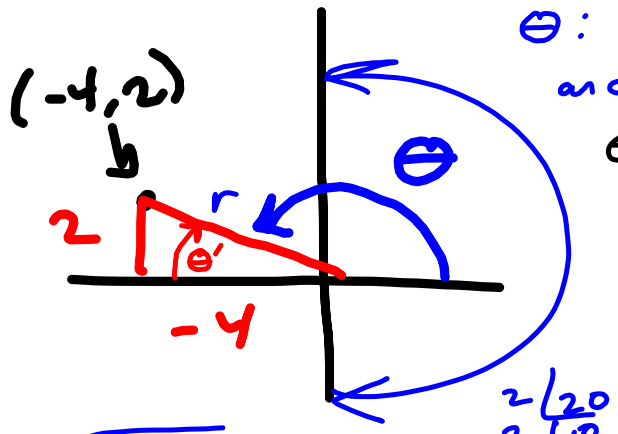
1. We convert $(x, y) = (-4, 2)$ to polar coordinates, (r, θ).

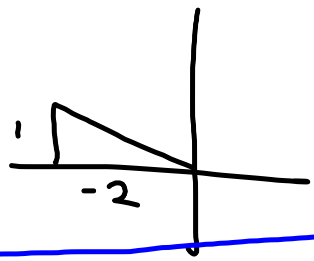
a. (10 pts) Assume $r > 0$ and $\theta \in [0, 360^0]$. Find the *exact* polar coordinates of the point. This may require leaving your answer with an 'arctan' in it. Use degrees for angle measures.



θ: $\arctan\left(-\frac{1}{2}\right)$

$\theta = 180^\circ - \theta'$

$= 180^\circ + \arctan\left(-\frac{1}{2}\right)$



$r = \sqrt{2^2 + 4^2}$

$= \sqrt{20} = 2\sqrt{5}$

$2\,|\,20$
$2\,|\,10$
5

$$\boxed{(r, \theta) = \left(2\sqrt{5}, 180^\circ + \arctan\left(-\tfrac{1}{2}\right)\right)}$$

b. (10 pts) Approximate your answer in part a. with 4 decimal-place accuracy.

$\arctan\left(-\frac{1}{2}\right) \approx -26.56505117^\circ \Rightarrow$

$\theta = 180^\circ + \arctan\left(-\frac{1}{2}\right)$

$\approx 153.4349488^\circ$

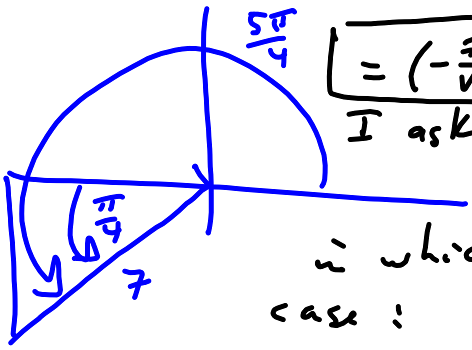
$r = 2\sqrt{5} \approx 4.472135954 \Rightarrow$

$$\boxed{(r, \theta) \approx (4.4721, 153.4349^\circ)}$$

2. (10 pts) Convert $(r,\theta)=\left(7, \frac{5\pi}{4}\right)$ to rectangular coordinates. Give an exact answer and a decimal answer, accurate to 4 decimal places.

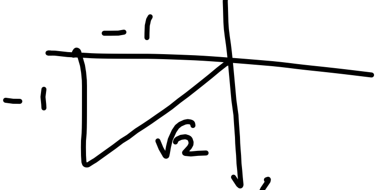
$$(x,y)=(r\cos\theta, r\sin\theta)$$

$$=\left(7\cos\frac{5\pi}{4}, 7\sin\frac{5\pi}{4}\right)$$



$\boxed{=\left(-\frac{7}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)}$ is fine, unless I ask for "simplified radical form":

in which case:



$$-\frac{7}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{-7\sqrt{2}}{2} \Longrightarrow (x,y)=\left(\frac{-7\sqrt{2}}{2}, \frac{-7\sqrt{2}}{2}\right)$$

