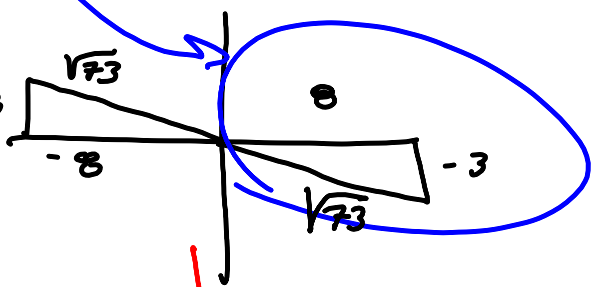
Test 2, FALL '17

① $\tan u = -\frac{3}{8}$, $\sin(u) < 0$



$3^2 + 8^2 = 9 + 64$

$= 73$

$\sin u = -\frac{3}{\sqrt{73}}$ $\quad$ $\csc u = -\frac{\sqrt{73}}{3}$

$\cos u = \frac{8}{\sqrt{73}}$ $\quad$ $\sec u = \frac{\sqrt{73}}{8}$

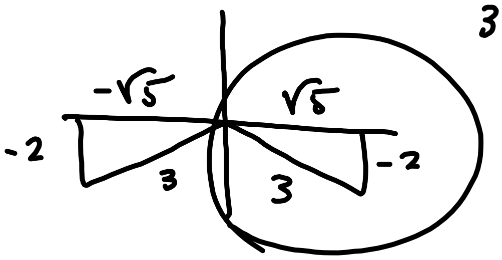
$\tan u = -\frac{3}{8}$ $\quad$ $\cot u = -\frac{8}{3}$

2, 3, 5, 7, | 11, 13, 17, 19, 23, 29, 31, 37

① $\tan u = -\frac{3}{8}$, $\sin(u) < 0$

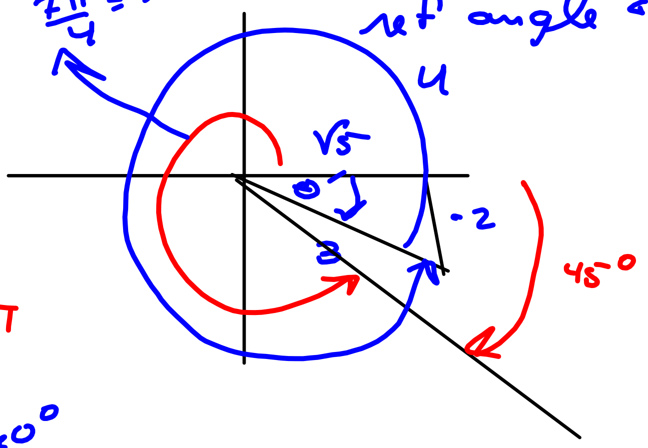
$\sin u = -\frac{2}{3}$ & $\tan u < 0$

$3^2 - 2^2 = 5$



$\frac{7\pi}{4} = 315^\circ$

$2 < \sqrt{5}$, so ref angle $< 45^\circ$



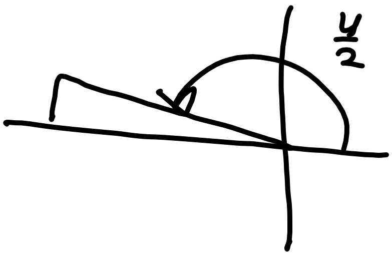
Where's $\frac{u}{2}$?

