122 §1.6 Solutions #s 1-11, 13-16, 20-27, 29-37

33 + 7 context = 40 poss.

(1) Tangent, cotangent and cosecant are ODD so their graphs are symmetric w.r.t. the origin. 1pt

(2) Tangent, cotangent and secant and cosecant all have vertical asymptotes 1pt

(3) To sketch secant or cosecant, first sketch its reciprocal function (cosine or sine, resp.) 1pt

(4) $f(x) = g(x)\sin(x) \Rightarrow g(x)$ is a DAMPING function 1pt

(16) The domain of $\cot(x)$ is $\{x \mid x \neq n\pi, n \in \mathbb{Z}\}$ 1pt

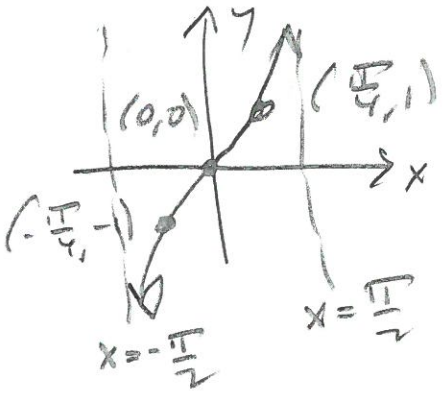
$= \mathbb{R} \setminus \{n\pi \mid n \in \mathbb{Z}\}$

