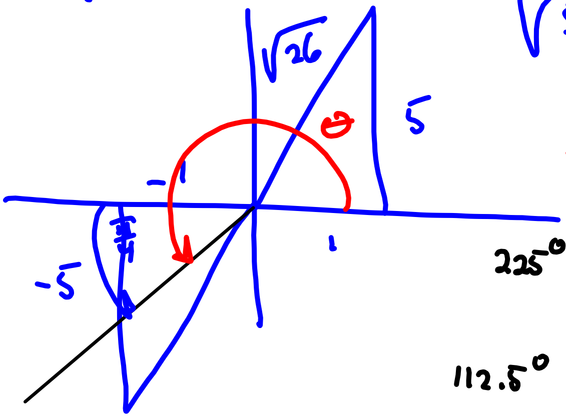
$\tan\theta = 5, \; \sin\theta < 0$

$\tan\theta = 5$

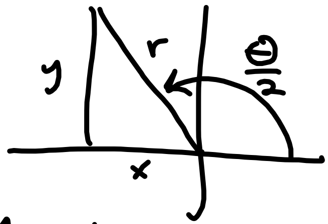
$\sqrt{5^2+1^2} = \sqrt{26}$



Find $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$ & $\tan\frac{\theta}{2}$

$225^\circ = \frac{5\pi}{4} < \theta < \frac{3\pi}{2} = 270^\circ$

$112.5^\circ = \frac{5\pi}{8} < \frac{\theta}{2} < \frac{3\pi}{4} = 135^\circ$



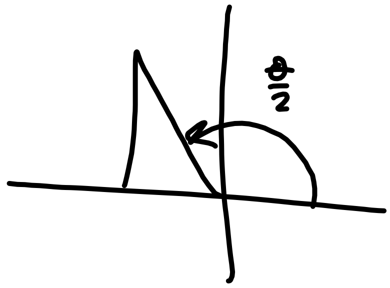
so cosine < 0
sine > 0

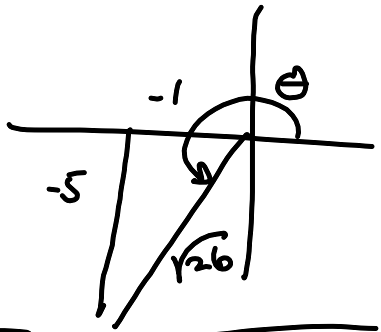
$\sin\frac{\theta}{2} = \frac{y}{r} = \frac{+}{+} = + > 0$

$\cos\frac{\theta}{2} = \frac{x}{r} = \frac{-}{+} = - < 0$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$





$$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\left(\frac{-1}{\sqrt{26}}\right)}{2}}$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{26}}}{2}} = \sqrt{\frac{\frac{1}{1}\cdot\frac{\sqrt{26}}{\sqrt{26}}+\frac{1}{\sqrt{26}}}{2}}$$

$$= \sqrt{\frac{\frac{\sqrt{26}+1}{\sqrt{26}}}{2}} = \sqrt{\frac{\sqrt{26}+1}{\sqrt{26}}\cdot\frac{1}{2}}$$

Simplified radical form

$$= \sqrt{\frac{\sqrt{26}+1}{2\sqrt{26}}} = \sqrt{\frac{(\sqrt{26}+1)}{2\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}}}$$

$$= \sqrt{\frac{26+\sqrt{26}}{2\cdot 26}} = \frac{\sqrt{26+\sqrt{26}}}{\sqrt{52}}\cdot\frac{\sqrt{52}}{\sqrt{52}}$$

$$= \frac{\sqrt{(26+\sqrt{26})(52)}}{52} = \frac{\sqrt{1352+52\sqrt{26}}}{52}$$

Scratch:

$$\sqrt{26+\sqrt{26}}\sqrt{52} = \sqrt{(26+\sqrt{26})(52)}$$

$$= \sqrt{(52)(26) + 52\sqrt{26}}$$

$$= 1352 + 52\sqrt{26}$$

$$\boxed{= \frac{\sqrt{1352+52\sqrt{26}}}{52} = \sin\frac{\theta}{2}}$$

$$\cos\frac{\theta}{2} = -\frac{\sqrt{1352-52\sqrt{26}}}{52}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos u}{\sin u} = \frac{1-\left(-\frac{1}{\sqrt{26}}\right)}{\frac{-5}{\sqrt{26}}} = \frac{1+\frac{1}{\sqrt{26}}}{\frac{-5}{\sqrt{26}}} = \frac{\frac{\sqrt{26}+1}{\sqrt{26}}}{\frac{-5}{\sqrt{26}}}$$

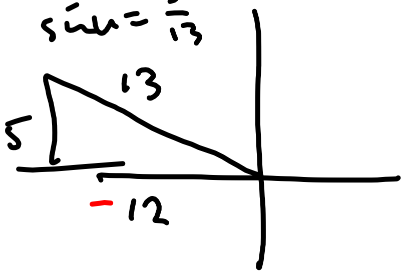
$$= \frac{\sqrt{26}+1}{\cancel{\sqrt{26}}}\cdot\left(-\frac{\cancel{\sqrt{26}}}{5}\right) = \frac{-\sqrt{26}-1}{5} = -\frac{\sqrt{26}+1}{5}$$

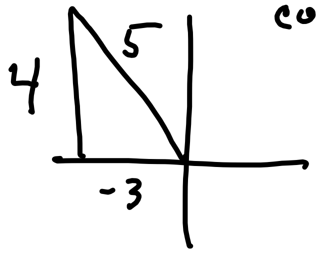
$$\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{\frac{\sqrt{1352+52\sqrt{26}}}{52}}{-\frac{\sqrt{1352-52\sqrt{26}}}{52}} = -\frac{\sqrt{1352+52\sqrt{26}}}{\sqrt{1352-52\sqrt{26}}} \quad ?!$$

$\sin u = \frac{5}{13}$, $\cos v = -\frac{3}{5}$

u & $v \in$ QII

$\sin u = \frac{5}{13}$ $\cos v = -\frac{3}{5}$





$$\sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$$

$$\sin(u+v) = \sin u \cos v + \sin v \cos u$$

$$= \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right)$$

v

$$= \frac{-15-48}{65} = \boxed{-\frac{63}{65} = \sin(u+v)}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$= \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$$

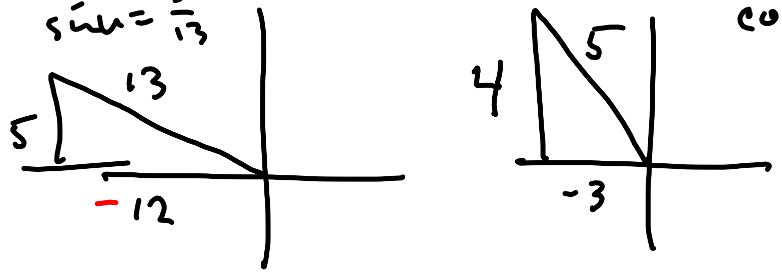
$$= \frac{36-20}{65} = \boxed{\frac{16}{65} = \cos(u+v)}$$

$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\frac{-63}{65}}{\frac{16}{65}} = -\frac{63}{65} \cdot \frac{65}{16} =$$

$$= \boxed{\frac{-63}{16} = \tan(u+v)}$$

=

$\sin u = \frac{5}{13}$ $\cos v = -\frac{3}{5}$



$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{-\frac{5}{12} + \left(-\frac{4}{3}\right)}{1 - \left(-\frac{5}{12}\right)\left(-\frac{4}{3}\right)}$$

$$= \frac{-\frac{5}{12} - \frac{4}{3}\cdot\frac{4}{4}}{1 - \frac{20}{36}} = \frac{\frac{-5-16}{12}}{\frac{36-20}{36}}$$

$$-\left(-\frac{5}{12}\right)\left(-\frac{4}{3}\right) = -\left(\frac{20}{36}\right)$$