$\frac{7\pi}{4} < u < 2\pi$

$315^\circ < u < 360^\circ$

$\frac{7\pi}{8} < \frac{u}{2} < \pi$

$157.5^\circ < \frac{u}{2} < 180^\circ$



$\frac{315}{2}$

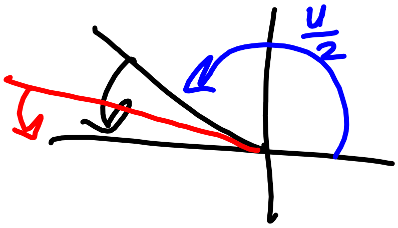
Quick way

You know $\frac{3\pi}{2} < u < 2\pi$

$270^\circ < u < 360^\circ$

$\frac{3\pi}{4} < \frac{u}{2} < \pi$

$135^\circ < \frac{u}{2} < 180^\circ$



$\frac{u}{2} \in QII$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\cos u}{2}}$$

$$= -\sqrt{\frac{1+\frac{\sqrt{5}}{3}}{2}} = -\sqrt{\frac{\frac{3+\sqrt{5}}{3}}{2}}$$

$$= -\sqrt{\frac{3+\sqrt{5}}{6}}$$

$$\left(\frac{3+\sqrt{5}}{3}\right)\left(\frac{1}{2}\right)$$

$$\sqrt{\frac{3+\sqrt{5}}{6}\cdot\frac{6}{6}} = \sqrt{\frac{18+6\sqrt{5}}{36}}$$

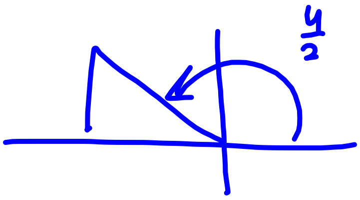
$$= \frac{-\sqrt{3+\sqrt{5}}}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}$$

$$\boxed{= \frac{-\sqrt{18+6\sqrt{5}}}{6}}$$

$$\sqrt{6}\sqrt{3+\sqrt{5}} = \sqrt{6(3+\sqrt{5})}$$

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{\sqrt{5}}{3}}{2}} = \sqrt{\frac{\frac{3-\sqrt{5}}{3}}{2}}$$

$\frac{u}{2}$



$$= \sqrt{\frac{3-\sqrt{5}}{6}}$$

$$= \sqrt{\frac{6\cdot 3-6\sqrt{5}}{6\cdot 6}}$$

$$= \boxed{\frac{\sqrt{18-6\sqrt{5}}}{6}}$$

$$1-\frac{\sqrt{5}}{3} = \frac{1}{1}\cdot\frac{3}{3}-\frac{\sqrt{5}}{3}$$

$$= \frac{3-\sqrt{5}}{3}$$

Don't simplify.

$$\tan\left(\frac{u}{2}\right) = \frac{\sqrt{18-6\sqrt{5}}}{6}\cdot\frac{6}{\sqrt{18+6\sqrt{5}}} = \boxed{\sqrt{\frac{18-6\sqrt{5}}{18+6\sqrt{5}}}}$$

Simplify $\tan\frac{u}{2}$ for bonus:

$$\frac{\sqrt{18-6\sqrt5}}{\sqrt{18+6\sqrt5}}\cdot\frac{\sqrt{18+6\sqrt5}}{\sqrt{18+6\sqrt5}}=\frac{\sqrt{324-180}}{18+6\sqrt5}$$

$$=\frac{\sqrt{144}}{18+6\sqrt5}=\frac{12}{18+6\sqrt5}\cdot\frac{18-6\sqrt5}{18-6\sqrt5}$$

$$\begin{array}{r}36\\ \underline{5}\\ \end{array}\qquad \begin{array}{r}324\\ \underline{180}\\ 144\end{array}$$

$$\frac{12(18-6\sqrt5)}{324-180}$$

$$=\frac{(216-72\sqrt5)}{144}$$

$6\sqrt5\cdot6\sqrt5$

$=36\cdot5$

$=180$

$2\,|\,144$, $2\,|\,72$, $2\,|\,36$, $2\,|\,18$, $3\,|\,9$, 3

$2\,|\,216$, $2\,|\,108$, $2\,|\,54$, $3\,|\,27$, $3\,|\,9$, 3

$2\,|\,72$, $2\,|\,36$, $2\,|\,18$, $3\,|\,9$, 3

$$=\frac{72(3-\sqrt5)}{72\cdot2}=\boxed{\frac{3-\sqrt5}{2}}$$

3) $3\csc^3 x - 6\csc^2 x - 4\csc x + 8 = 0$

Let $u = \csc(x)$

$3u^3 - 6u^2 - 4u + 8 = 0$

Cubic Factoring on test ?!

