Test is Thursday. Most of you were thru 1.7, last week, so we're on pace, here.

Evaluating an Inverse Trigonometric Function

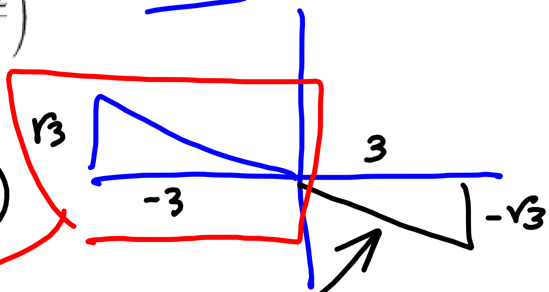
In Exercises 5–18, evaluate the expression without using a calculator.

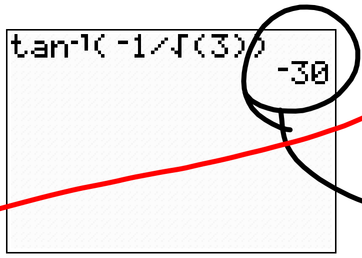
5. $\arcsin \frac{1}{2}$
6. $\arcsin 0$
7. ~~$\arccos \frac{1}{2}$~~
8. $\arccos 0$
9. $\boxed{\arctan \frac{\sqrt{3}}{3}}$
10. $\arctan 1$
11. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
12. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

§ $\tan\theta = -\frac{\sqrt{3}}{3}$ & $\sin\theta > 0$

Find the values of the 6 trigs.

2 pics

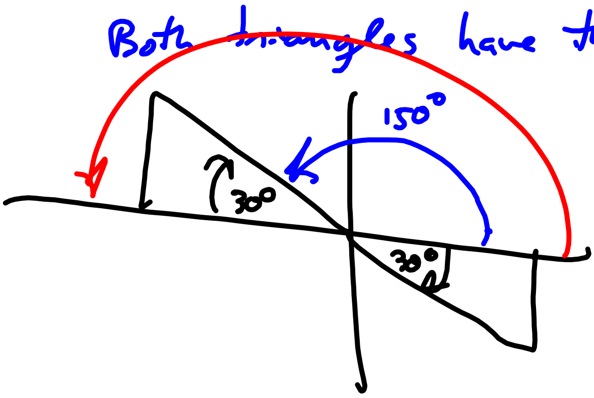




Your calculator only sees $\theta = -30°$

This The One that satisfies $\tan\theta = -\frac{\sqrt{3}}{3}$ AND $\sin\theta > 0$

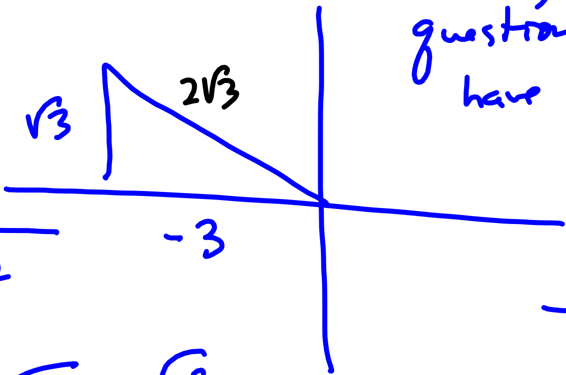
Both triangles have the same $\theta' = 30°$



So we want $180° - 30°$ gives $150°$

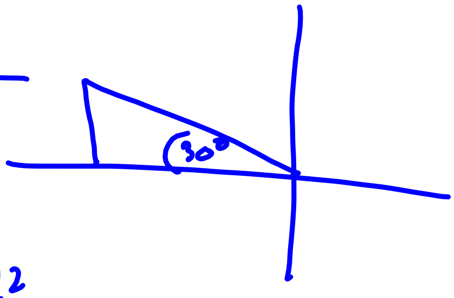
$\theta = 150^\circ$

Answering the actual question, once you have the picture.



