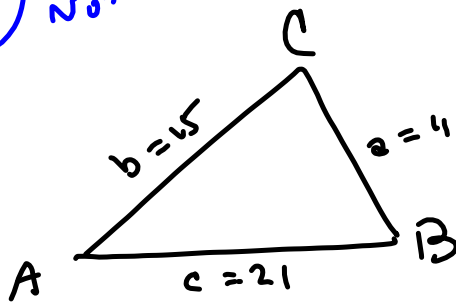
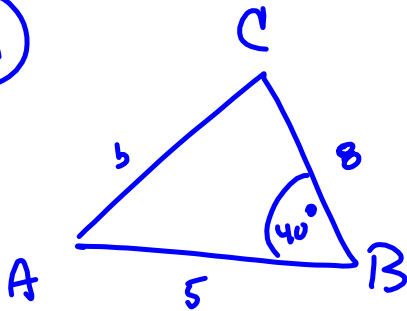


(#13) Not asked ~~*sigh*~~ 4, 15, 21



(31)



$$b^2 = 2^2 + c^2 - 2ac \cos B$$

$$b^2 = 8^2 + 5^2 - 2(5)(8) \cos 40^\circ$$

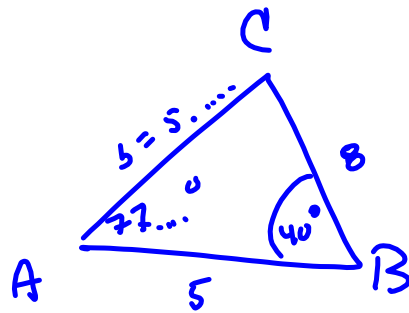
$$= 64 + 25 - 80 \cos 40^\circ$$

$$= 89 - 80 \cos 40^\circ$$

$$89 - 80 \cdot \cos\left(\frac{40 \cdot \text{Pi}}{180}\right) \approx 89 - 80 \cos\left(\frac{2}{9} \pi\right)$$

$$\text{evalf}(\%) \approx 27.71644455 \approx b^2$$

$$\Rightarrow b \approx 5.264640971$$

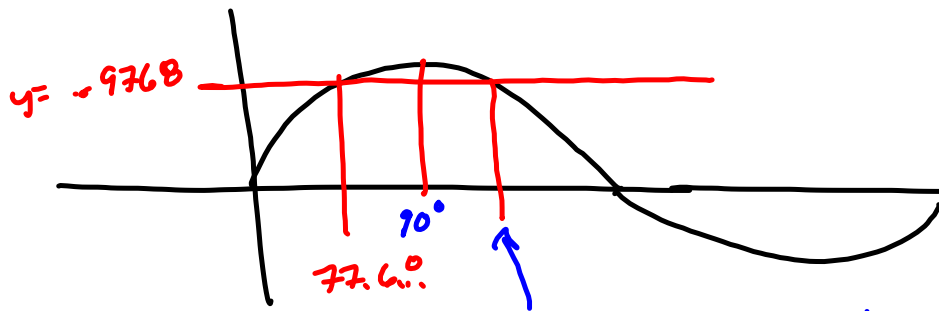


$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin A = \frac{a \sin B}{b} \approx \frac{8 \sin 40^\circ}{5.264640971} \approx 0.9767619302$$

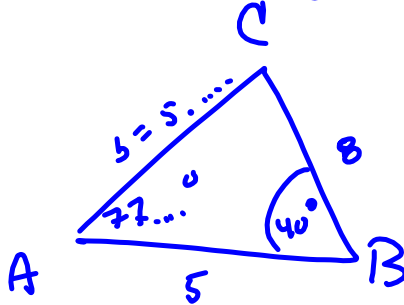
$$\Rightarrow A \approx \sin^{-1}(0.9767619302)$$

$$\approx 77.62394369^\circ$$



102.3760563 is book answer for A.

why?

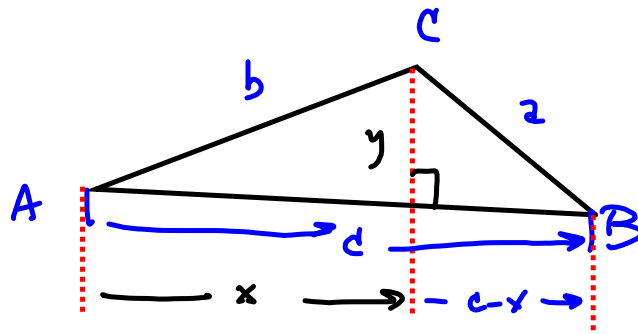


By previous work, this puts C close to 63° . But C needs to be the smallest angle, since $c = 5$ is the shortest side.

