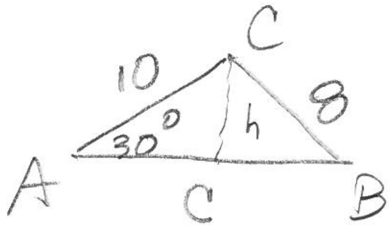
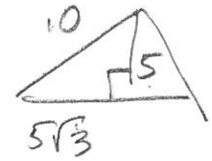
122 TEST 4 Spring 2017

1) a)





$\frac{h}{10} = \sin 30° = \frac{1}{2} \Rightarrow$

$h = 5$ & $h < a < b \Rightarrow$ 2 possibilities

b) $c^2 = a^2 + b^2 - 2ab\cos\theta$

$= 8^2 + 10^2 - 2(8)(10)\cos C$

Can't be done!

c) $\frac{\sin B}{10} = \frac{\sin 30°}{8}$

$\sin B = \frac{10(\frac{1}{2})}{8} = \frac{5}{8} \rightarrow$

$\boxed{B \approx 38.68218745°}$

So $C = 180° - 30° - B \approx \boxed{111.3178125° \approx C}$

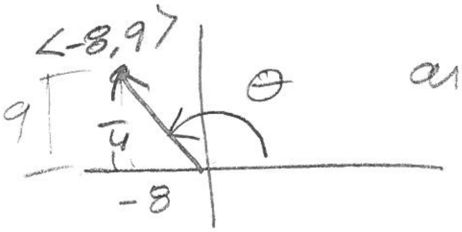
b) $c^2 = 164 - 160\cos C$

$\approx 222.1665383 \rightarrow \boxed{c \approx 14.90525204\text{ cm}}$

122 T4

2 a $\bar{u} = \langle -5-3, 4-(-5) \rangle = \boxed{\langle -8, 9 \rangle = \bar{u}}$

b $\|\bar{u}\| = \sqrt{(-8)^2 + 9^2} = \sqrt{64+81} = \boxed{\sqrt{145} = \|\bar{u}\|}$

c 

$\arctan\left(-\frac{9}{8}\right)$

$5 \lfloor 145$
29

$\approx -48.36646066^\circ$

So ref angle is ϕ

So $\theta = 180^\circ - |\text{previous}|$

$\approx 180^\circ - 48. ...^\circ$

≈ 131.6335393

$\approx \boxed{131.6335^\circ \approx \theta}$

3 a $\bar{u} = \langle -3, 7 \rangle = \boxed{-3\bar{i} + 7\bar{j}}$

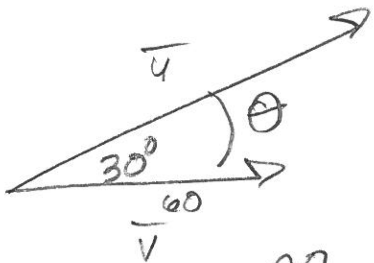
b Resultant.

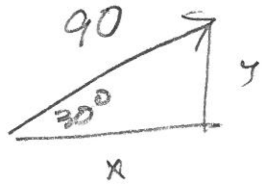
~~c~~

122 T3

(4) $\|\bar{u}\| = 90\text{ N}$, $\|\bar{v}\| = 60\text{ N}$

(a)





$x = 90\cos 30^\circ$

$= \frac{90\sqrt{3}}{2} = 45\sqrt{3}$

$y = 90\sin 30^\circ$

$= 90 \cdot \frac{1}{2} = 45$

$\bar{u} =$

$\boxed{\bar{u} = \langle 45\sqrt{3}, 45 \rangle}$

$\boxed{\bar{v} = \langle 60, 0 \rangle}$

(b) $\boxed{\bar{u} + \bar{v} = \langle 45\sqrt{3} + 60, 45 \rangle}$

(c) $\arctan\left(\frac{45}{45\sqrt{3}+60}\right) \approx 18.06753729^\circ$

$\approx \boxed{18.0675^\circ} \approx \theta$

122 T3

(5) $f(x) = 5x^3 - 23x^2 + 77x - 39$

(a) $f(2)$:

$$\begin{array}{r|rrrr} 2 & 5 & -23 & 77 & -39 \\ & & 10 & -26 & 102 \\ \hline & 5 & -13 & 51 & \boxed{63 = f(2)} \end{array}$$

(b) $2+3i$ is a zero:

$$\begin{array}{r|rrrr} 2+3i & 5 & -23 & 77 & -39 \\ & & 10+15i & -71-9i & 39 \\ \hline & 5 & -13+15i & 6-9i & 0 \end{array}$$