$\sqrt{(\sqrt{3})^2 + (3)^2}$

$\sqrt{3+9} = \sqrt{12}$



$\frac{2\sqrt{3}}{\sqrt{3}}$

$2\underline{|12}$
$2\underline{|6}$
3

$\sqrt{12} = 2\sqrt{3}$

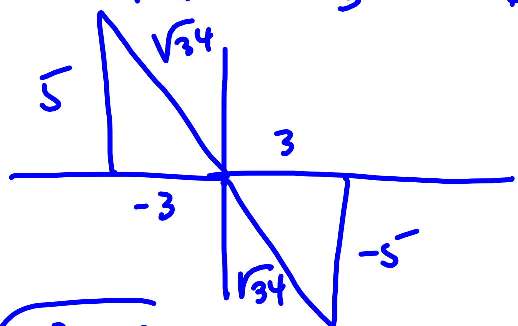
$\sin\theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$ $\csc\theta = 2$

$\cos\theta = -\frac{3}{2\sqrt{3}} = \frac{-\sqrt{3}}{2}$ $\sec\theta = \frac{-2\sqrt{3}}{3}$

$\tan\theta = -\frac{\sqrt{3}}{3}$ $\cot\theta = -\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}\cdot 3}{3} = -\sqrt{3}$

I also like to have you find θ.

$\tan\theta = -\frac{5}{3}$ & $\sin\theta > 0$



$\sqrt{5^2 + 3^2} = \sqrt{34}$

$\sin\theta = \frac{5}{\sqrt{34}}$ $\csc\theta = \frac{\sqrt{34}}{5}$

$\cos\theta = \frac{-3}{\sqrt{34}}$ $\sec\theta = -\frac{\sqrt{34}}{3}$

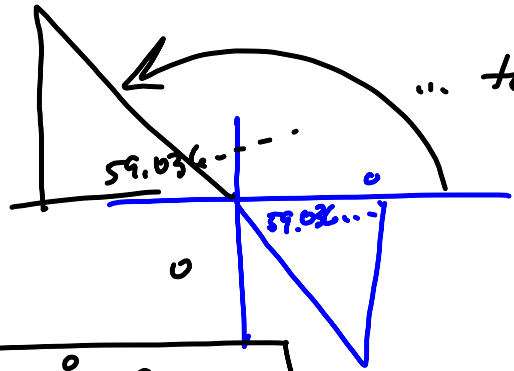
$\tan\theta = -\frac{5}{3}$ $\cot\theta = -\frac{3}{5}$

Solve $\tan\theta = -\frac{5}{3}$, $\sin\theta > 0$ in degrees & radians, for $\theta \in [0, 2\pi] = [0^\circ, 360^\circ]$

... to 4 decimal places.

```
                  -30
tan-1(-5/3)
       -59.03624347
180+Ans
        120.9637565
Ans*π/180
        2.111215827
```

```
Ans-π
       -1.030376827
```



$\theta \approx \boxed{120.9637565^\circ}$

$\theta \approx 2.111215829$

$\approx \boxed{2.1112 \approx \theta}$

Find ALL Sol'ns to $\tan\theta = -\frac{5}{3}$.

$\theta \approx -59.0362^\circ + 360^\circ n$, $\forall n \in \mathbb{Z}$ For every integer n in $\mathbb{Z}$ = integers.

$\theta \approx -1.0304 + 2n\pi$

OR

$\theta \approx 120.9638^\circ + 360^\circ n$, $\forall n \in \mathbb{Z}$.

$2.1112 + 2n\pi$

Short answer, when answers are 180° apart:

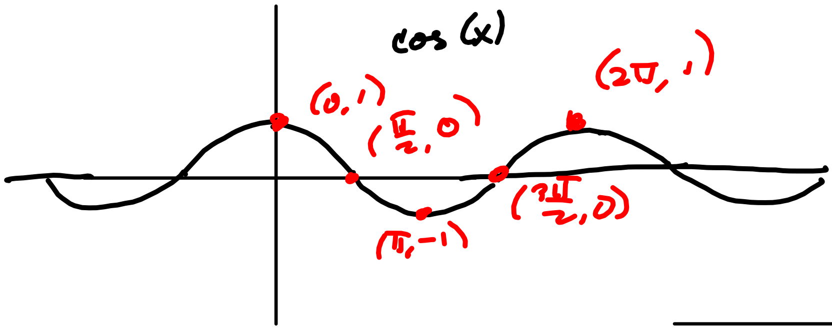
$\theta \approx -59.0362^\circ + 180^\circ n$, $\forall n \in \mathbb{Z}$.