The resolution to this is the $102.37\dots^\circ$ that also satisfies

$$\sin A \approx 0.9768$$

(See picture above.)



Pythagoras said that

$$\begin{aligned} a^2 &= (c-x)^2 + y^2 \\ &= (x-c)^2 + y^2 \end{aligned}$$

$$= (b \cos A - c)^2 + (b \sin A)^2$$

$$(r-s)^2 = r^2 - 2rs + s^2$$

$$= \underline{b^2 \cos^2 A} - 2(b \cos A)c + c^2 + \underline{b^2 \sin^2 A}$$

$$= b^2 (\cos^2 A + \sin^2 A) - 2bc \cos A$$

$$a^2 = b^2 - 2bc \cos A$$

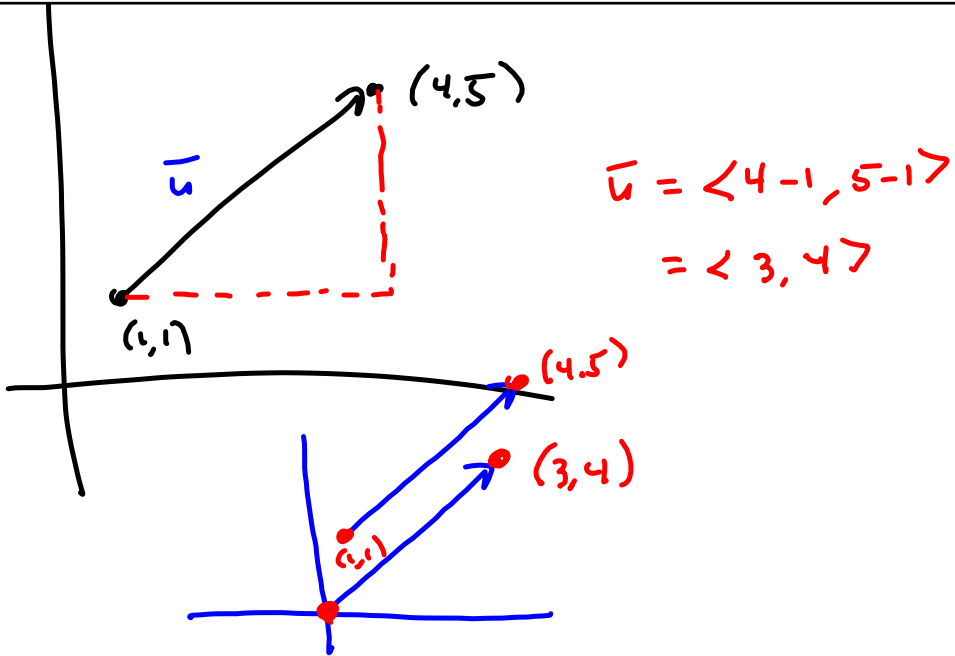
$$(r+s)^2 = r^2 + 2rs + s^2$$

$$\frac{x}{b} = \cos A$$

$$x = b \cos A$$

$$\frac{y}{b} = \sin A$$

$$y = b \sin A$$



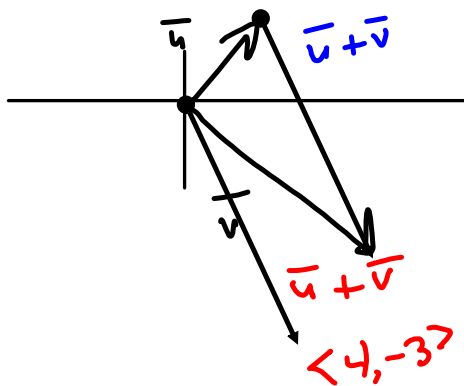
$$\vec{u} + \vec{v}$$

$$\vec{u} = \langle u_1, u_2 \rangle = \langle 1, 2 \rangle$$

$$\vec{v} = \langle v_1, v_2 \rangle = \langle 3, -5 \rangle$$

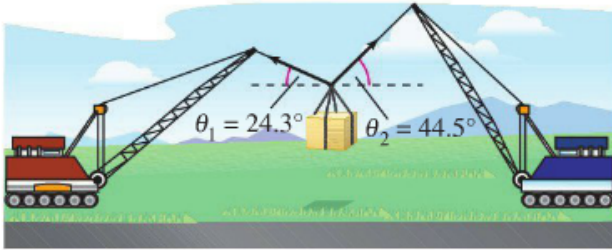
$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

