MAT 122
200 Points

Test 4 – Fall, 2017

(5 pts) Name KEY
NO GRAPHING CALCULATORS!!!

10-point deduction for each of the following: Faint writing, Lack of margin, Problems out of order, Illegibile work. Work on the back of any page will receive zero points. Other than that, we're golden. :o)

1. We convert $(x, y) = (4, -2)$ to polar coordinates, (r, θ).

   a. (15 pts) Assume $r > 0$ and $\theta \in [0, 360^0)$. Find the *exact* polar coordinates of the point. This may require leaving your answer with an 'arctan' in it. Use degrees for angle measures.

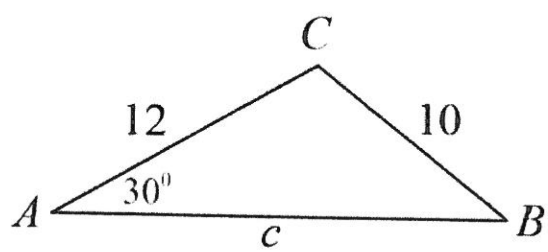
2. (15 pts) Convert $(r, \theta) = \left(8, \dfrac{11\pi}{6}\right)$ to rectangular coordinates. Give an exact answer and a decimal answer, accurate to 4 decimal places.

3. (15 pts) Sketch the graph of $r = 7\cos\theta$.

4. Consider the triangle in the figure on the right. Assume lengths are in miles.

   a. (15 pts) Find Angle B. Round final answer to 4 decimal places.

   b. (15 pts) Find side c. Round final answer to 4 decimal places.



Bonus 1. (5 pts) Find angle C. Round final answer to 4 decimal places.

5. Let $f(x) = 3x^3 - 10x^2 + 31x + 26$.

   a. (10 pts) Use synthetic division to show that $x = 2 + 3i$ is a solution of the equation $f(x) = 0$.

   b. (10 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

6. (10 pts) Find $\sin\left(\dfrac{u}{2}\right), \cos\left(\dfrac{u}{2}\right)$ and $\tan\left(\dfrac{u}{2}\right)$, given that $\cos(u) = -\dfrac{3}{7}$ and $\sin(u) < 0$.

MOAR Bonus Answer up to 3 of the following, for up to 30 bonus points.

Bonus 2. (10 pts) Find all solutions of the equation $2\sin^2(2x) - 1 = 0$ in the interval $[0, 2\pi)$.

Bonus 3. Let $z = 16\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right)$.

a. (10 pts) Express z in standard form.

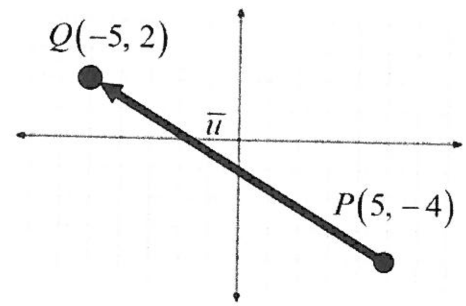
b. (10 pts) Find the principal 3rd root of z, i.e., find $\sqrt[3]{z}$. Leave z in trigonometric form for this.

c. (10 pts) Now, find the other *two* 3rd roots of z, in trigonometric form.

d. (10 pts) Finally, let $w = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$, and find the trigonometric form of the product $z \cdot w$.

Bonus 4. (10 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to $\cos(\arctan(3x))$.

Bonus 5. (10 pts) Find the direction angle of $\overline{u}$, where $\overline{u}$ is the vector corresponding to the directed line segment $\overrightarrow{PQ}$ in the figure on the right. Use degrees, rounded to 4 places.



Bonus 6. (10 pts) Build a *cosine* function that achieves its maximum height of $y = 50$ meters at time $x = 3$ seconds and its minimum height of $y = -30$ meters at $x = 27$ seconds.

Bonus 7. (10 pts) Write $z = 6 - 6\sqrt{3}\,i$ in trigonometric form, rounded to 4 decimal places. Use an angle $\theta \in [0, 2\pi)$.