$3u^2(u-2) - 4(u-2) = 0$

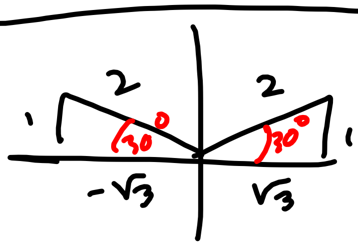
$(u-2)(3u^2-4) = 0$

$u - 2 = 0$ OR $3u^2 - 4 = 0$

$u = 2$

$\csc x = 2$

$\sin x = \frac{1}{2}$



$x = 30°, 150°$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\pi - \frac{\pi}{6}$

$(\sqrt{3}u)^2 = 2^2$

$(\sqrt{3}u - 2)(\sqrt{3} + 2)$

$\sqrt{3}u - 2 = 0$ OR $\sqrt{3}u + 2 = 0$

$\sqrt{3}u = 2$

$u = \frac{2}{\sqrt{3}}$

$\csc x = \frac{2}{\sqrt{3}}$

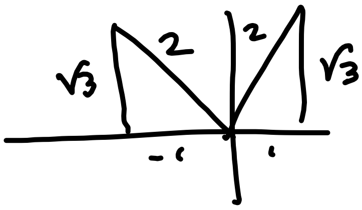
$\sin x = \frac{\sqrt{3}}{2}$

$\sqrt{3}u = -2$

$u = -\frac{2}{\sqrt{3}}$

$\csc x = -\frac{2}{\sqrt{3}}$

$\sin x = -\frac{\sqrt{3}}{2}$



$$3u^2 - 4 = 0$$
$$3u^2 = 4$$
$$u^2 = \frac{4}{3}$$
$$u = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$$
$$\csc x = \pm\frac{2}{\sqrt{3}}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$

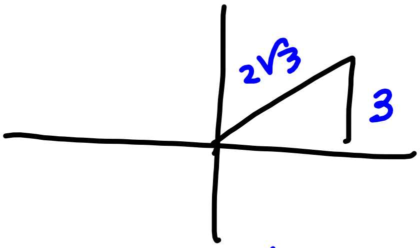
$$3u^2 - 4 = 0$$
$$3u^2 + 0u - 4 = 0$$
$$a = 3,\ b = 0,\ c = -4$$
$$b^2 - 4ac = 0^2 - 4(3)(-4)$$
$$= 48$$
$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{0 \pm \sqrt{48}}{2(3)}$$

2 | 48
2 | 24
2 | 12
2 | 6
3

$$= \frac{\pm 4\sqrt{3}}{6}$$
$$= \pm\frac{2\sqrt{3}}{3}$$
$$\csc x = \pm\frac{2\sqrt{3}}{3}$$

Like a machine

$$\boxed{\sin x = \pm\frac{3}{2\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \pm\frac{3\sqrt{3}}{2 \cdot 3} = \pm\frac{\sqrt{3}}{2}$$

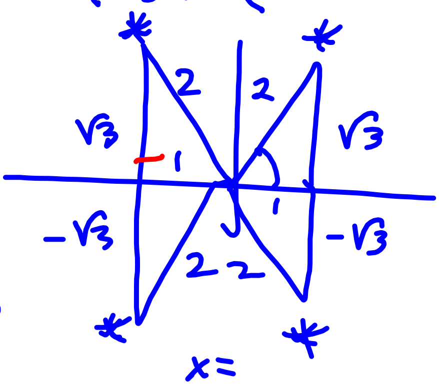


$$\arcsin\left(\frac{3}{2\sqrt{3}}\right) = 60^0$$

Degrees mode!

$$\sin^{-1}(3/(2*\sqrt{3})) = 60^0$$

Tell Marcos to be quiet.



