$x = \cos\theta$, $y = 2\sin(2\theta) = 2(2\sin\theta\cos\theta)$

$= 4\sin\theta\cos\theta$

$= 4\sin\theta\, x$

θ	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	2
$\frac{\pi}{3}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	0	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$-\sqrt{3}$
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	-2
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$
π	-1	0

This will repeat.

$4\sin\frac{\pi}{6}\cos\frac{\pi}{6} = 4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$4\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 4\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 2$

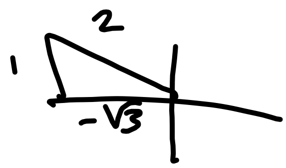
$4\sin\frac{\pi}{3}\cos\frac{\pi}{3} = 4\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \sqrt{3}$

$4\sin\frac{\pi}{2}\cos\frac{\pi}{2}$

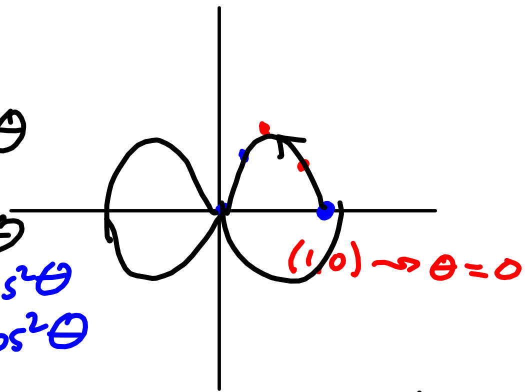
$4\sin\frac{2\pi}{3}\cos\frac{2\pi}{3} = 4\cdot\frac{\sqrt{3}}{2}\cdot -\frac{1}{2} = -\sqrt{3}$

$4\sin\frac{3\pi}{4}\cos\frac{3\pi}{4}$

$4\sin\frac{5\pi}{6}\cos\frac{5\pi}{6} = 4\cdot\frac{1}{2}\cdot -\frac{\sqrt{3}}{2}$