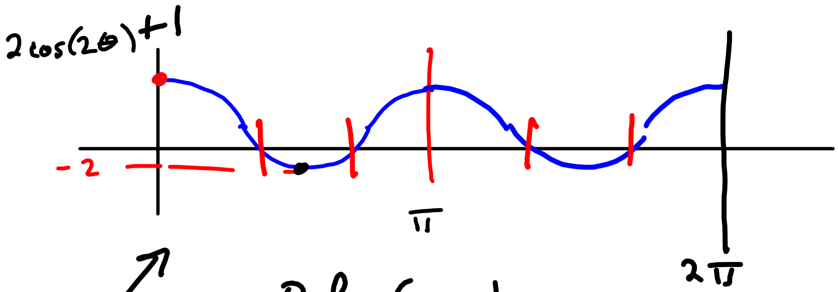
4.949747467

$$\boxed{(x,y)\approx(-4.9497, -4.9497)}$$

3. (10 pts) Sketch the graph of $r = 7\sin\theta$.

Try $1+2\cos(2\theta) = r$

Rectangular coords



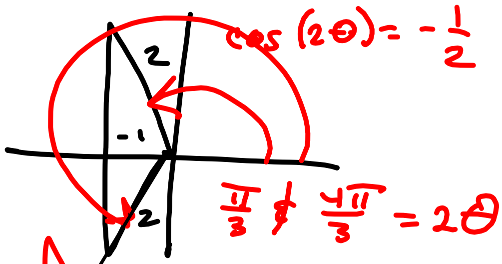
Polar Graph

Find the zeros:

$1+2\cos(2\theta) = 0$

$2\cos(2\theta) = -1$

$\cos(2\theta) = -\frac{1}{2}$



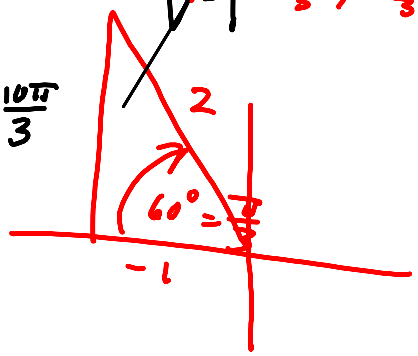
Since

$0 \le \theta \le 2\pi$, $2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

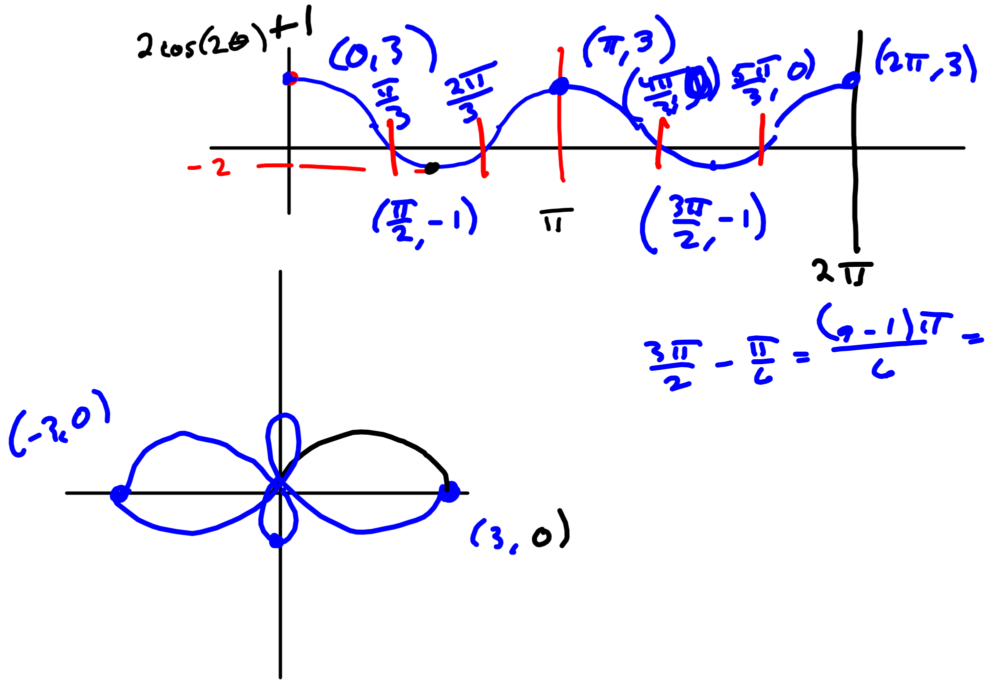
$0 \le 2\theta \le 4\pi$

$2\pi + \frac{2\pi}{3} = \frac{6\pi+2\pi}{3} = \frac{8\pi}{3}$

$2\pi + \frac{4\pi}{3} = \frac{10\pi}{3}$

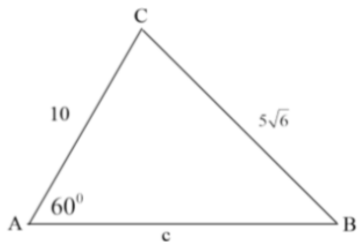


$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ is where it's zero



$$\frac{3\pi}{2} - \frac{\pi}{6} = \frac{(9-1)\pi}{6} = \frac{4\pi}{3}$$

4. (20 pts) Solve the triangle in the figure. Assume lengths are in miles.
Round your final answers to 2 places

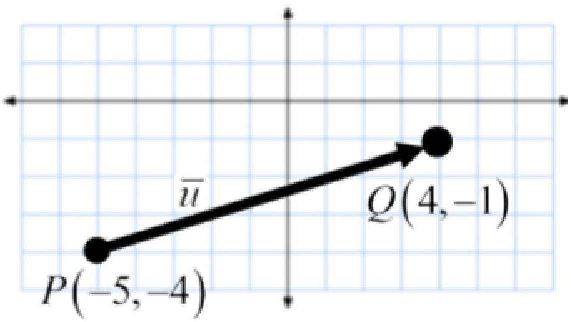


Bonus 1. (10 pts) Give the *exact* value of side c.

5. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\overline{u}$:

a. (10 pts) Express the vector $\overline{u} = \overrightarrow{PQ}$ in component form.

$$\overline{u} = \langle 4-(-5), -1-(-4) \rangle$$
$$= \langle 9, 3 \rangle$$



b. (10 pts) Compute the magnitude of $\overline{u}$. Leave your answer in simplified radical form.

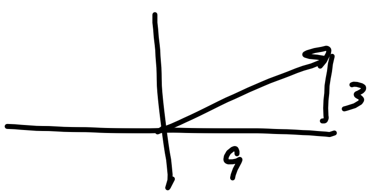
$$\|\overline{u}\| = \sqrt{9^2+3^2} = \sqrt{90} = 3\sqrt{10}$$

c. (10 pts) Express $\overline{u}$ as a linear combination of the canonical (standard) unit vectors $\overline{i}$ and $\overline{j}$.

$$\langle 9, 3 \rangle = 9\langle 1, 0 \rangle + 3\langle 0, 1 \rangle$$
$$= 9\overline{i} + 3\overline{j}$$

. (10 pts) Find the direction angle of $\overline{u}$. Use degrees, rounded to 4 places.

$$\theta = \arctan\left(\tfrac{1}{3}\right)$$
$$\approx$$



6. Let $f(x)=2x^3-19x^2+62x-70$.
   a. (10 pts) Use synthetic division to show that $x=3+i$ is a solution of the equation $f(x)=0$.