122 E4 FALL '17

1. (4, −2)

a) $4^2+2^2=20$

$\sqrt{20}=2\sqrt{5}$

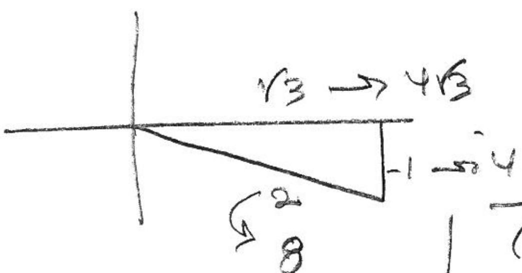
$\Rightarrow$ ~~$\boxed{\left(2\sqrt{5},\ \arctan\left(-\tfrac{1}{2}\right)\right)}$~~

oops! Want $0 \le \theta < 360^\circ$, so

$\boxed{\left(2\sqrt{5},\ 360^\circ + \arctan\left(-\tfrac{1}{2}\right)\right)}$

15 pts

2. $(r, \theta) = \left(8, \frac{11\pi}{6}\right)$



$\boxed{(x, y) = (4\sqrt{3}, -4) \approx (6.9282, -4.0000)}$

15 pts

3. $r = 7\cos\theta$

$r^2 = 49x^2$

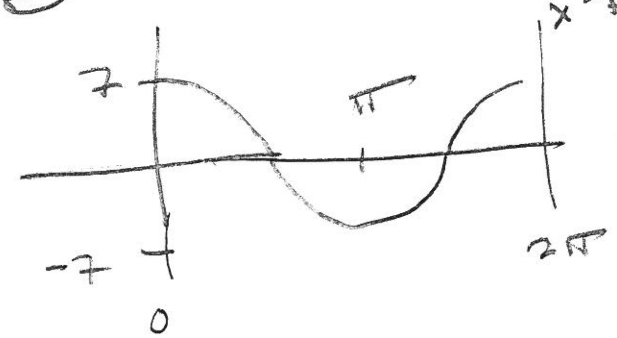
$x^2+y^2 = 49x^2$

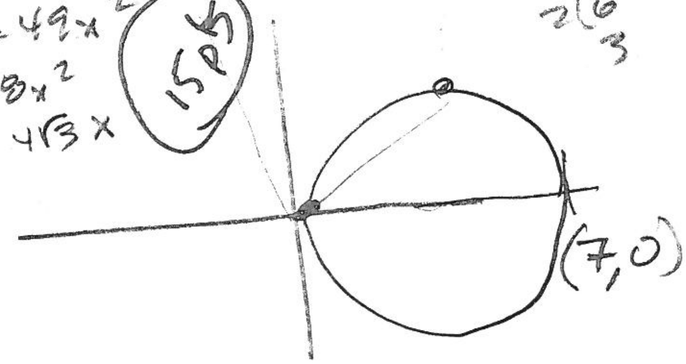
$y^2 = 48x^2$

$y = \pm 4\sqrt{3}\,x$

15 pts

2|48
2|24
2|12
2|6
3





122 E4

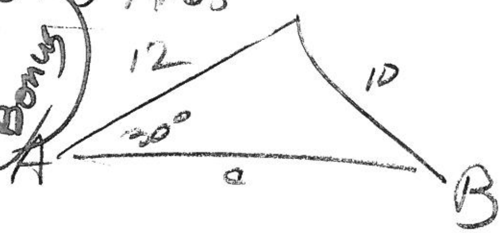
(4) $\frac{\sin B}{12} = \frac{\sin 30^\circ}{10} \longrightarrow \sin B = \frac{12 \sin 30^\circ}{10} = \frac{6}{10} = \frac{3}{5}$

$\longrightarrow B \approx 36.86989765^\circ$

(a) 15pts $\approx \boxed{36.8699^\circ \approx B}$

(b) 15pts $C \approx 180^\circ - 30^\circ - 36.86989765^\circ$

$\approx \boxed{113.1301024 \approx C}$ 5pts Bonus



$c^2 = a^2 + b^2 - 2ab\cos C$

$\approx 10^2 + 12^2 - 2(10)(12)\cos C$

$\approx 100 + 144 - 240(-.392820323)$

$\approx 244 + 94.27687753$

$\approx 338.2768775 \longrightarrow$

$\longrightarrow \boxed{c \approx 18.3923 \text{ mi}}$

$c = \frac{a}{\sin A} \cdot \sin C$

$\approx \left(\frac{10}{\frac{1}{2}}\right)(.9196152420)$

$\approx 20(.9196152420)$

Bonus 5pts $\boxed{C \approx 113.1301^\circ}$

5 (a) 10 pts $f(x) = 3x^3 - 10x^2 + 31x + 26$

$$\begin{array}{r|rrrr} 2+3i & 3 & -10 & 31 & 26 \\ & & 6+9i & -35+ & -26 \\ \hline & 3 & -4+9i & -4+6i & 0 \end{array}$$