$$= \langle 4, -3 \rangle$$

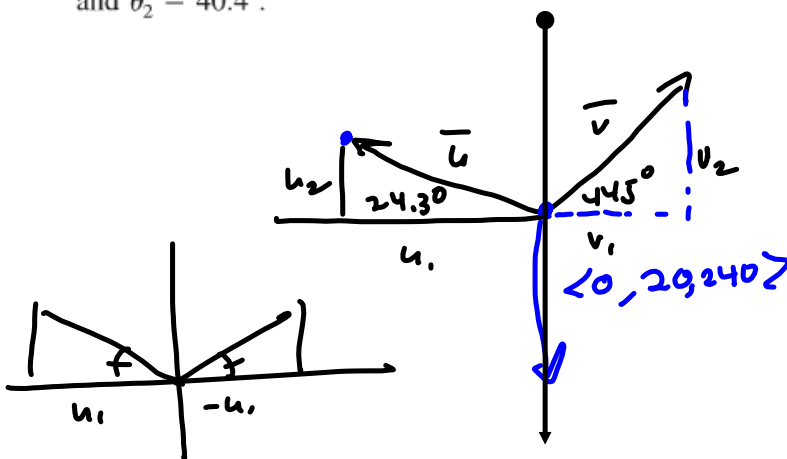


$\vec{u} + \vec{v}$ is called the RESULTANT

89. **Cable Tension** The cranes shown in the figure are lifting an object that weighs 20,240 pounds. Find the tension in the cable of each crane.



90. **Cable Tension** Repeat Exercise 89 for $\theta_1 = 35.6^\circ$ and $\theta_2 = 40.4^\circ$.



We want
the resultant
to have an
upward component
of 20,240 lbs

We know that $\vec{u} + \vec{v} = \langle 0, 2040 \rangle$

$$u_1 + v_1 = 0$$

$$u_2 + v_2 = 2040$$

$$\frac{u_2}{\|\vec{u}\|} = \sin 24.3^\circ$$

$$v_2 = \|\vec{v}\| \sin 44.5^\circ$$

$$u_2 = \|\vec{u}\| \sin 24.3^\circ$$

$$-\frac{u_1}{\|u\|} = \cos 24.3^\circ \quad \frac{v_1}{\|v\|} = \cos 44.5^\circ$$

$$u_1 = -\|u\| \cos 24.3^\circ \quad v_1 = \|v\| \cos 44.5^\circ$$

$$\bar{u} + \bar{v} =$$

$$\langle -\|u\| \cos 24.3^\circ + \|v\| \cos 44.5^\circ, \|u\| \sin 24.3^\circ + \|v\| \sin 44.5^\circ \rangle$$

$$= \langle 0, 2040 \rangle$$

$$\approx \left\langle 0.9114032766 \|\bar{u}\| + 0.7132504491 \|\bar{v}\|, \right.$$

$$0.4115143587 \|\bar{u}\| + 0.7009092643 \|\bar{v}\| \rangle$$

$$\text{So, } -.9114... \|\bar{u}\| + .71... \|\bar{v}\| = 0$$

$$\|\bar{u}\| = \frac{-.71... \|\bar{v}\|}{-.9114...} \approx 0.7825849077 \|\bar{v}\|$$

$$\text{So } .4115... \|\bar{u}\| + .7009... \|\bar{v}\| = 20240$$

$$(.4115...) (.782... \|\bar{v}\|) + .7009... \|\bar{v}\| = 20240$$

$$2b \|\bar{v}\| + c \|\bar{v}\| = 20240$$

$$(2b+c) \|\bar{v}\| = 20240$$

$$\|\bar{v}\| = \frac{20240}{2b+c}$$

$$a = .4115...$$

$$b = .782...$$

$$c = .7009...$$