(b) $x = \cos\theta$ $\quad y = 4\sin\theta\cos\theta$



$x^2 + y^2 = \cos^2\theta + 16\sin^2\theta\cos^2\theta$

$x^2 \quad + 16(1-\cos^2\theta)\cos^2\theta$

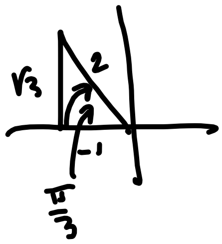
Hmmmmm

$(4x)^2 + y^2 = 16\cos^2\theta + 16\sin^2\theta\cos^2\theta$

$= 16\cos^2\theta(1+\sin^2\theta)$

$= 16\cos^2\theta(1+1-\cos^2\theta)$

$= 16\cos^2\theta(2-\cos^2\theta)$



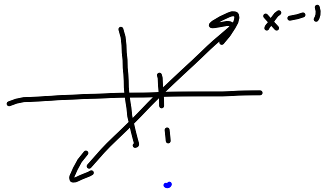
$$= 32\cos^2\theta - 16\cos^4\theta$$

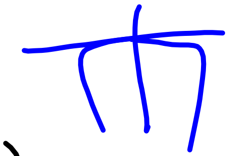
$$16x^2 + y^2 = 32x^2 - 16x^4$$

$$y^2 = -16x^4 + 16x^2$$

$$y = \pm\sqrt{-16x^4 + 16x^2}$$

$$= \pm 4\sqrt{-x^4 + x^2} = \pm 4\sqrt{-x^2(x^2-1)}$$

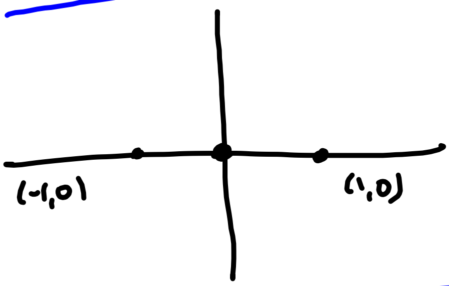


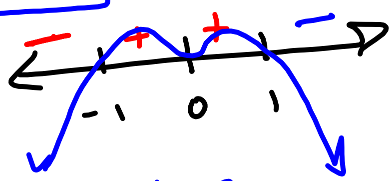


$$-x^2(x+1)(x-1)$$

$$-x^4 + x^2$$

$$\boxed{-x^2(x^2-1) = -x^2(x-1)(x+1)}$$



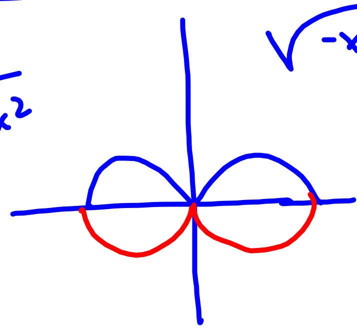


$-x^4 + x^2$ Rough

$\sqrt{-x^2+x}$

$+\sqrt{-x^4+x^2}$

$-\sqrt{-x^4+x^2}$

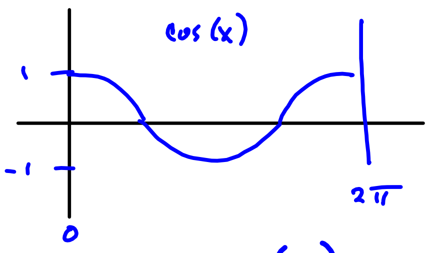


$r = 2\cos(2\theta) + 1$ Polar Graph.

