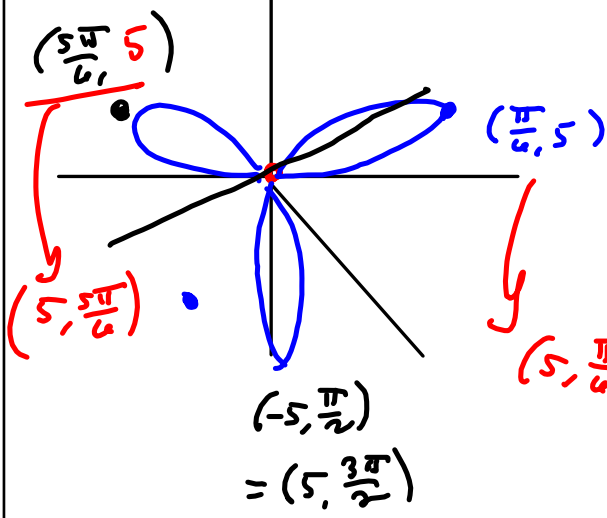
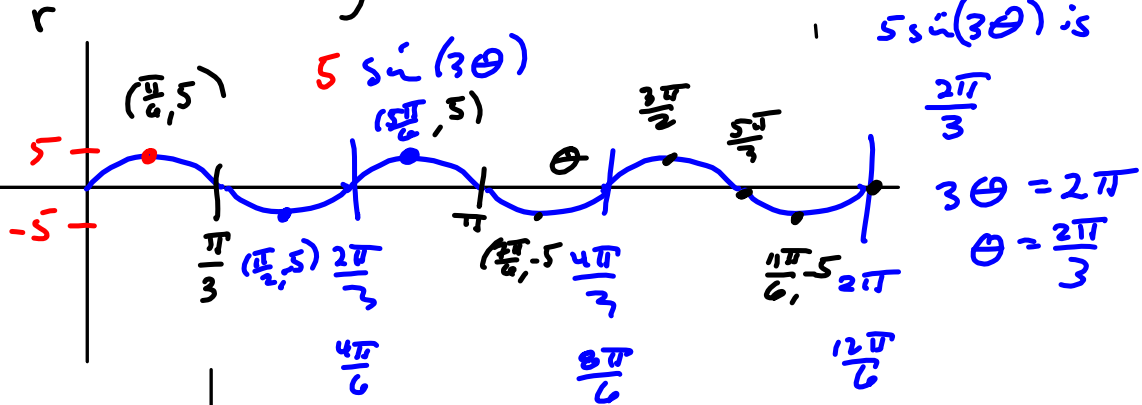


Polar Graphs. An n-petal rose.

Bonus for symmetry checks.

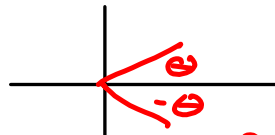
Something like $5 \sin(3\theta)$ Period of $5 \sin(3\theta)$ is



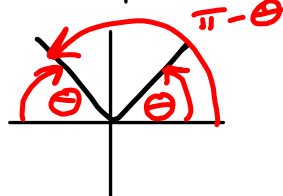
These are all (θ, r) 's instead of (r, θ) 's
 $(5, \frac{\pi}{4})$, idiot.

Check for 3 kinds of symmetry

① Polar (x-) axis

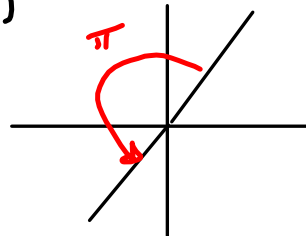


② $\theta = \frac{\pi}{2}$ (y-) axis



or
 $(-r, -\theta)$

③ Pole (origin)



$\theta + \pi$
or
 $-r$

$$r = 5 \sin 3\theta$$

① Polar axis : $r = 5 \sin(3(-\theta))$
 $= -5 \sin(3\theta)$

$$\textcircled{2} \quad \theta = \frac{\pi}{2}: \quad = -15 \sin \theta \quad \text{No.}$$

$$r = 5 \sin 3\theta$$

$$r = 5 \sin (3(\pi - \theta))$$

$$= 5 \sin (3\pi - 3\theta)$$

$$= 5 [\sin 3\pi \cos (-3\theta) + \cos (3\pi) \sin (-3\theta)]$$

$$= 5 (-1)(-\sin (3\theta)) = 5 \sin 3\theta \quad \text{Yes.}$$



$$-r = 5 \sin (3(-\theta))$$

$$-r = 5 \sin (3\theta)$$

$$r = 5 \sin (3\theta)$$

Polar axis: $-\theta$;

Pole: $\pi + \theta, -r$

$\theta = \frac{\pi}{2}: \pi - \theta, (-r, -\theta)$

Vectors and the Law of Cosines.

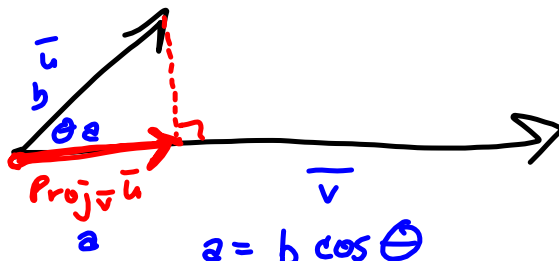
Angle between vectors:

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

$$\vec{u} = \langle u_1, u_2 \rangle$$

$$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2}$$

$$\vec{v} = \langle v_1, v_2 \rangle$$



$$\text{Proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} = \|\vec{u}\| \cos \theta = \|\vec{u}\| \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

$\frac{\vec{v}}{\|\vec{v}\|}$ is a unit vector
in the direction of \vec{v} .

$$\begin{aligned} a \text{ as a vector is } & \left(\|\vec{u}\| \cos \theta \right) \frac{\vec{v}}{\|\vec{v}\|} = \cancel{\|\vec{u}\|} \frac{\vec{u} \cdot \vec{v}}{\cancel{\|\vec{u}\|} \|\vec{v}\|} \frac{\vec{v}}{\|\vec{v}\|} \\ & = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} \end{aligned}$$