$(-4+9i)(2+3i) = -8 - 12i + 18i - 27$

$= -35 + 6i$

$-2(2+3i)(2-3i) = -2(2^2+3^2) = -2(4+9) = -26$

(b) 10 pts

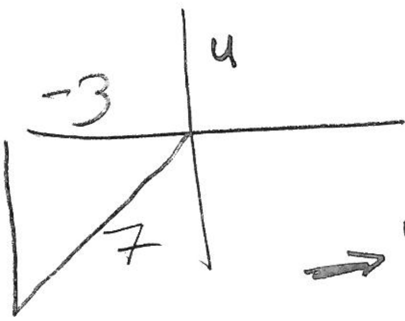
$$\begin{array}{r|rrr} 2-3i & 3 & -4+9i & -4+6i \\ & & 6-9i & 4-6i \\ \hline & 3 & 2 & 0 \end{array}$$

So, $\boxed{f(x) = (x-(2+3i))(x-(2-3i))(3x+2)}$

$\cdot 122 \quad E4$

6 10pts

$\cos u = -\frac{3}{7}, \ \sin u < 0$



$-\sqrt{58}$

$49+9=58 \mid 2$
29

$\pi < u < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$

$\Longrightarrow \frac{u}{2} \in QII$

$\sin > 0$
$\cos < 0$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\left(-\frac{3}{7}\right)}{2}}$$

$$= \sqrt{\frac{1+\frac{3}{7}}{2}} = \sqrt{\frac{\frac{10}{7}}{2}} = \boxed{\sqrt{\frac{5}{7}} = \frac{\sqrt{35}}{7} = \sin\left(\frac{u}{2}\right)}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\left(-\frac{3}{7}\right)}{2}} = \boxed{-\sqrt{\frac{2}{7}} = -\frac{\sqrt{14}}{7} \approx \cos\left(\frac{u}{2}\right)}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sqrt{35}}{7} \cdot \left(-\frac{7}{\sqrt{14}}\right) = -\sqrt{\frac{35}{14}} \boxed{= -\sqrt{\frac{5}{2}} = -\frac{\sqrt{10}}{2} = \tan\left(\frac{u}{2}\right)}$$

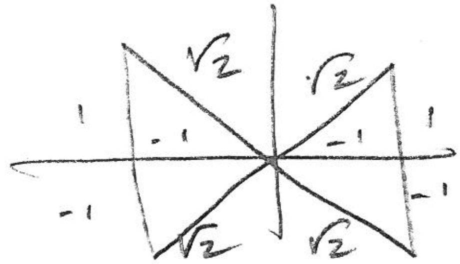
122 E4

B2 $2\sin^2(2x) - 1 = 0$

$\sin^2(2x) = \frac{1}{2}$

$\sin(2x) = \pm \frac{1}{\sqrt{2}}$

2x pic =



$0 \le x < 2\pi \longrightarrow$

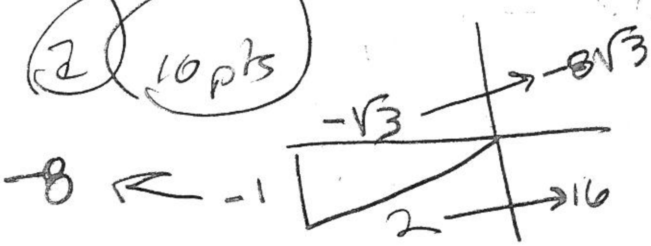
$0 \le 2x < 4\pi$

$2x \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4} \right\}$

$\rightarrow \boxed{x \in \left\{ \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \right\}}$

B3 $z = 16\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

(a) 10pts



$(x, y) = (-8\sqrt{3}, -8)$

so $\approx -13.85640646 - 8i$

$\boxed{z = -8\sqrt{3} - 8i}$

$= x + yi$

(b) 10pts

$\sqrt[3]{z} = \boxed{16^{\frac{1}{3}}\left(\cos\frac{7\pi}{18} + i\sin\frac{7\pi}{18}\right)}$

$\approx 2.519842100\left(\cos\left(\frac{7\pi}{18}\right) + i\sin\left(\frac{7\pi}{18}\right)\right)$

/122 E4

B3 c $\frac{2\pi}{3}$ = increment $= \frac{12\pi}{18}$

$$\frac{7\pi}{18} + \frac{12\pi}{18} = \frac{19\pi}{18}$$

$$\frac{19\pi}{18} + \frac{12\pi}{18} = \frac{31\pi}{18}$$

Check: $\frac{31\pi}{18} + \frac{12\pi}{18} = \frac{43\pi}{18} = \left(\frac{36+7}{18}\right)\pi$