(7) The range of $\sec(x)$ is $(-\infty, -1] \cup [1, \infty)$ 1pt

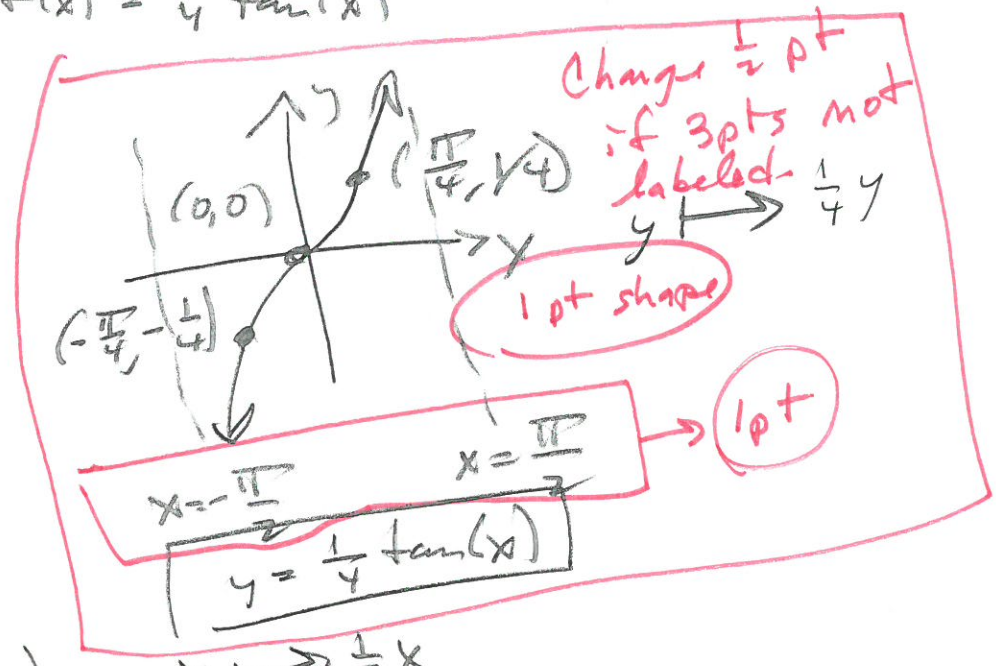
(8) The period of $\csc(x)$ is 2π 1pt

~~(9) Sketch~~

(9) Sketch $y = f(x) = \frac{1}{4}\tan(x)$

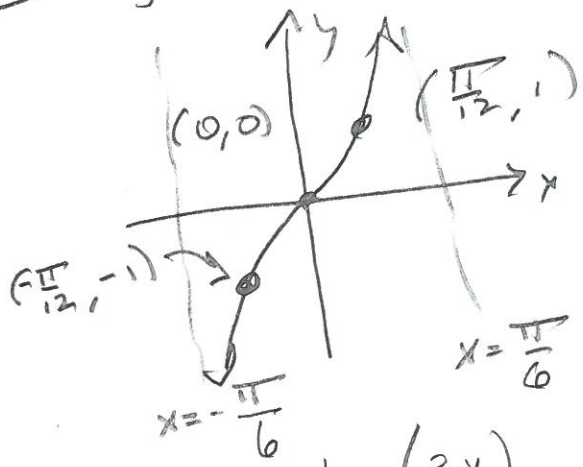


$\tan(x)$



$x \mapsto \frac{1}{3}x$

(10) $y = \tan(3x)$



$y = \tan(3x)$

$x \mapsto \frac{1}{3}x$

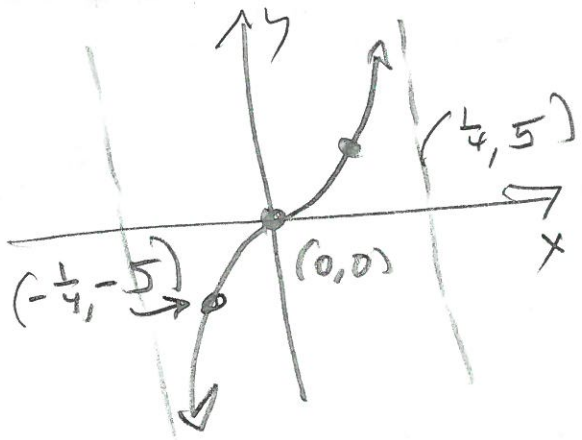
2 pts

Same criteria

122 §1.6

(12) Sketch $y = -5\tan(\pi x)$

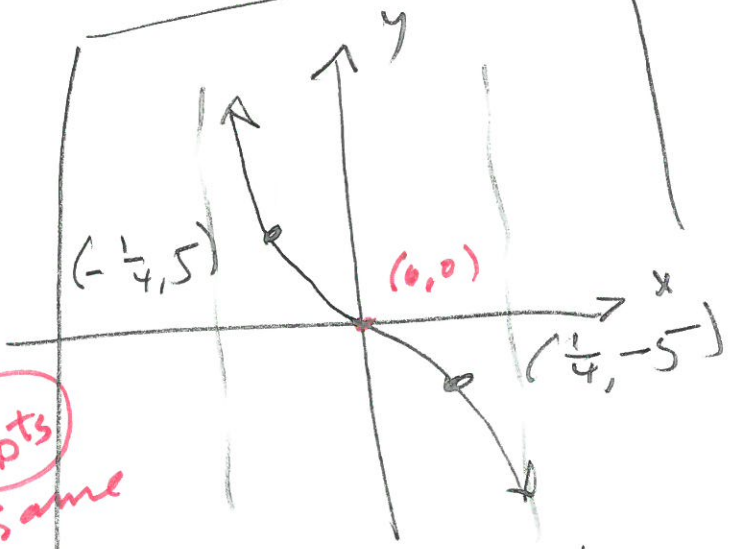
$y \mapsto -5y$ $\quad x \mapsto \frac{1}{\pi}x$



$y = 5\tan(\pi x)$

Forgot the "−"!

$\frac{\pi}{4} \div \pi = \frac{1}{4}$



2 pts same

$x = -\frac{1}{2}$ $\quad x = \frac{1}{2}$

$y = -5\tan(\pi x)$

(13) Sketch $y = 5\csc(6x)$

$5\sin(6x)$?