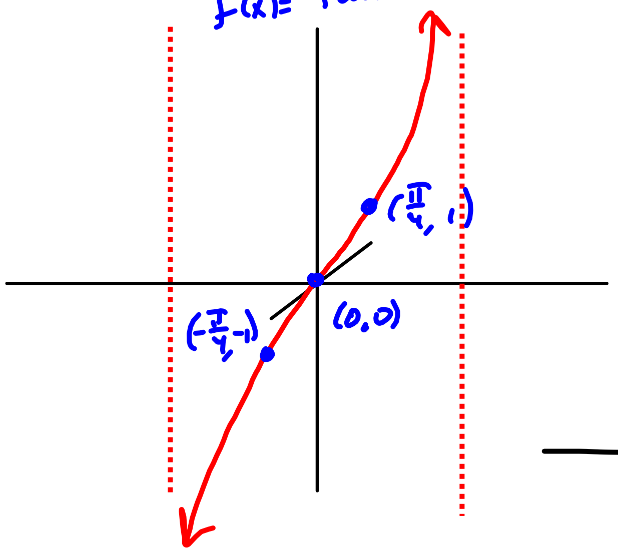
b. (10 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

The fundamental theorem says you can always find ONE root. The factor theorem says with that root, you can split off a linear factor. Then the fundamental theorem says you can find one root of the *depressed polynomial* that is left after splitting off the linear factor. Rinse and repeat:

The fundamental theorem says ALL polynomials of degree n have n (possibly repeated) complex roots (which includes real numbers which are just uninteresting complex numbers.).