(c)

$$\begin{array}{r|rrr} 2-3i & 5 & -13+15i & 6-9i \\ & & 10-15i & -6+9i \\ \hline & 5 & -3 & 0 \end{array}$$

$f(x) = (x-(2+3i))(x-(2-3i))(5x-3)$

$(2+3i)(-13+15i) = -26+30i-39i+45i^2$

$= -26-9i-45$

$= -71-9i$

$(6-9i)(2+3i) = 3(2-3i)(2+3i) = 3(2^2+3^2)$

$= 3(13) = 39$

122 T 3

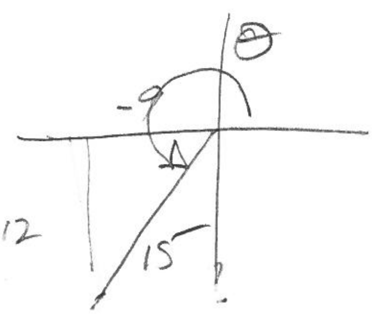
(6) $z = -9 - 12i$

(a) $z + \bar{z} = 2\,\mathrm{Re}(z) = \boxed{-18 = z + \bar{z}}$

$z\bar{z} = 9^2 + 12^2 = 81 + 144 = 225 = z\bar{z}$

$3 \,|\, 225$
$3 \,|\, 75$
$5 \,|\, 25$
5

(b)



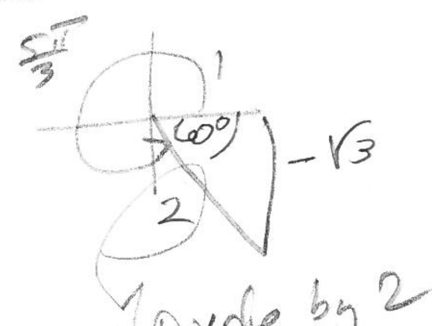
$\theta = \arctan\left(\frac{12}{9}\right) + 180^\circ$

$\approx 233.301024^\circ \longrightarrow$

$\boxed{z \approx 15\left(\cos(233^\circ) + i\sin(233^\circ)\right)}$

(7) $z = \cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)$

$\boxed{= \frac{1}{2} - \frac{\sqrt{3}}{2}i}$



122 T3

(7) (b) $\sqrt[5]{z} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ $k=0$

(c) Increment is $\frac{2\pi}{5}$

$\frac{\pi}{3} + \frac{2\pi}{5} = \frac{5\pi \cancel{+} 6\pi}{15} = \frac{11\pi}{15}$ $k=1$

$\frac{11\pi}{15} + \frac{6\pi}{15} = \frac{17\pi}{15}$ $k=2$

$\frac{17\pi + 6\pi}{15} = \frac{23\pi}{15}$ $k=3$

$\frac{23\pi + 6\pi}{15} = \frac{29\pi}{15}$ $k=4$

CHECK: $k=5$

$\frac{29\pi + 6\pi}{15} = \frac{35\pi}{15} = \frac{30\pi}{15} + \frac{5\pi}{15} = 2\pi + \frac{\pi}{3}$, so full circle

$\cos\frac{11\pi}{15} + i\sin\frac{11\pi}{15}$

$\cos\frac{17\pi}{15} + i\sin\frac{17\pi}{15}$

$\cos\frac{23\pi}{15} + i\sin\frac{23\pi}{15}$

$\cos\frac{29\pi}{15} + i\sin\frac{29\pi}{15}$

122 T3

(7) (d) $z^3 = \cos(5\pi) + i\sin(5\pi)$

$= \cos\pi + i\sin\pi$

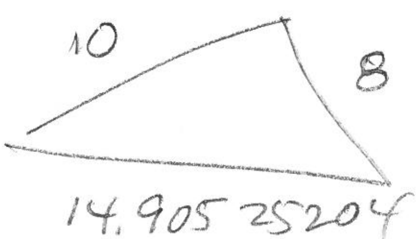
$= \boxed{-1 = z^3}$!