$= 2\pi + \frac{7\pi}{18}$ ✓

So, the other 2 are:

$$\boxed{\begin{array}{l} \sqrt[3]{16}\left(\cos\left(\frac{19\pi}{18}\right) + i\sin\left(\frac{19\pi}{18}\right)\right) \\ \sqrt[3]{16}\left(\cos\left(\frac{31\pi}{18}\right) + i\sin\left(\frac{31\pi}{18}\right)\right) \end{array}}$$

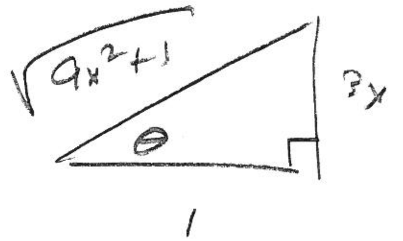
d 10pts $w = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\longrightarrow$ $\boxed{zw = 32\left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right)}$

$$\frac{\pi}{6} + \frac{7\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

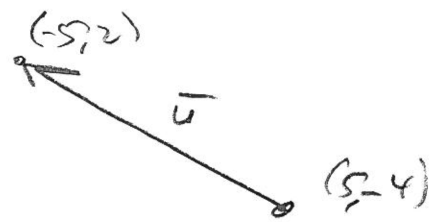
· 122 E4

B4 $\cos(\arctan(3x)) = \cos\theta$ $\boxed{= \dfrac{1}{\sqrt{9x^2+1}}}$



10 pts

B5



$\bar{u} = \langle -5-5,\ 2-(-4) \rangle$

$= \langle -10, 6 \rangle$

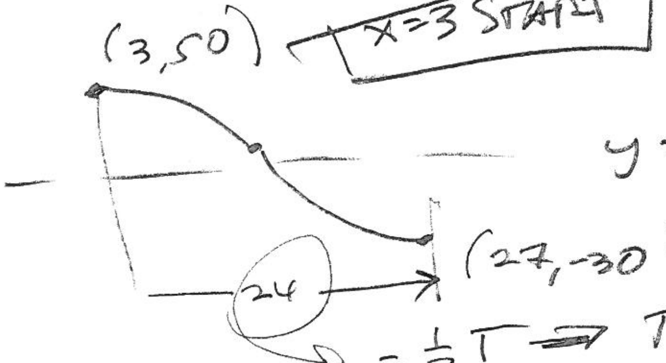
$\arctan\left(\frac{6}{-10}\right) = \arctan\left(-\frac{3}{5}\right)$

So $\theta = 180^\circ + \arctan\left(-\frac{3}{5}\right)$

$\approx 149.0362435^\circ$

$\approx \boxed{149.0362^\circ \approx \theta}$

B6



$\boxed{x = 3 \text{ START}}$

$\frac{50-30}{2} = \frac{20}{2} = \boxed{10 = \text{mid}}$

$y = \frac{50+30}{2} = \frac{80}{2} = \boxed{40 = A}$

$= \frac{1}{2}T \Rightarrow T = 48$ & "$\frac{\pi}{24}$"

$$\boxed{f(x) = 40\cos\left(\frac{\pi}{24}(x-3)\right) + 10}$$

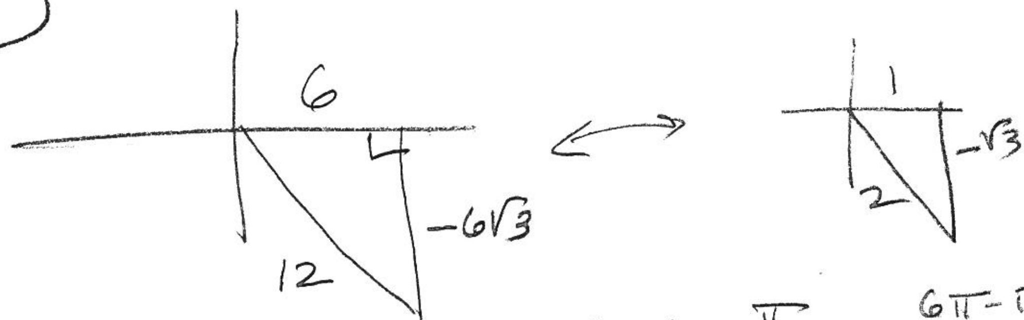
122 E4

B7

10pts

$z = 6 - 6\sqrt{3}\, i$

$\theta \in [0, 2\pi)$



$$2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$

So, $\boxed{z = 12\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)}$