$x \mapsto \frac{1}{6}x$

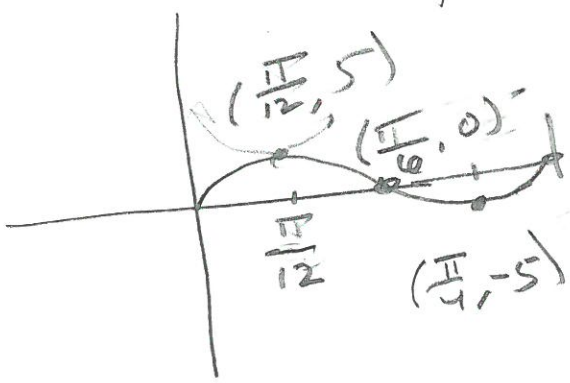
$y \mapsto 5y$

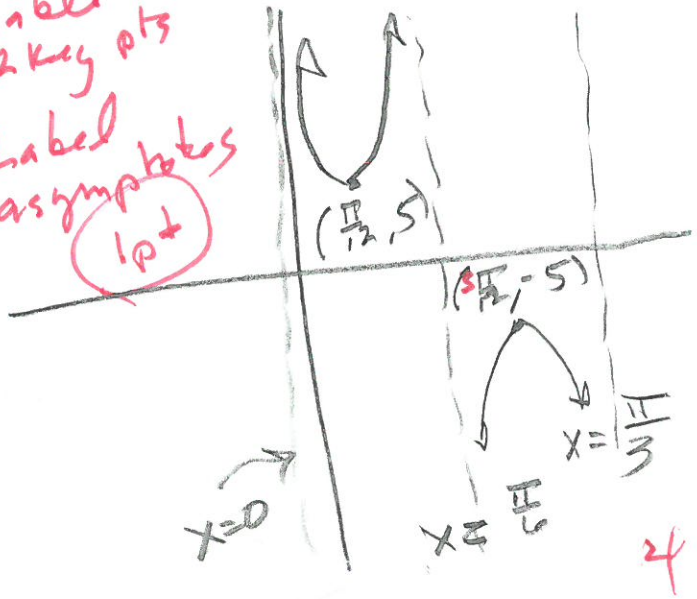
1 pt

Label 2 key pts

Label asymptotes

1 pt





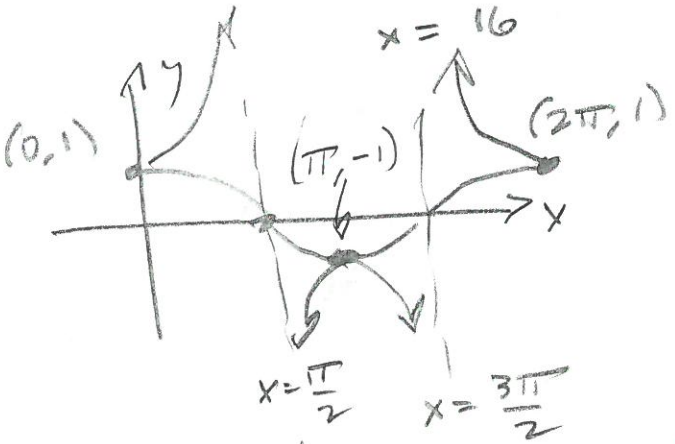
122 §1.6

I'd like an old-school sketch, even though they ask for graphing utility for this #21:

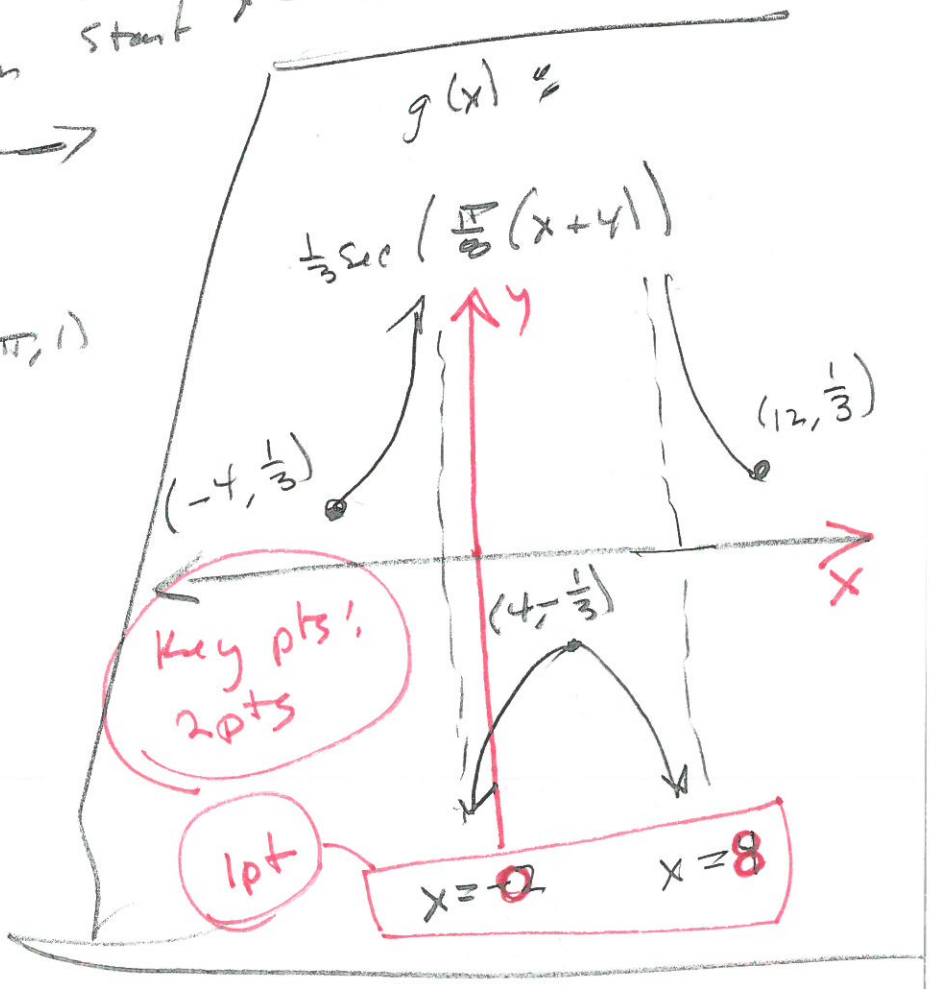
21 $y = \frac{1}{3}\sec\left(\frac{\pi}{8}x + \frac{\pi}{2}\right)$

$= \boxed{\frac{1}{3}\sec\left(\frac{\pi}{8}(x+4)\right) = g(x)}$

$y \mapsto \frac{1}{3}y$; Period $= 16$, from $\frac{\pi}{8}x = 2\pi$; start $x = -4$



cos(x) & sec(x)



122 5.1.6

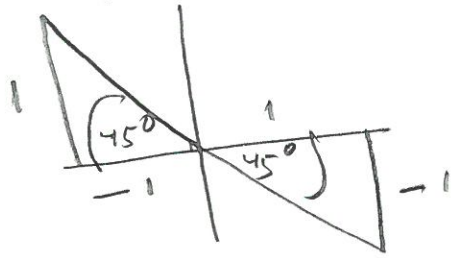
(23) It says use a graph, b/c tan/3 is a graphing section, but the main skill is equation-solving.

$\cot(x) = -1 \longrightarrow \tan(x) = \frac{1}{-1} = -1$ is more intuitive.

$\tan(x) = -1 \longrightarrow x = 180^\circ - 45^\circ = 135^\circ$

OR $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

OR $-2\pi + \frac{3\pi}{4} = -\frac{5\pi}{4}$



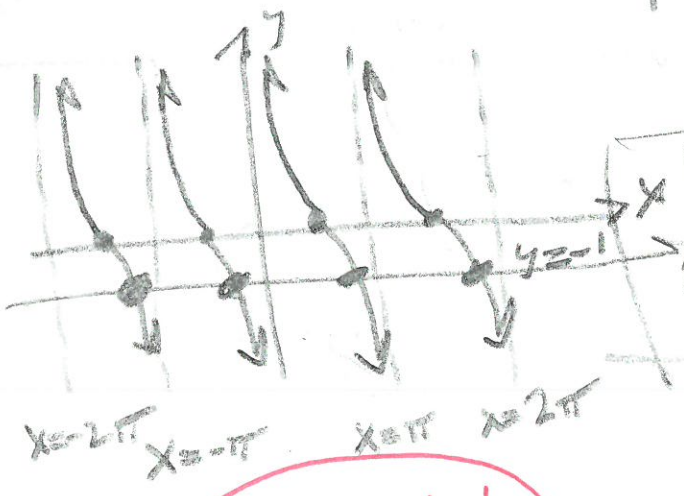
$-\frac{\pi}{4} = x = -45^\circ$ OR

$x = 360^\circ - 45^\circ = 315^\circ = \frac{7\pi}{4}$

So $x \in \left\{-\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

OR $\{-225^\circ, -45^\circ, 135^\circ, 315^\circ\}$

Picture:



Pic: 1 pt

Answers in degrees or (π-) radians, either one.