(e) $zw = \left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\left(5\left(\cos\frac{4\pi}{7} + i\sin\left(\frac{4\pi}{7}\right)\right)\right)$

$= 5\left(\cos\frac{47\pi}{21} + i\sin\frac{47\pi}{21}\right)$

$\frac{5\pi}{3} + \frac{4\pi}{7} = \frac{(35+12)\pi}{21} = \frac{47\pi}{21}$

$\boxed{\text{B 1}}$



$s = \frac{10 + 8 + 14.\ldots}{2}$

≈ 81.45262602, so

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

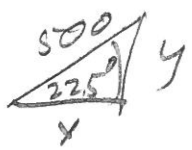
$\approx \sqrt{28448641.63}$

$\approx \boxed{5333.726805 \text{ cm}^2}$

122 T 3

B2

a



$x = 500 \cos 22.5^\circ \approx 461.9397663$

$\boxed{x \approx 462 \ \frac{m}{s}}$

$y = 500 \sin 22.5^\circ \approx 191.3417162$

$\boxed{y \approx 191 \ \frac{m}{s}}$

b

$$500 \cos 22.5 = 500\sqrt{\frac{1+\cos 45^\circ}{2}}$$

$$= 500\sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = 500\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

$$= 500\sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{250\sqrt{2+\sqrt{2}} \ \frac{m}{s} = x}$$

$$y = 500 \sin 22.5 = 500\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = 500\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \boxed{250\sqrt{2-\sqrt{2}} \ \frac{m}{s} = y}$$

122 T3

B2 c $S = \frac{1}{2}gt^2 + v_0 t + s_0$

$= -4.9t^2 + 250\sqrt{2-\sqrt{2}}\,t + 0$

$= -4.9\left(t^2 - \frac{250\sqrt{2-\sqrt{2}}}{4.9}t\right) \overset{\text{SET}}{=} 0$

$\Rightarrow t\left(t - \frac{250\sqrt{2-\sqrt{2}}}{4.9}\right) = 0$

$\rightarrow t = 0.$ or $t = \frac{250\sqrt{2-\sqrt{2}}}{4.9}$

$\approx 39.04932983\ s$

$\rightarrow$ Horizontal distance $v_x t = \text{distance}$

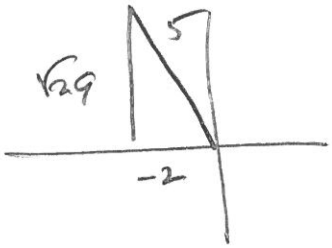
$\approx (500\cos(22.5^\circ))(39.04932983)$

$= (250\sqrt{2+\sqrt{2}})(39.04932983) \approx 18038.4383$

$\approx \boxed{18{,}038\ m}$ $\approx$ Distance traveled.

122 T3

B3 $\cos u = -\frac{2}{5}$ & $\sin u > 0$



$90^\circ < u < 135^\circ$

$180^\circ < 2u < 270^\circ$ Q III

$\sin(2u) = 2 \sin u \cos u$

$= 2\left(\frac{\sqrt{29}}{5}\right)\left(\frac{-2}{5}\right)$

$= \boxed{\frac{-4\sqrt{29}}{25} = \sin(2u)}$

$\cos(2u) = 2\cos^2 u - 1$

$= 2\left(-\frac{2}{5}\right)^2 - 1$

$= 2\left(\frac{4}{25}\right) - 1$

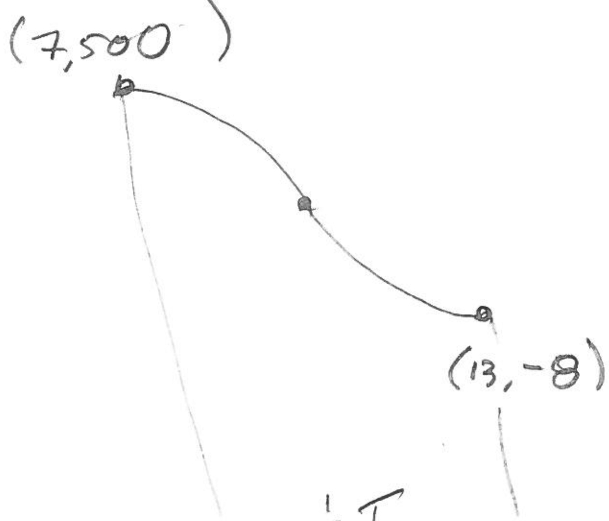
$= \frac{+8 - 25}{25} = \boxed{-\frac{17}{25} = \cos(2u)}$

$\tan(2u) = \frac{\sin(2u)}{\cos(2u)} = \left(\frac{-4\sqrt{29}}{25}\right)\left(-\frac{25}{17}\right)$

