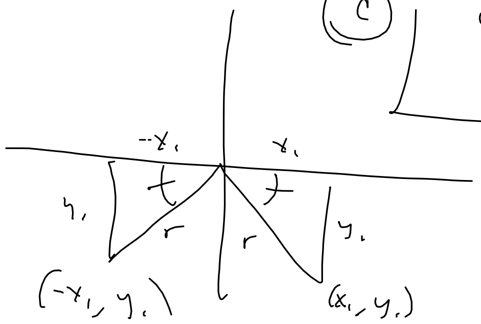
$t = -60^\circ$ (clockwise)

$\pi - t = 180^\circ - (-60^\circ) = 180^\circ + 60^\circ = 240^\circ$

Mirror image across the y-axis.

If (x_1, y_1) corresponds to t, then $(-x_1, y_1)$ " " $\pi - t$.

(c) Conjecture: $\cos(\pi - t) = -\cos t$

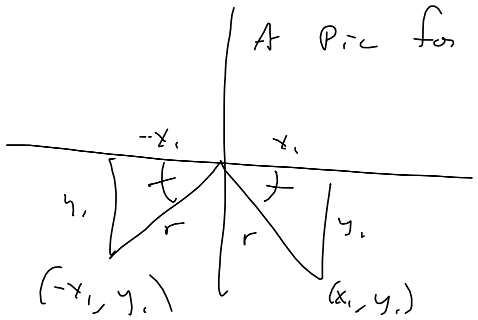


(b) Same for sine: conjecture. $\sin(\pi - t) = \sin(t)$

(a) Conjecture (x_1, y_1) corresponds to $t \to (-x_1, y_1)$ " " $\pi - t$ on the unit circle.

(55) Context..

A Pic for t, $\pi - t$



(a) Conjecture: (x_1, y_1) corresponds to $t \Rightarrow$ $(-x_1, y_1)$ " " $\pi - t$ on the unit circle.

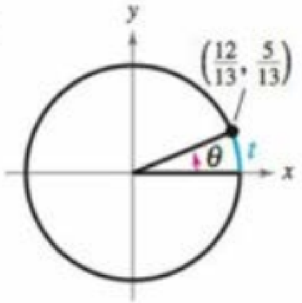
(b) Same for sine? conjecture. $\boxed{\sin(\pi - t) = \sin(t)}$

(c) Conjecture: $\boxed{\cos(\pi - t) = -\cos t}$

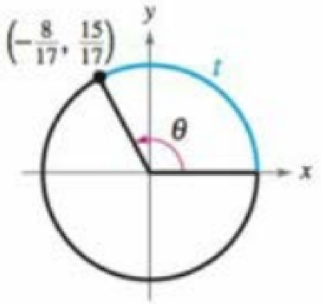
Determining Values of Trigonometric Functions

In Exercises 5–8, determine the exact values of the six trigonometric functions of the real number t.

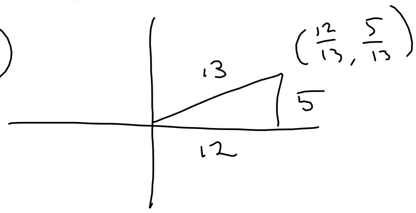
5.

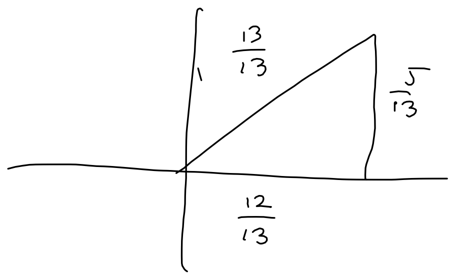


6.



5





$\sin\theta = \frac{5}{13}$ $\quad$ $\csc\theta = \frac{13}{5}$

$\cos\theta = \frac{12}{13}$ $\quad$ $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\quad$ $\cot\theta$ $\frac{12}{5}$

$\cot = \frac{12}{5}$

without its argument,

$\sin\theta = \frac{\pi}{13}$

sine is just a sin.

57. Verifying Expressions Are Not Equal Verify that $\cos 2t \neq 2\cos t$ by approximating $\cos 1.5$ and $2\cos 0.75$.

Let $t = 0.75$. We show $\cos(2t) \neq 2\cos(t)$

Proof:

$2\cos(.75) \approx 1.463377738$

$\cos(2t) \approx .0707372017$

Not even CLOSE to the same!

```
cos(.75)
       .7316888689
cos(1.5
       .0707372017
2cos(.75
       1.463377738
```

Just takes one counterexample to prove a conjecture is false.

Proving a conjecture to be true is much more difficult.

No lines

One side only — Every Page

Margin —— Write 122 in top left corner

Print Name in top RIGHT Corner (Front Page).

Context: Work stands on its own.

I don't have to open book to know what's asked.

Determining Values of Trigonometric Functions

In Exercises 5–8, determine the exact values of the six trigonometric functions of the real number *t*.

#'s 5-8 Find exact values of trig. func. of $t \in \mathbb{R}$

Jan 22-9:27 AM