$\theta \approx 2.1112 + n\pi$, $\forall n \in \mathbb{Z}$.

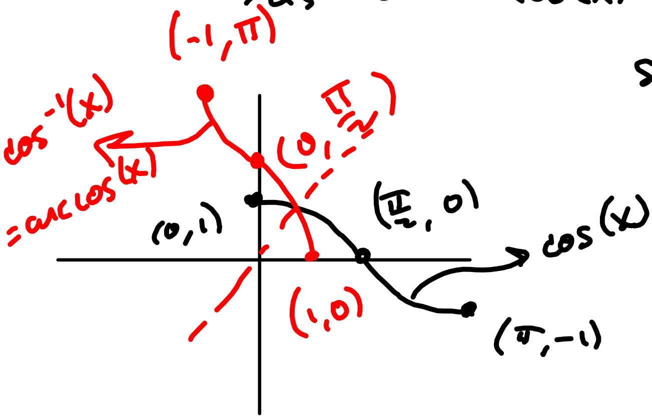
Graphing an Inverse Trigonometric Function In Exercises 19 and 20, use a graphing utility to graph f, g, and $y = x$ in the same viewing window to verify geometrically that g is the inverse function of f. (Be sure to restrict the domain of f properly.)

19. $f(x) = \cos x, \quad g(x) = \arccos x$

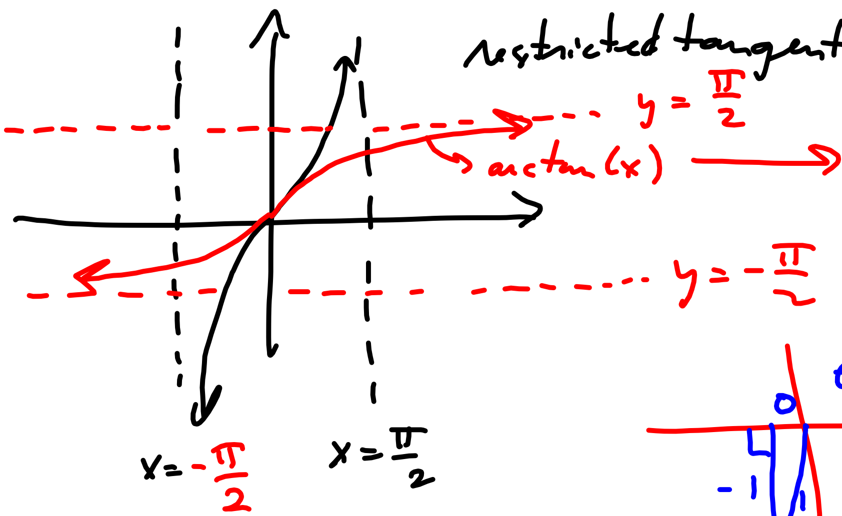
20. $f(x) = \tan x, \quad g(x) = \arctan x$



restricted cos(x) & arccos(x) on same graph

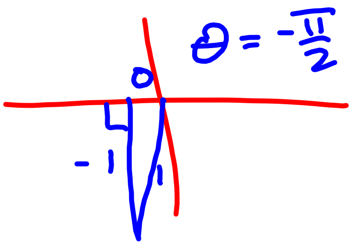


This is how you get D & R of the inverses, by knowing how to the trigs.



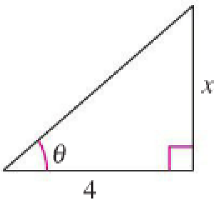
$D = (-\infty, \infty)$

$R = (-\frac{\pi}{2}, \frac{\pi}{2})$

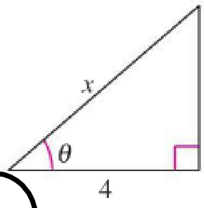


Using an Inverse Trigonometric Function In Exercises 41–46, use an inverse trigonometric function to write θ as a function of x.