$= \boxed{\frac{4\sqrt{29}}{17} = \tan(2u)}$

122 T3

B 4



$\frac{1}{2}T = 13-7 = 6 \rightarrow \boxed{T=12}$

STARTS @ $\cancel{x}$=7

$12b = 2\pi$

$b = \frac{\pi}{6}$

$* \cos\left(\frac{\pi}{6}(x-7)\right) + *$

MIDLINE: $\frac{500 + -8}{2} = \frac{492}{2} = 246 = y$

$* \cos\left(\frac{\pi}{6}(x-7)\right) + 246$

AMP: $\frac{500-(-8)}{2} = \frac{508}{2} = 254$

$$f(x) = 254\cos\left(\frac{\pi}{6}(x-7)\right) + 246$$

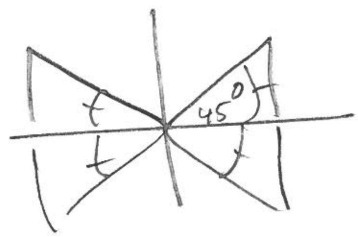
122 T 3

B5

$$2\sin^2(3x) - 1 = 0 \qquad 0 \le x < 2\pi$$

$$\sin^2(3x) = \frac{1}{2} \qquad 0 \le 3x < 6\pi \;!$$

$$\sin(3x) = \pm\frac{1}{\sqrt{2}}$$



$$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$$

That's the easy part.

But there are more solutions

Take 3x's to 6π or take x's to 2π

$$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4},$$

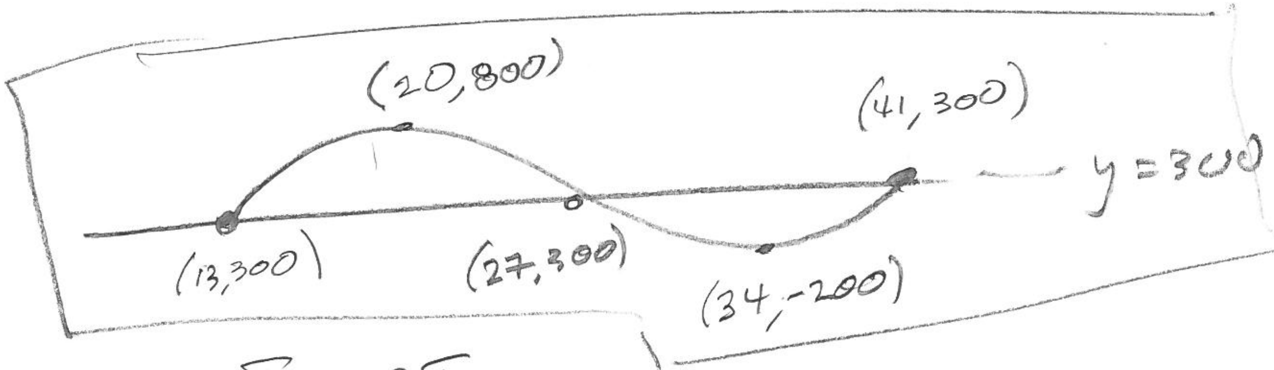
$$\frac{21\pi}{4}, \frac{23\pi}{4}, \cancel{\frac{25\pi}{4} > 6\pi}$$

So
$$x \in \left\{ \frac{2n-1}{12}\pi \;\middle|\; n = 1, 2, \ldots, 12 \right\}$$

$$= \left\{ \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12} \right\}$$

122 T3

B6 $f(x) = 500\sin\left(\frac{\pi}{14}(x - 13)\right) + 300$



$T: \frac{\pi}{14}x = 2\pi$

$x = 28 = T$

$\frac{T}{4} = 7$, so add 7 to each x-coord, proceeding to the right.

B7

122 T3

B7 $\bar{u} = \cancel{B} \langle -7, 7 \rangle$

$\bar{v} = \langle 15, 5\sqrt{3} \rangle$

a $\theta = ?$

b $\text{proj}_{\bar{v}} \bar{u} = ?$

a $\cos\theta = \dfrac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \dfrac{-7(15) + 7(5\sqrt{3})}{\sqrt{49+49}\sqrt{225+75}}$

$= \dfrac{-105 + 35\sqrt{3}}{(7\sqrt{2})(10\sqrt{3})} = \dfrac{-105 + 35\sqrt{3}}{70\sqrt{6}}$

$\rightarrow \boxed{\theta = 105^{0}}$

b $\text{proj}_{\bar{v}} \bar{u} = \dfrac{\bar{u} \cdot \bar{v}}{\|\bar{v}\|^2} \bar{v} = \dfrac{-105 + 35\sqrt{3}}{300} \langle 15, 5\sqrt{3} \rangle$

$= \dfrac{-21 + 7\sqrt{3}}{60} \langle 15, 5\sqrt{3} \rangle$

$= \dfrac{-21 + 7\sqrt{3}}{20} \langle 3, \sqrt{3} \rangle$

$\text{proj}_{\bar{v}} \bar{u}$: $\boxed{\approx \langle -2.218911087, -1.281088913 \rangle}$