2 pts

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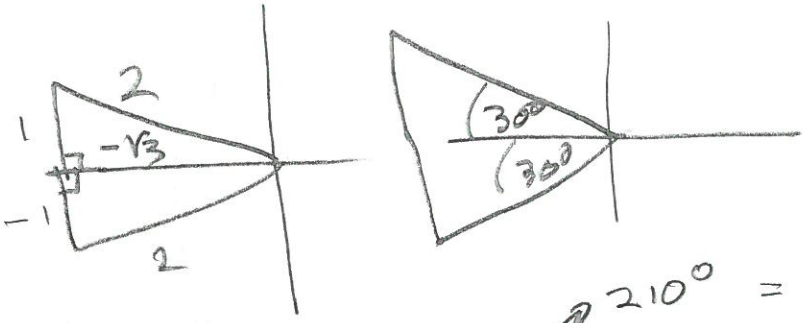
(24) $\sec(x) = -\frac{2\sqrt{3}}{3} \Rightarrow \cos(x) = -\frac{3}{2\sqrt{3}}$

$= -\frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3}}{2 \cdot 3} = -\frac{\sqrt{3}}{2}$ Aha! 30-60!

$1-2=\sqrt{3}$!



So $x = 180^\circ \pm 30^\circ$ → $210^\circ = \frac{7\pi}{6}$, $150^\circ = \frac{5\pi}{6}$

And for the negative angles, subtract 360°

from the previous!

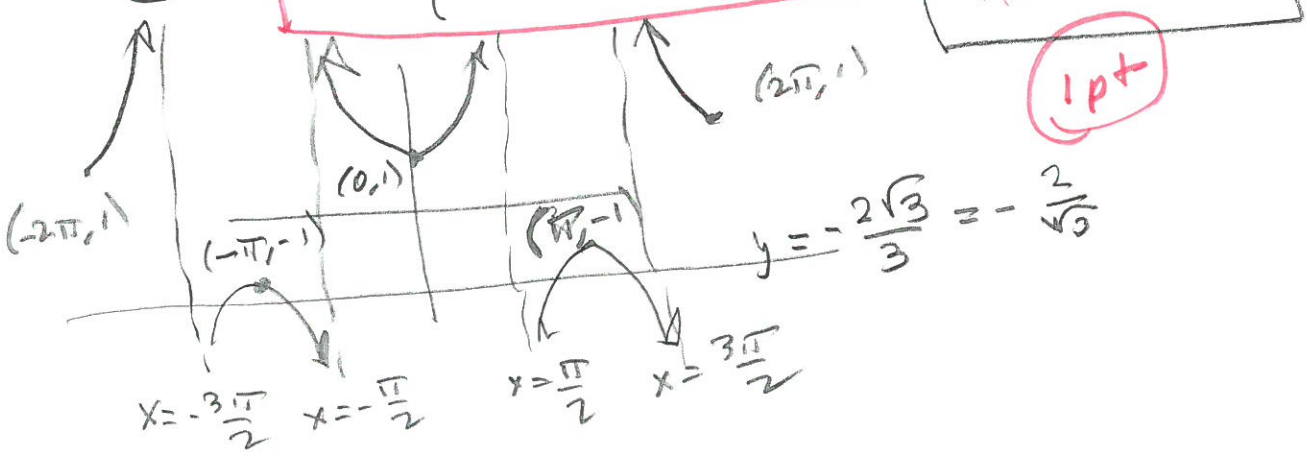
$210^\circ - 360^\circ = -150^\circ = -\frac{5\pi}{6}$

$150^\circ - 360^\circ = -210^\circ = -\frac{7\pi}{6}$

$x \in \left\{ \pm\frac{5\pi}{6}, \pm\frac{7\pi}{6} \right\}$ → 2pts

Sketch

1pt



122 §1.6

(26) Is $f(x) = x^2 - 3\sec x$ odd or even

TOOLKIT: $\sin\theta, \csc\theta, \tan\theta, \cot\theta$ are ODD.