60^0, $180^0 - 60^0 = 120^0$, $x =$ $180^0 + 60^0 = 240^0$, $360^0 - 60^0 = 300^0$

$$\boxed{\begin{array}{l} x = 60^\circ, 120^\circ, 240^\circ, 300^\circ \\ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \end{array}}$$

4b $\frac{\pi}{6} + 2n\pi, \quad n \in \mathbb{Z}$

$\frac{5\pi}{6} + 2n\pi, \quad n \in \mathbb{Z}$

$\frac{\pi}{3}$ " "

$\frac{2\pi}{3}$ " "

$\frac{4\pi}{3}$ " " $\rightarrow \frac{\pi}{3} + n\pi, \quad n \in \mathbb{Z}$

$\frac{5\pi}{3}$ " " $\rightarrow \frac{2\pi}{3} + n\pi, \quad n \in \mathbb{Z}$

$27x^3 - 8 = (3x)^3 - 2^3$

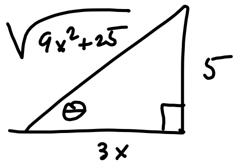
$= (3x-2)((3x)^2 + 6x + 4)$

$= (3x-2)(9x^2 + 6x + 4)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$\tan\left(\arccos\left(\frac{3x}{\sqrt{9x^2+25}}\right)\right) = \tan\theta = \frac{5}{3x}$



$\sqrt{5^2+(3x)^2} = \sqrt{25+9x^2}$

#6 is unnecessary duplication

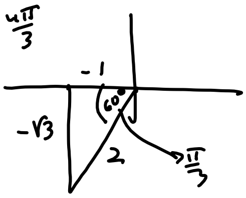
#7 $\sin\left(\frac{19\pi}{12}\right) =$

Sum identity : $\sin(u+v) = \sin u\cos v + \sin v\cos u$

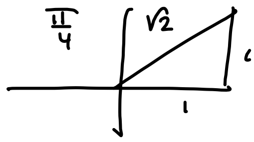
$\frac{19}{12} = \frac{18+1}{12} = \frac{17}{12}+\frac{2}{12} = \frac{16}{12}+\frac{3}{12} = \frac{4}{3}+\frac{1}{4}$

$\frac{19\pi}{12} \quad \frac{4\pi}{3}+\frac{\pi}{4}$

$\frac{4\pi}{3}$



$\frac{\pi}{4}$



$= \sin\frac{4\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{4\pi}{3}$

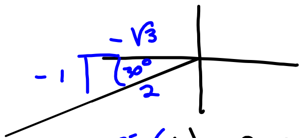
$= -\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\left(-\frac{1}{2}\right) = \boxed{\frac{-\sqrt{3}-1}{2\sqrt{2}}}$ is ok $\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

Part b. :

$\frac{u}{2} = \frac{19\pi}{12} = \frac{1}{2}\cdot\frac{19\pi}{6}$

$\pi < u < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$

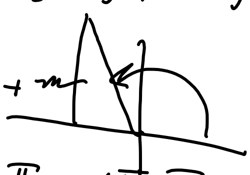


$\frac{19\pi}{6}\cdot\frac{180}{\pi}$

$\sin(u) = 2\sin\frac{19\pi}{6}\cos\frac{19\pi}{6}$

Thinking wrong. ½-angle, silly.

I found u's position, but forgot it's $3\pi +$ more



$3\pi < \frac{19\pi}{6} < \frac{7\pi}{2}$

$\boxed{\frac{3\pi}{2} < \frac{19\pi}{12} < \frac{7\pi}{4}}$

$3\pi + \frac{\pi}{2} = \frac{6\pi+\pi}{2}$

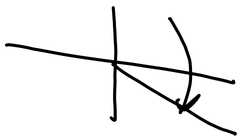
This is the hard way!

So $\frac{19\pi}{12} \in$ QIV

WE KNOW $\frac{u}{2} = \frac{19\pi}{12}$ & that's easy to locate, directly

$\sin\frac{19\pi}{12} = -\sqrt{\frac{1-\cos\frac{19\pi}{6}}{2}}$

$= \boxed{-\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}}$



$\sin < 0$