$7\tan\left(3x-\frac{9}{2}\right)-5$

$f(x)=\tan(x)$

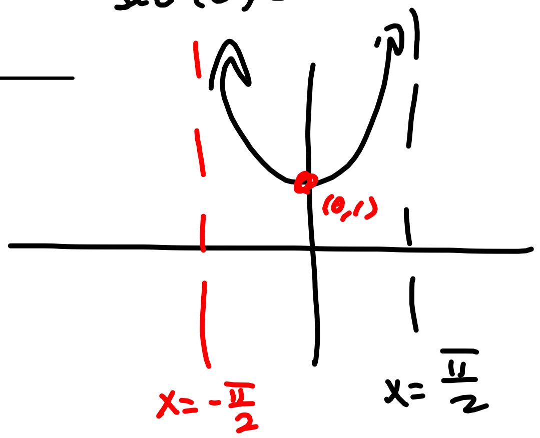


$\frac{d}{dx}[\tan x]$ Calculus

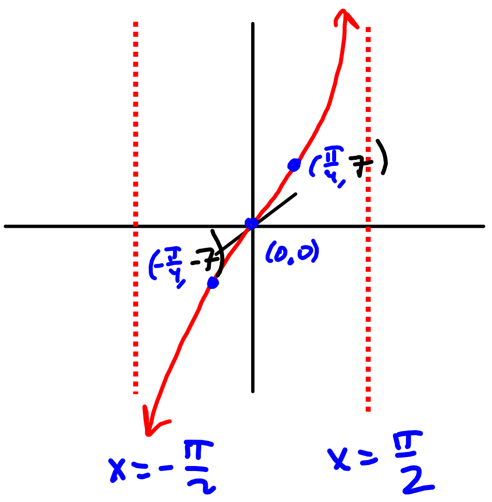
$=\sec^2(x)$

$\sec^2(0)=$

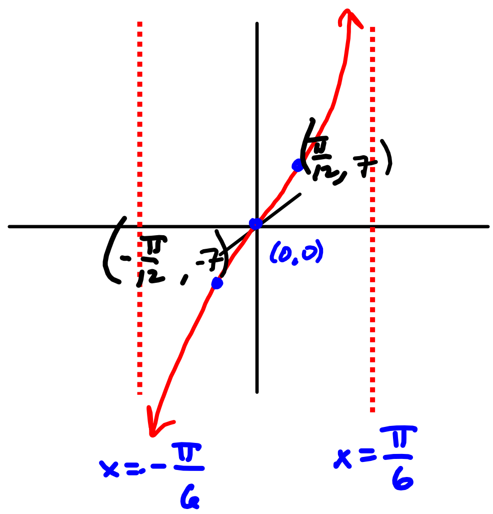
Sec² slope of tangent @ origin is m=1



$7\tan(x)=7f(x)$



$7\tan(3x)=7f(3x)$



$g(x) = 7\tan\left(3x - \frac{9}{2}\right) - 5$

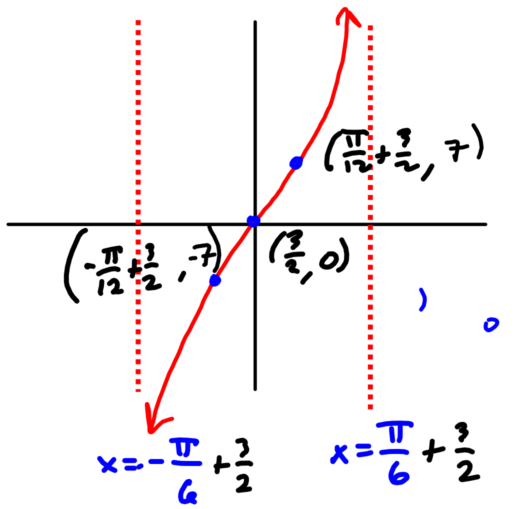
$3x - \frac{9}{2} = 3\left(x - \frac{3}{2}\right)$

$\dfrac{\frac{9}{2}}{3} = \frac{9}{2} \cdot \frac{1}{3} = \frac{3}{2}$

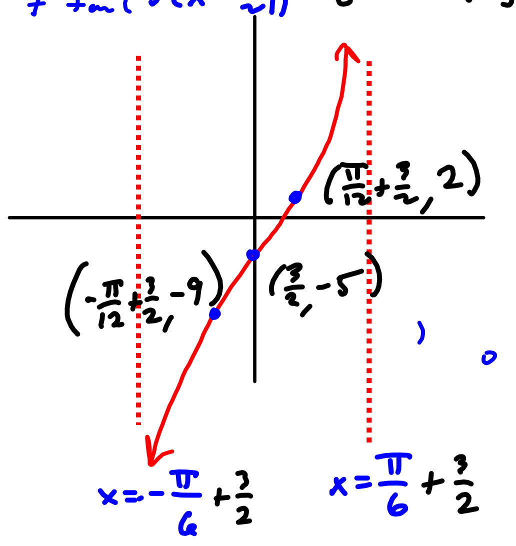
Ahhhhh.

Delay (shift right) by $\frac{3}{2}$

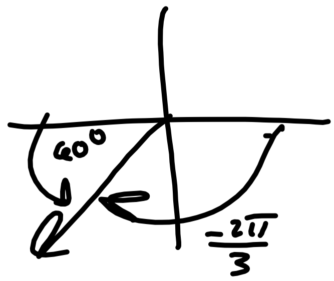
$7\tan\left(3\left(x - \frac{3}{2}\right)\right) = 7f\left(3\left(x - \frac{3}{2}\right)\right)$



$7\tan\left(3\left(x - \frac{3}{2}\right)\right) - 5 = 7f\left(3\left(x - \frac{3}{2}\right)\right) - 5$



$$z = 16\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$$



$= -120^\circ$

$$16^{\frac{1}{4}} = \left(2^4\right)^{\frac{1}{4}} = 2$$

4th roots, so $\dfrac{-\frac{2\pi}{3}}{4} = -\frac{2\pi}{3}\cdot\frac{1}{4}$

$= -\frac{\pi}{6}$

$$\boxed{\sqrt[4]{z} = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)}$$

Increment is $\frac{2\pi}{4} = \frac{\pi}{2} = \frac{3\pi}{6}$

$-\frac{\pi}{6} + \frac{\pi}{2} = \frac{-\pi + 3\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$ *

$\frac{2\pi}{6} + \frac{3\pi}{6} = \frac{5\pi}{6}$ *

$\sqrt{2}$

$\frac{5\pi}{6} + \frac{3\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$ *

$-\sqrt{2}$

Other roots are

$2\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

$2\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$

$2\left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right)$

$$r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right)$$

$k = 0, 1, 2, \ldots, n-1$