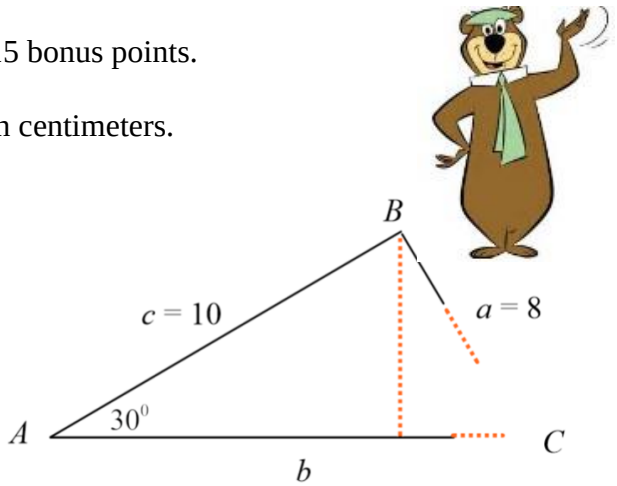
MAT 122 Test 4 – Spring, 2018 Name_____
100 Points NO GRAPHING CALCULATORS!!!

I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.

1. Let $f(x)=x^3-7x^2+25x-39$
   a. (10 pts) Use synthetic division to find $f(-3)$.
   b. (10 pts) Use synthetic division to show that $x=2+3i$ is a solution of the equation $f(x)=0$.
   c. (10 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.
2. Let $z=-3+3\sqrt{3}\,i$
   a. (10 pts) Find $z+\bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.
   b. (10 pts) Express z in trigonometric form.
3. Let $z=8\left(\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right)$.
   a. (10 pts) Express z in standard form.
   b. (10 pts) Find the principal 3rd root of z, i.e., find $\sqrt[3]{z}$. Leave z in trigonometric form for this.
   c. (10 pts) Now, find the *other* 3rd roots of z, in trigonometric form.
   d. (10 pts) Find the trigonometric form of z^2.
   e. (10 pts) Finally, let $w=3\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$, and find the trigonometric form of the product $z\cdot w$.

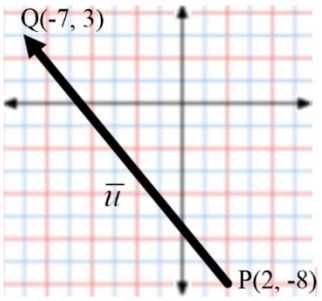
Bonus Section: Answer up to three (3) 5-pointers for up to 15 bonus points.

B1 Consider the triangle in the figure. Assume lengths are in centimeters.



   a. (5 pts) Show that there are 2 possible triangles from this information.
   b. (5 pts) Use the law of sines to find *one* of the possible values of angle *C*. Round your result to 4 decimal places. Then find the *other* possible value, rounded to 4 decimal places.

B2 (5 pts) Consider the directed line segment $\bar{u} = \overrightarrow{PQ}$ in the figure on the right. Find the direction angle of $\bar{u}$. Use degrees, rounded to 4 places.



B3 (5 pts) Find $\sin\left(\frac{u}{2}\right), \cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, given that $\tan(u) = \frac{2}{5}$ and $\sin(u) < 0$.

B4 (5 pts) Build a sine function that achieves its maximum height of $y = 120$ meters at time $x = 3$ seconds and its minimum height of $y = -100$ meters at $x = 15$ seconds.

B5 (5 pts) Find all solutions of the equation $4\sin^2(2x) - 1 = 0$ in the interval $[0, 2\pi)$.