ODD + EVEN = "+" + "−" = neither

(ODD)(EVEN) = (−)(+) = − = odd

(ODD)(ODD) = (−)(−) = + = even

(EVEN)(EVEN) = (+)(+) = + = even

ODD + ODD = "−" + "−" = − = ODD

EVEN + EVEN = "+" + "+" = + = Even

$x^2 \rightsquigarrow +$

$\sec(x) \rightsquigarrow +$

Sum of evens is even

1pt

Don't sweat graph.

(27) $g(x) = x\csc x = x \cdot \frac{1}{\sin x} = (-)\left(\frac{1}{-}\right) =$ + EVEN

1pt

(29) We graph $f(x) = \cos^2\left(\frac{\pi x}{2}\right) = \frac{1}{2}\left(1 + \cos(\pi x)\right)$

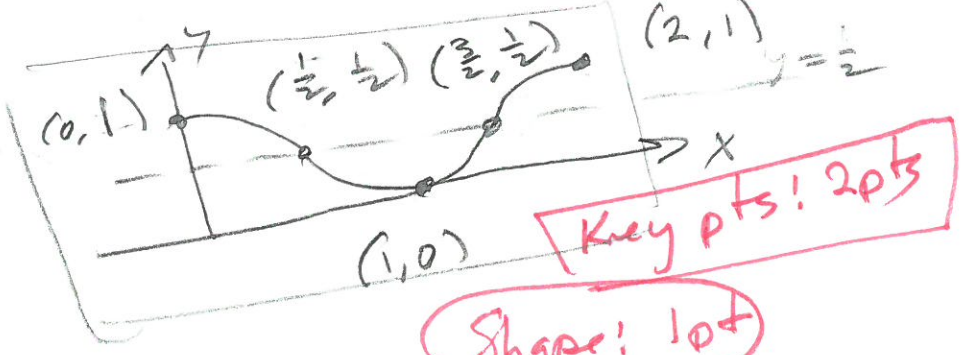
$= \frac{1}{2} + \frac{1}{2}\cos(\pi x)$

mid: $y = \frac{1}{2}$

period: $\pi x = 2\pi$

@ $x = 2$

Amp: $\frac{1}{2}$



Key pts: 2pts

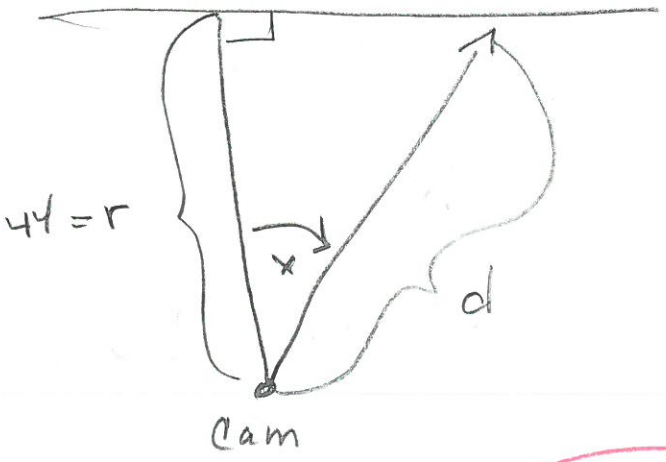
Shape: 1pt

122 §1.6

NOTE TO STUDENTS: I didn't get #31 copied. #s 32-35 are good to SEE, but you want to see them explained.

(36) A tv camera is $r = 44$m from the street down which a parade is marching, Left-to-right. Write the distance d (in meters) to a spot in the parade as a function of the angle x in diagram

$\forall\, x \in (-\frac{\pi}{2}, \frac{\pi}{2})$.



(or $\sec(x) = \frac{d}{r}$)

$\cos(x) = \frac{r}{d}$ 1pt

Want $d = d(x)$, so, isolate the d:

$d = \frac{r}{\cos x} = r \sec x = 44 \sec x$

$d = d(x) = 44 \sec(x)$ 3pts

Sketch: 1pt