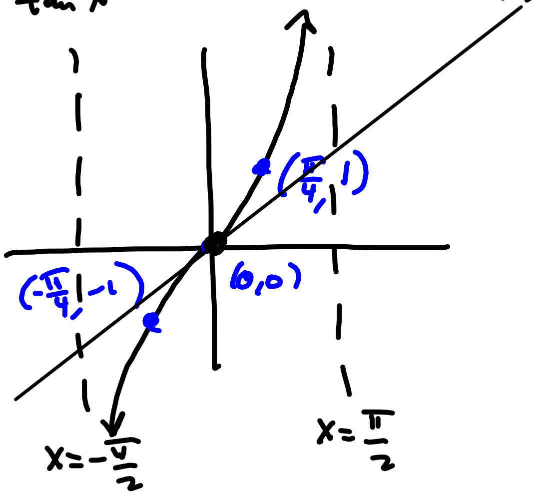
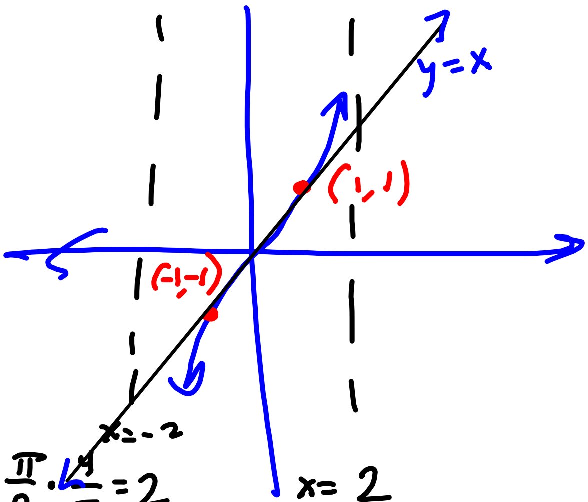
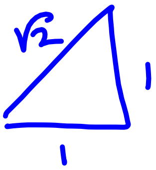
2.3#97 Find smallest possible fixed point of ... $f(x) = \tan\left(\frac{\pi x}{4}\right)$

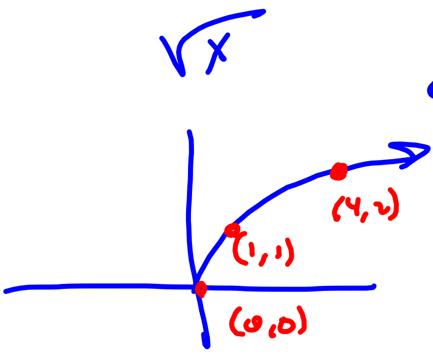


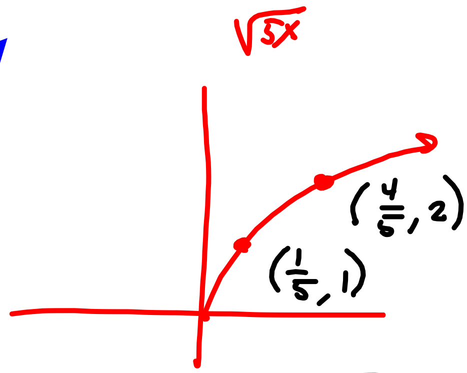


$\frac{\pi}{4} = 45°$



$$\frac{\frac{\pi}{2}}{\frac{\pi}{4}} = \frac{\pi}{2} \cdot \frac{4}{\pi} = 2$$





$$\sqrt{5\left(\tfrac{4}{5}\right)} = \sqrt{4} = 2$$

$$f(x) = \tan\left(\tfrac{\pi}{4}x\right) = x$$

Looking for points where $y = x$,
i.e., points on the graph of the line $y = x$

A fixed point of $f(x)$ is a value c, such that

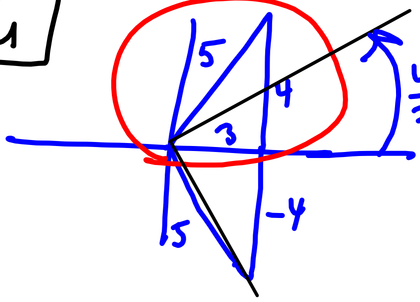
$$f(c) = c \longrightarrow$$

$$f(c) - c = 0$$

Given $\cos u = \frac{3}{5}$, find $\cos\frac{u}{2}$, $\sin\frac{u}{2}$ & $\tan\frac{u}{2}$

and $\sin u > 0$

$\boxed{u}$



$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$

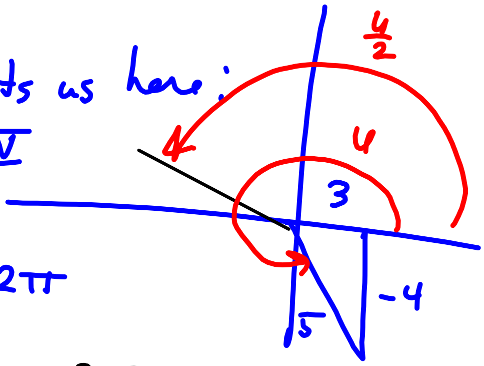
$= \sqrt{\frac{1+\cos u}{2}}$, b/c $\frac{u}{2} \in QI$

Given $\boxed{\cos u = \frac{3}{5} \;\&\; \sin u < 0}$ - - - -

$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$ $\quad \to QIV$

$\cos u = \frac{3}{5}$ & $\sin u < 0$ puts us here:

QIV



$270^\circ = \frac{3\pi}{2} < u < 360^\circ = 2\pi$

$\frac{3\pi}{4} < \frac{u}{2} < \pi$ $\qquad \frac{u}{2} \in QII$, so

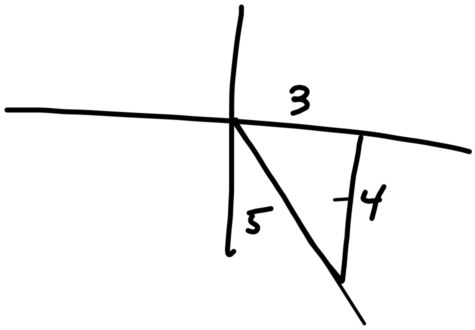
$\sin\frac{u}{2} > 0 \qquad \sin\frac{u}{2} = +\sqrt{\frac{1-\cos u}{2}}$

$\cos\frac{u}{2} < 0 \qquad \cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}}$

20% on the $\frac{1}{2}$-angle question is choosing the "+" or the "−"

Similar to 2u in terms of going from
sin u & cos u to the sin(2u), cos(2u)
You want to know what quadrant you're in.

$\cos u = \frac{3}{5}$, $\sin u < 0$. what quadrant is
2u in?



$\frac{7\pi}{4} < u < 2\pi$

$\frac{7\pi}{2} < 2u < 4\pi$

So $2u \in Q\,\overline{IV}$

So $\sin 2u < 0$

& $\cos 2u > 0$