1. $\dfrac{\sin u}{\cos u} =$ ________ 2. $\dfrac{1}{\csc u} =$ ________ 3. $\dfrac{1}{\tan u} =$ ________

4. $\sec\left(\dfrac{\pi}{2} - u\right) =$ ________ 5. $1 +$ ________ $= \csc^2 u$ 6. $\cot(-u) =$ ________

Skills and Applications

Using Identities to Evaluate a Function **In Exercises 7–14, use the given values to find the values (if possible) of all six trigonometric functions.**

7. $\sin x = \dfrac{1}{2}, \ \cos x = \dfrac{\sqrt{3}}{2}$

8. $\csc \theta = \dfrac{25}{7}, \ \tan \theta = \dfrac{7}{24}$

9. $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{3}{5}, \quad \cos x = \dfrac{4}{5}$

10. $\sin(-x) = -\dfrac{1}{3}, \quad \tan x = -\dfrac{\sqrt{2}}{4}$

11. $\sec x = 4, \quad \sin x > 0$

12. $\csc \theta = -5, \quad \cos \theta < 0$

13. $\sin \theta = -1, \quad \cot \theta = 0$

14. $\tan \theta$ is undefined, $\quad \sin \theta > 0$

Factoring a Trigonometric Expression **In Exercises 29–32, factor the trigonometric expression. There is more than one correct form of each answer.**

29. $3 \sin^2 x - 5 \sin x - 2$ 30. $6 \cos^2 x + 5 \cos x - 6$

31. $\cot^2 x + \csc x - 1$ 32. $\sin^2 x + 3 \cos x + 3$

Multiplying Trigonometric Expressions **In Exercises 33 and 34, perform the multiplication and use the fundamental identities to simplify. There is more than one correct form of each answer.**

33. $(\sin x + \cos x)^2$ 34. $(2 \csc x + 2)(2 \csc x - 2)$

Simplifying a Trigonometric Expression **In Exercises 35–44, use the fundamental identities to simplify the expression. There is more than one correct form of each answer.**

35. $\cot \theta \sec \theta$ 36. $\tan(-x) \cos x$

Factoring a Trigonometric Expression In Exercises 21–28, factor the expression and use the fundamental identities to simplify. There is more than one correct form of each answer.

21. $\tan^2 x - \tan^2 x \sin^2 x$

22. $\sin^2 x \sec^2 x - \sin^2 x$

23. $\dfrac{\sec^2 x - 1}{\sec x - 1}$

24. $\dfrac{\cos x - 2}{\cos^2 x - 4}$

25. $1 - 2\cos^2 x + \cos^4 x$

26. $\sec^4 x - \tan^4 x$

27. $\cot^3 x + \cot^2 x + \cot x + 1$

28. $\sec^3 x - \sec^2 x - \sec x + 1$

more than one correct form of each answer.

45. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$

46. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$

47. $\tan x - \dfrac{\sec^2 x}{\tan x}$

48. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$

Rewriting a Trigonometric Expression In Exercises 49 and 50, rewrite the expression so that it is not in fractional form. There is more than one correct form of each answer.

49. $\dfrac{\sin^2 y}{1 - \cos y}$

50. $\dfrac{5}{\tan x + \sec x}$

Trigonometric Functions and Expressions In Exercises 51 and 52, use a graphing utility to determine which of the six trigonometric functions is equal to the expression. Verify your answer algebraically.

51. $\cos x \cot x + \sin x$

52. $\dfrac{1}{\sin x}\left(\dfrac{1}{\cos x} - \cos x\right)$

Trigonometric Substitution In Exercises 53–56, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

53. $\sqrt{9 - x^2}, \quad x = 3 \cos \theta$

54. $\sqrt{49 - x^2}, \quad x = 7 \sin \theta$

55. $\sqrt{x^2 - 4}, \quad x = 2 \sec \theta$

56. $\sqrt{9x^2 + 25}, \quad 3x = 5 \tan \theta$

Trigonometric Substitution In Exercises 57 and 58, use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$. Then find $\sin \theta$ and $\cos \theta$.