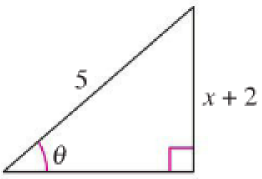
41.



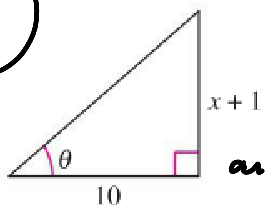
42.



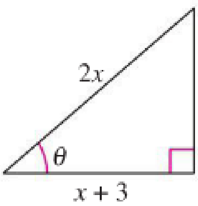
43.



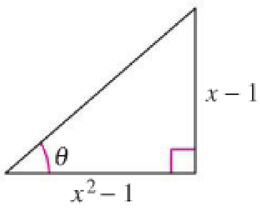
44.



45.



46.



$\tan\theta = \frac{x+1}{10}$

$\arctan(\tan\theta) = \arctan\left(\frac{x+1}{10}\right)$

$\boxed{\theta = \arctan\left(\frac{x+1}{10}\right)}$

is fine, as long as θ stays nice.

Using Inverse Properties In Exercises 47–52, use the properties of inverse trigonometric functions to evaluate the expression.

47. $\sin(\arcsin 0.3)$

48. $\tan(\arctan 45)$ $= 45$

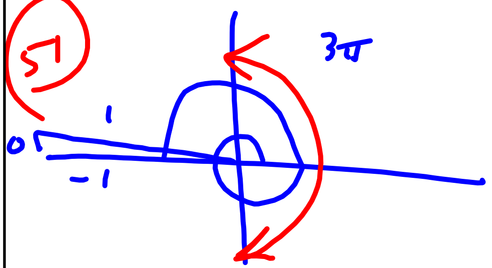
49. $\cos[\arccos(-0.1)]$

50. $\sin[\arcsin(-0.2)]$ $= -.2$

51. $\arcsin(\sin 3\pi)$

52. $\arccos\left(\cos \frac{7\pi}{2}\right)$

51

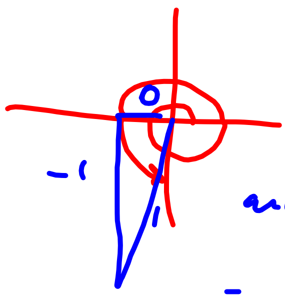


$\sin(3\pi) = 0$

$\arcsin(0) = 0$

52

$\frac{7\pi}{2} = \frac{6\pi + \pi}{2} = 3\pi + \frac{\pi}{2}$

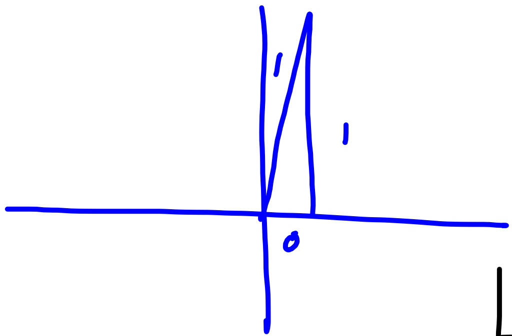


$\cos\left(\frac{7\pi}{2}\right) = 0$

so

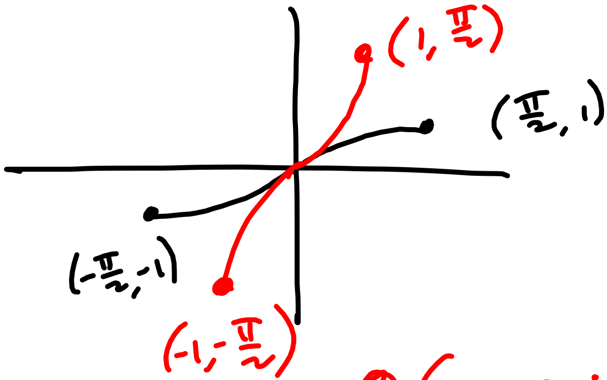
$\arccos\left(\cos \frac{7\pi}{2}\right)$