Graph in Rectangular coordinates

$y = 2\cos(2x) + 1$

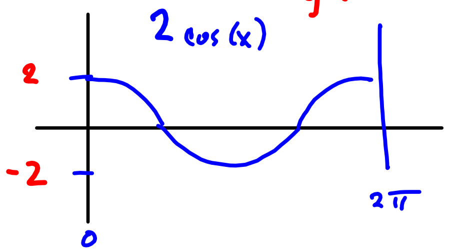
$\cos(x)$



$2f(x)$

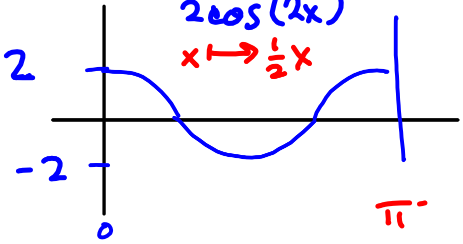
$y \longmapsto 2y$

$2\cos(x)$



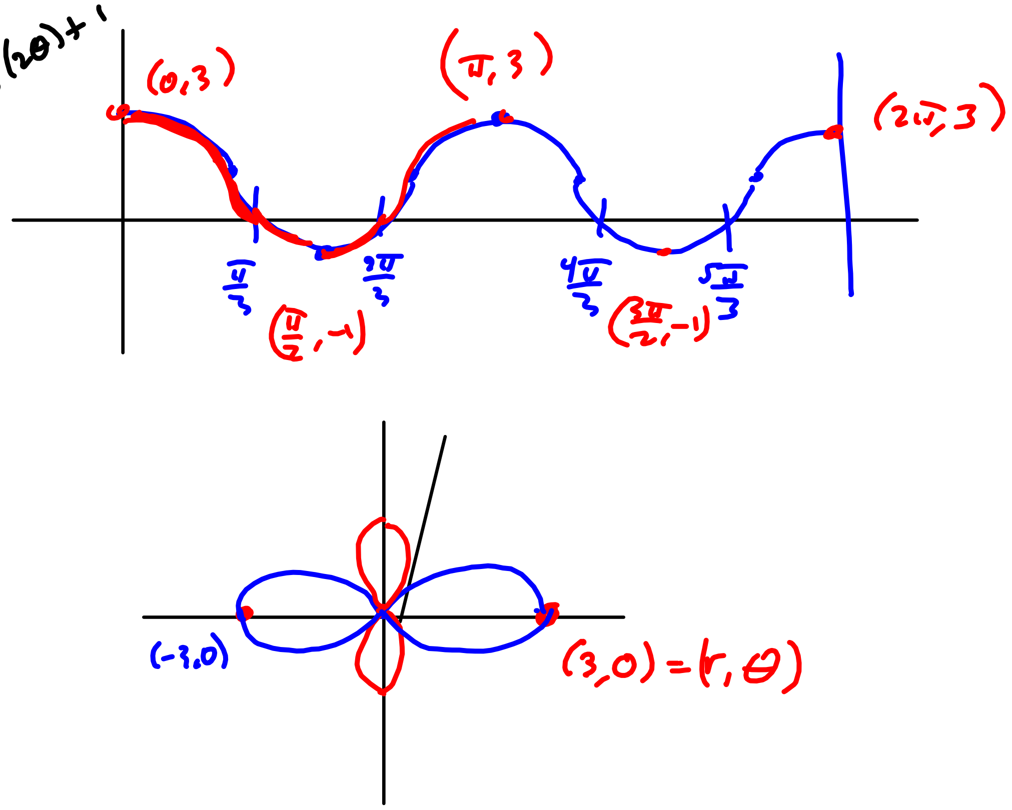
$2\cos(2x)$

$x \longmapsto \frac{1}{2}x$



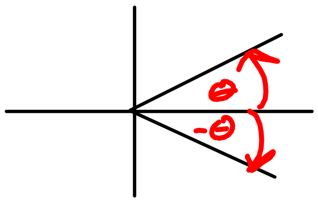
$2\cos(2x) + 1$

$r = 2\cos(2\theta) + 1$

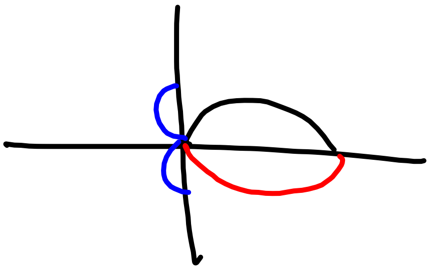


3 Kinds of Symmetry

① About the polar axis (used to call it the x-axis)



Check: Replace θ by $-\theta$ & see if you get an equivalent equation (identical)



$r = 2\cos(2\theta) + 1$

$r = 2\cos(2(-\theta)) + 1$

$= 2\cos(-2\theta) + 1$

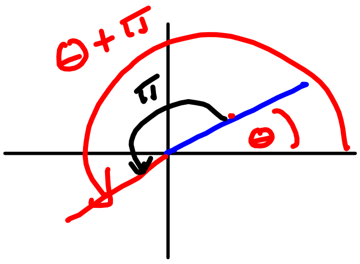
$= 2\cos(2\theta) + 1$ Same.

So, symmetric about the polar axis

② Symmetry thru the pole. 2 tests.

(i) Replace r by $-r$ & see if identical.

(ii) Replace θ by $\theta + \pi$ & see if ident.

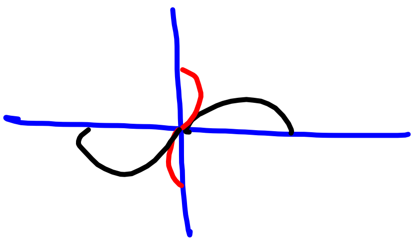


(i) $-r = 2\cos(2\theta) + 1$? Nah.

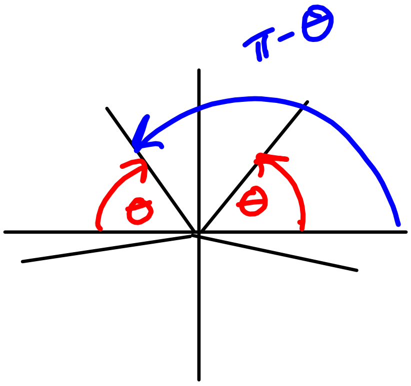
(ii) $r = 2\cos(2(\theta + \pi)) + 1$

$= 2\cos(2\theta + 2\pi) + 1$

$= 2\cos(2\theta) + 1$ Yeah!



3. About the line $\theta = \frac{\pi}{2}$



Replace θ by $\pi - \theta$

$$
\begin{aligned}
r &= 2\cos(2(\pi - \theta)) + 1 \\
&= 2\cos(2\pi - 2\theta) + 1 \\
&= 2\cos(-2\theta) + 1 \\
&= 2\cos(2\theta) + 1
\end{aligned}
$$

Sweet!