57. $3 = \sqrt{9 - x^2}, \quad x = 3 \sin \theta$

58. $-5\sqrt{3} = \sqrt{100 - x^2}, \quad x = 10 \cos \theta$

62. Rate of Change The rate of change of the function $f(x) = \sec x + \cos x$ is given by the expression $\sec x \tan x - \sin x$. Show that this expression can also be written as $\sin x \tan^2 x$.

Exploration

True or False? In Exercises 63 and 64, determine whether the statement is true or false. Justify your answer.

63. The even and odd trigonometric identities are helpful for determining whether the value of a trigonometric function is positive or negative.

64. A cofunction identity can transform a tangent function into a cosecant function.

Finding Limits of Trigonometric Functions In Exercises 65 and 66, fill in the blanks.

65. As $x \to \left(\dfrac{\pi}{2}\right)^-$, $\tan x \to$ ☐ and $\cot x \to$ ☐.

66. As $x \to \pi^+$, $\sin x \to$ ☐ and $\csc x \to$ ☐.

Determining Identities In Exercises 67 and 68, determine whether the equation is an identity, and give a reason for your answer.

67. $\dfrac{(\sin k\theta)}{(\cos k\theta)} = \tan \theta, \quad k$ is a constant.

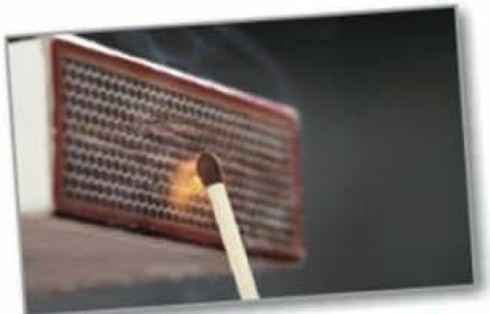
58. $-5\sqrt{3} = \sqrt{100 - x^2}, \quad x = 10\cos\theta$

Solving a Trigonometric Equation **In Exercises 59 and 60, use a graphing utility to solve the equation for θ, where $0 \le \theta < 2\pi$.**

59. $\sin\theta = \sqrt{1 - \cos^2\theta}$

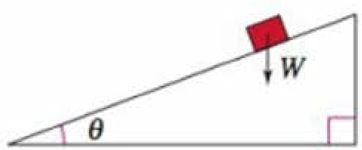
60. $\sec\theta = \sqrt{1 + \tan^2\theta}$

61. **Friction** The forces acting on an object weighing W units on an inclined plane positioned at an angle of θ with the horizontal (see figure) are modeled by



$$\mu W\cos\theta = W\sin\theta$$

where μ is the coefficient of friction. Solve the equation for μ and simplify the result.



67. $\dfrac{(\sin k\theta)}{(\cos k\theta)} = \tan\theta, \quad k$ is a constant.

68. $\sin\theta\csc\theta = 1$

69. **Trigonometric Substitution** Use the trigonometric substitution $u = a\tan\theta$, where $-\pi/2 < \theta < \pi/2$ and $a > 0$, to simplify the expression $\sqrt{a^2 + u^2}$.

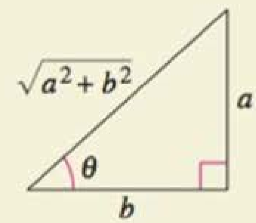
70. **HOW DO YOU SEE IT?**

Explain how to use the figure to derive the Pythagorean identities

$\sin^2\theta + \cos^2\theta = 1,$

$1 + \tan^2\theta = \sec^2\theta,$

and $\quad 1 + \cot^2\theta = \csc^2\theta.$



Discuss how to remember these identities and other fundamental trigonometric identities.

71. **Writing Trigonometric Functions in Terms of Sine** Write each of the other trigonometric functions of θ in terms of $\sin\theta$.

72. **Rewriting a Trigonometric Expression** Rewrite the following expression in terms of $\sin\theta$ and $\cos\theta$.

$$\frac{\sec\theta(1 + \tan\theta)}{\sec\theta + \csc\theta}$$