$$= \frac{\frac{-21}{12}}{\frac{16}{36}} = -\frac{21}{12}\cdot\frac{36}{16} = -\frac{21\cdot 3}{16} = -\frac{63}{16}$$

$$\frac{\frac{-7}{4}}{\frac{4}{9}} = -\frac{7}{4}\cdot\frac{9}{4}$$

4
$\cancel{8}$
$\frac{16}{\cancel{36}}$
$\cancel{48}$
9

$$\cos(5x)\cos(2x) = \frac{1}{2}\left[\cos(7x) + \cos(3x)\right]$$

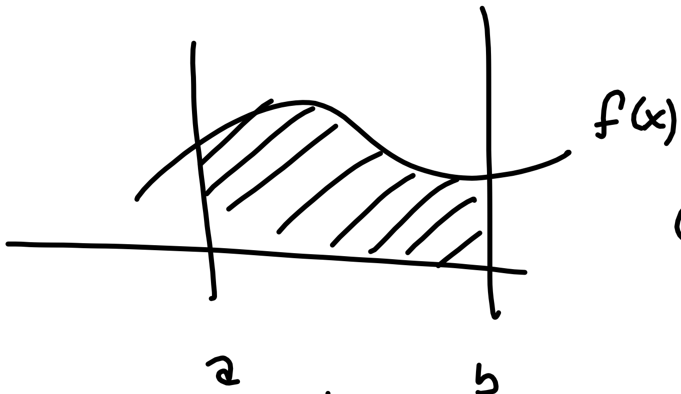
By Formula:

$$\cos u \cos v = \frac{1}{2}\left[\cos(u+v) + \cos(u-v)\right]$$

Calculus:

Area under a curve $\rightarrow$ $\displaystyle\int \cos(5x)\cos(2x)\,dx$ is impossible to work with

but $\displaystyle = \frac{1}{2}\int\left(\cos(7x) + \cos(3x)\right)dx$ is doable.



Calc Preview!

$$\text{Area} = \int_a^b f(x)\,dx$$

$3\sec^4(x)-16\sec^2(x)+16=0$.

Let $u=\sec(x)$

$$3u^4-16u^2+16=0$$

Let $v=u^2$

$$3v^2-16v+16=0$$

$$=3v^2-12v-4v+16$$

$$=3v(v-4)-4(v-4)$$

$$3v\,☺-4\,☺$$

$$=☺(3v-4)=(v-4)(3v-4)=0$$

$(3)(16)=48$

$-16=-15-1$ 15

$=-14-2$ 28

$=-13-3$ 39

$=-12-4$ 48 !

$v-4=0$

$v=4$

$u^2=4$

$u=\pm 2$

$3v-4=0$

$3u^2-4=0$

$3u^2=4$

$u^2=\frac{4}{3}$

$\sqrt{u^2}=\sqrt{\frac{4}{3}}$

$|u|=\sqrt{\frac{4}{3}}$

$u=\pm\sqrt{\frac{4}{3}}=\pm\frac{2}{\sqrt{3}}$

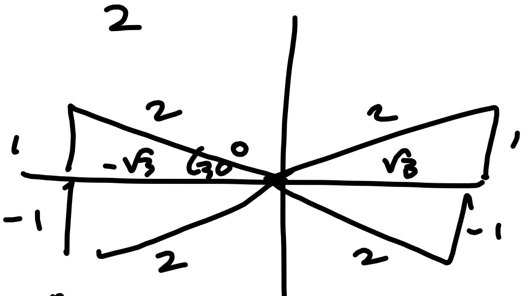
$\sec\theta=\pm\frac{2}{\sqrt{3}}$

$\cos\theta=\pm\frac{\sqrt{3}}{2}$

$\sqrt{(-3)^2}=3$

$\sqrt{3^2}=3$

$$\sqrt{u^2}=\begin{cases}u & \text{if } u\geq 0\\ -u & \text{if } u<0\end{cases}$$



$\theta=30^\circ, 150^\circ, 210^\circ, 330^\circ$

$=\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$