$= \frac{\pi}{2}$



Restricted cosine means arccosine only sees from $[0, \pi]$

Restricted Sine & Arcsine



$\mathcal{R}(\text{arcsine}) = [-\frac{\pi}{2}, \frac{\pi}{2}] = \mathcal{D}(\text{sine}^*)$

$\mathcal{D}(\text{arcsine}) = [-1, 1] = \mathcal{R}(\text{sine}^*)$

*The RESTRICTED sine.

Evaluating a Composition of Functions In Exercises 53–64, find the exact value of the expression. (*Hint*: Sketch a right triangle.)

$\boxed{\textbf{53. } \sin\left(\arctan \tfrac{3}{4}\right)}$

54. $\sec\left(\arcsin \tfrac{4}{5}\right)$

55. $\cos(\tan^{-1} 2)$

56. $\sin\left(\cos^{-1} \dfrac{\sqrt{5}}{5}\right)$

57. $\cos\left(\arcsin \tfrac{5}{13}\right)$

58. $\csc\left[\arctan\left(-\tfrac{5}{12}\right)\right]$

59. $\sec\left[\arctan\left(-\tfrac{3}{5}\right)\right]$

60. $\tan\left[\arcsin\left(-\tfrac{3}{4}\right)\right]$

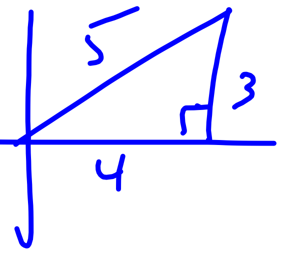
61. $\sin\left[\arccos\left(-\tfrac{2}{3}\right)\right]$

62. $\cot\left(\arctan \tfrac{5}{8}\right)$

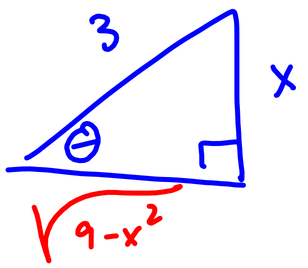
63. $\csc\left(\cos^{-1} \dfrac{\sqrt{3}}{2}\right)$

64. $\sec\left[\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)\right]$

$$\sin\left(\arctan\left(\tfrac{3}{4}\right)\right) = \tfrac{3}{5}$$



$$\tan\left(\arcsin\left(\tfrac{x}{3}\right)\right) = \tan\theta$$



$$\sqrt{3^2 - x^2} = \sqrt{9-x^2}$$

$$\rightarrow \tan\theta = \frac{x}{\sqrt{9-x^2}}$$

Writing an Expression In Exercises 65–74, write an algebraic expression that is equivalent to the given expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

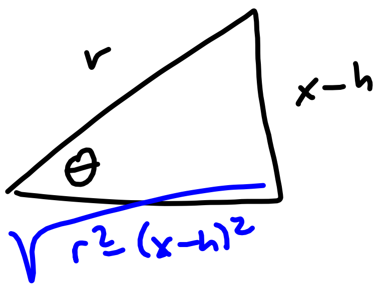
65. $\cot(\arctan x)$ **66.** $\sin(\arctan x)$

67. $\cos(\arcsin 2x)$ **68.** $\sec(\arctan 3x)$

69. $\sin(\arccos x)$ **70.** $\sec[\arcsin(x - 1)]$

71. $\tan\left(\arccos \frac{x}{3}\right)$ **72.** $\cot\left(\arctan \frac{1}{x}\right)$

73. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$ $\boxed{\textbf{74. } \cos\left(\arcsin \frac{x - h}{r}\right)}$ $= \cos\theta$



$\sqrt{r^2 - (x-h)^2}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{r^2 - (x-h)^2}}{r}$

Sketching the Graph of a Function In Exercises 83–88, sketch a graph of the function.

