In Exercises 1 and 2, fill in the blanks.

1. An equation that is true for all real values in its domain is called an ________.
2. An equation that is true for only some values in its domain is called a ________ ________.

In Exercises 3–8, fill in the blank to complete the fundamental trigonometric identity.

3. $\dfrac{1}{\cot u} =$ ________ **4.** $\dfrac{\cos u}{\sin u} =$ ________ **5.** $\sin^2 u +$ ________ $= 1$

6. $\cos\left(\dfrac{\pi}{2} - u\right) =$ ________ **7.** $\csc(-u) =$ ________ **8.** $\sec(-u) =$ ________

Skills and Applications

Verifying a Trigonometric Identity In Exercises 9–50, verify the identity.

9. $\tan t \cot t = 1$ **10.** $\sec y \cos y = 1$

11. $\cot^2 y(\sec^2 y - 1) = 1$

12. $\cos x + \sin x \tan x = \sec x$

13. $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$

14. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$

15. $\cos^2 \beta - \sin^2 \beta = 1 - 2\sin^2 \beta$

16. $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$

17. $\dfrac{\tan^2 \theta}{\sec \theta} = \sin \theta \tan \theta$ **18.** $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$

19. $\dfrac{\cot^2 t}{\csc t} = \dfrac{1 - \sin^2 t}{\sin t}$ **20.** $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$

33. $\tan\left(\dfrac{\pi}{2} - \theta\right) \tan \theta = 1$ **34.** $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan$

35. $\dfrac{\tan x \cot x}{\cos x} = \sec x$ **36.** $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$

37. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$

38. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$

39. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

40. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$

41. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$

Section 2.2

19. $\dfrac{\cot^2 t}{\csc t} = \dfrac{1 - \sin^2 t}{\sin t}$

20. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$

21. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x \sqrt{\sin x}$

22. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$

23. $\dfrac{\cot x}{\sec x} = \csc x - \sin x$

24. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$

25. $\sec x - \cos x = \sin x \tan x$

26. $\sec x(\csc x - 2 \sin x) = \cot x - \tan x$

27. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$

28. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$

29. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$

30. $\dfrac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$

31. $\dfrac{1}{\cos x + 1} + \dfrac{1}{\cos x - 1} = -2 \csc x \cot x$

32. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$

41. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$

42. $\sqrt{\dfrac{1 - \cos \theta}{1 + \cos \theta}} = \dfrac{1 - \cos \theta}{|\sin \theta|}$

43. $\cos^2 \beta + \cos^2\left(\dfrac{\pi}{2} - \beta\right) = 1$

44. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = 1$

45. $\sin t \csc\left(\dfrac{\pi}{2} - t\right) = \tan t$

46. $\sec^2\left(\dfrac{\pi}{2} - x\right) - 1 = \cot^2 x$

47. $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1 - x^2}}$

48. $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$

49. $\tan\left(\sin^{-1} \dfrac{x - 1}{4}\right) = \dfrac{x - 1}{\sqrt{16 - (x - 1)^2}}$

50. $\tan\left(\cos^{-1} \dfrac{x + 1}{2}\right) = \dfrac{\sqrt{4 - (x + 1)^2}}{x + 1}$

Error Analysis **In Exercises 51 and 52, describe the error(s).**

51. $(1 + \tan x)[1 + \cot(-x)]$
$= (1 + \tan x)(1 + \cot x)$
$= 1 + \cot x + \tan x + \tan x \cot x$
$= 1 + \cot x + \tan x + 1$
$= 2 + \cot x + \tan x$

52. $\dfrac{1 + \sec(-\theta)}{\sin(-\theta) + \tan(-\theta)} = \dfrac{1 - \sec\theta}{\sin\theta - \tan\theta}$
$= \dfrac{1 - \sec\theta}{(\sin\theta)[1 - (1/\cos\theta)]}$
$= \dfrac{1 - \sec\theta}{\sin\theta(1 - \sec\theta)}$
$= \dfrac{1}{\sin\theta} = \csc\theta$

Determining Trigonometric Identities **In Exercises 53–58, (a) use a graphing utility to graph each side of the equation to determine whether the equation is an identity, (b) use the *table* feature of the graphing utility to determine whether the equation is an identity, and (c) confirm the results of parts (a) and (b) algebraically.**

53. $(1 + \cot^2 x)(\cos^2 x) = \cot^2 x$

54. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$

66. Shadow Length

The length s of a shadow cast by a vertical gnomon (a device used to tell time) of height h when the angle of the sun above the horizon is θ can be modeled by the equation

$$s = \frac{h \sin(90° - \theta)}{\sin\theta}.$$

(a) Verify that the expression for s is equal to $h \cot\theta$.

(b) Use a graphing utility to complete the table. Let $h = 5$ feet.

θ	15°	30°	45°	60°	75°	90°
s						

(c) Use your table from part (b) to determine the angles of the sun that result in the maximum and minimum lengths of the shadow.

(d) Based on your results from part (c), what time of day do you think it is when the angle of the sun above the horizon is 90°?

54. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$

55. $2 + \cos^2 x - 3\cos^4 x = \sin^2 x(3 + 2\cos^2 x)$

56. $\tan^4 x + \tan^2 x - 3 = \sec^2 x(4\tan^2 x - 3)$

57. $\dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x}{1 - \cos x}$

58. $\dfrac{\cot \alpha}{\csc \alpha + 1} = \dfrac{\csc \alpha + 1}{\cot \alpha}$

Verifying a Trigonometric Identity **In Exercises 59–62, verify the identity.**

59. $\tan^5 x = \tan^3 x \sec^2 x - \tan^3 x$

60. $\sec^4 x \tan^2 x = (\tan^2 x + \tan^4 x)\sec^2 x$

61. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$

62. $\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$

Using Cofunction Identities **In Exercises 63 and 64, use the cofunction identities to evaluate the expression without using a calculator.**

63. $\sin^2 25° + \sin^2 65°$

64. $\tan^2 63° + \cot^2 16° - \sec^2 74° - \csc^2 27°$

65. **Rate of Change** The rate of change of the function $f(x) = \sin x + \csc x$ with respect to change in the variable x is given by the expression $\cos x - \csc x \cot x$. Show that the expression for the rate of change can also be written as $-\cos x \cot^2 x$.

Exploration

True or False? **In Exercises 67–69, determine whether the statement is true or false. Justify your answer.**

67. There can be more than one way to verify a trigonometric identity.

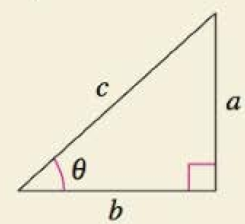
68. The equation $\sin^2 \theta + \cos^2 \theta = 1 + \tan^2 \theta$ is an identity because $\sin^2(0) + \cos^2(0) = 1$ and $1 + \tan^2(0) = 1$.

69. $\sin x^2 = \sin^2 x$

70. **HOW DO YOU SEE IT?** Explain how to use the figure to derive the identity

$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$$

given in Example 1.



Think About It **In Exercises 71–74, explain why the equation is not an identity and find one value of the variable for which the equation is not true.**

71. $\sin \theta = \sqrt{1 - \cos^2 \theta}$

72. $\tan \theta = \sqrt{\sec^2 \theta - 1}$

73. $1 - \cos \theta = \sin \theta$

74. $1 + \tan \theta = \sec \theta$