$f(x) = 50\sin\left(\frac{\pi}{6}x + \frac{7\pi}{6}\right) - 11$

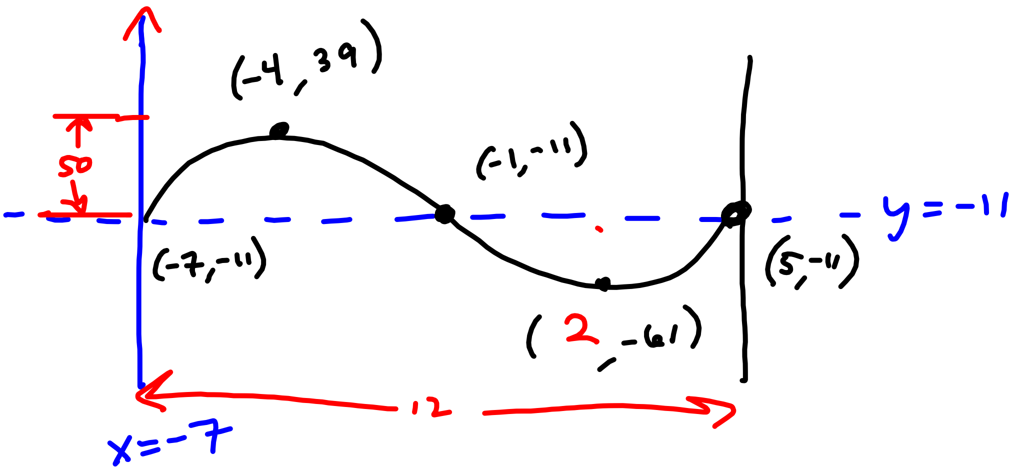
$= 50\sin\left(\frac{\pi}{6}(x+7)\right) - 11 \to y = -11$ midline

Amp: 50

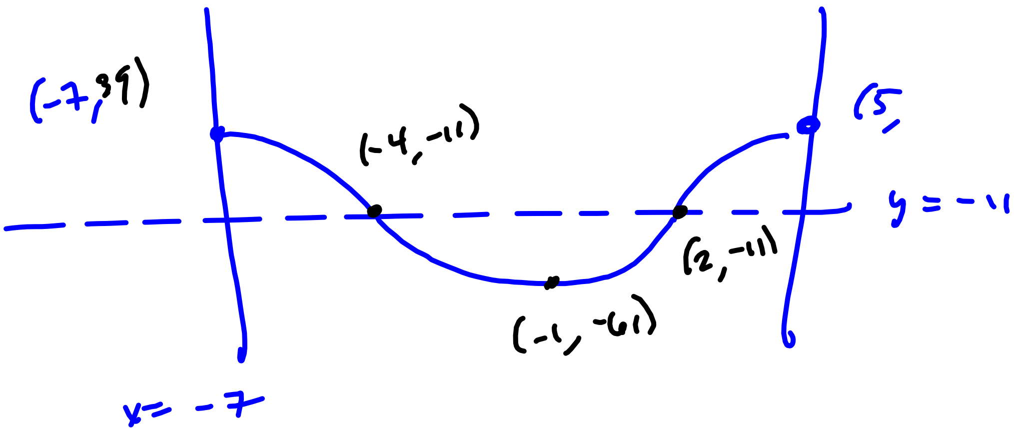
$x = -7$ starts

$\frac{\pi}{6}x = 2\pi$

$x = 12$ = Period.



$50\cos\left(\frac{\pi}{6}(x+7)\right) - 11$



Graphing an Inverse Trigonometric Function In Exercises 89–94, use a graphing utility to graph the function.

89. $f(x) = 2 \arccos(2x)$
90. $f(x) = \pi \arcsin(4x)$
91. $f(x) = \arctan(2x - 3)$
92. $f(x) = -3 + \arctan(\pi x)$
93. $f(x) = \pi - \sin^{-1}\left(\frac{2}{3}\right)$
94. $f(x) = \frac{\pi}{2} + \cos^{-1}\left(\frac{1}{\pi}\right)$