$expand((2 \cdot x - \text{sqrt}(3) \cdot I - 2) \cdot (2 \cdot x + \text{sqrt}(3) \cdot I - 2) \cdot (x-5) \cdot (x+2))$

$$= 4x^4 - 20x^3 - 9x^2 + 59x - 70$$

Decent Survey of Theory in
Test-Prep Videos
Test3
Test3 Takehome

Graph suggests $x=-2$ & $x=5$ are good guesses.

harryzaims.com is being upgraded..)

$(x+2)(4x^3-20x^2+47x-35)$

$$\begin{array}{r|rrrrr} -2 & 4 & -20 & -9 & 59 & -70 \\ & & -8 & 56 & -94 & 70 \\ \hline 5 & 4 & -28 & 47 & -35 & 0 \\ & & 20 & -40 & 35 & \\ \hline & 4 & -8 & 7 & 0 & \end{array}$$

Sweet! Sweet!

$(x+2)(x-5)(4x^2-8x+7)$

$4x^2-8x+7$

$a=4,\ b=-8,\ c=7$

$$b^2-4ac = (-8)^2 - 4(4)(7)$$

$$= 64 - 112 = -48 \Rightarrow 2 \text{ nonreal roots (zeros)}$$

$$\begin{array}{r} 7 \\ 4\ \ 16 \\ 7 \\ \hline 11\ \ 2 \end{array}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{8 \pm \sqrt{-48}}{2(4)}$$

$$= \frac{8 \pm 4i\sqrt{3}}{8}$$

$$= \frac{4(2 \pm i\sqrt{3})}{8}$$

$$= \frac{2 \pm i\sqrt{3}}{2}$$

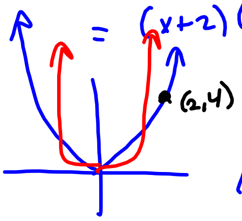
$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$\sqrt{48} = 2 \cdot 2\sqrt{3}$
$= 4\sqrt{3}$

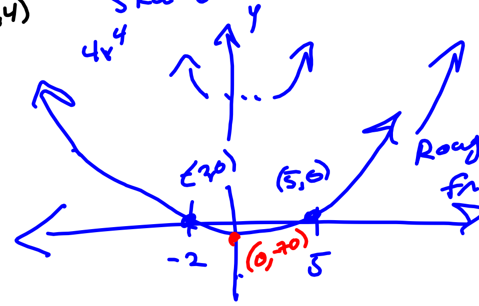
$$4(x+2)(x-5)\left(x-\left(\frac{2+\sqrt{3}i}{2}\right)\right)\left(x-\left(\frac{2-\sqrt{3}i}{2}\right)\right)$$

$= 4x^4 +$ smaller stuff

$$= (x+2)(x-5)(2x-(2+i\sqrt{3}))(2x-(2-i\sqrt{3}))$$

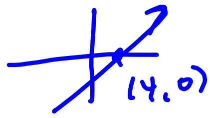


Sketch



Rough sketch from this info.

Rational Functions



$$\frac{expand((2\cdot x+3)\cdot(3\cdot x-2))}{expand((5\cdot x-20)\cdot(x+7))} \qquad \frac{6x^2+5x-6}{5x^2+15x-140}$$

$\boxed{\text{Domain: } \mathbb{R} \setminus \{-7,4\}}$ $x \neq 4$ and $x \neq -7$

is equivalent to $x \neq 4$ or 7

$$\frac{1}{x+7}$$

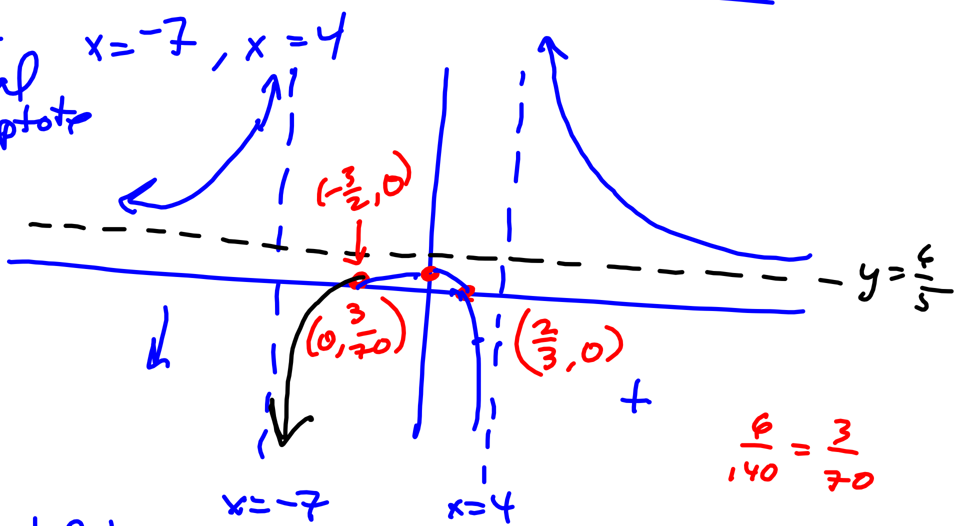
$= (-\infty,-7) \cup (-7,4) \cup (4,\infty)$

$= \{x \mid x \neq -7, 4\}$

$$\frac{6x^2+5x-6}{5x^2+15x-140}$$

V.A. $x=-7$, $x=4$

Vertical Asymptote



$\frac{6}{140} = \frac{3}{70}$

End Behavior

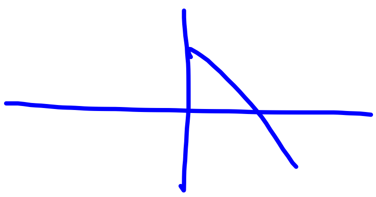
$$\frac{6x^2+\text{smaller}}{3x^2+\text{smaller}} \xrightarrow{x \to BIG} \frac{6x^2}{5x^2} = \frac{6}{5}$$

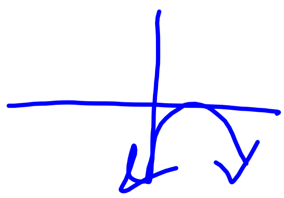
H.A.: $y=\frac{6}{5}$

Horizontal Asymptote.

y-int: $(0,+\frac{3}{70})$

x-int: $(-\frac{3}{2},0)$, $(\frac{2}{3},0)$





$$\frac{6x^2+5x-6}{5x^2+15x-140}$$

$$\begin{array}{r l} & \frac{6}{5} \quad r \; -13x+22 \\ 5x^2+15x-140 & \overline{)\,6x^2+5x-6} \\ & -\left(6x^2+18x-28\right) \\ \hline & -13x+22 \end{array}$$

$$\left(\frac{6}{\cancel{5}}\right)\left(\overset{28}{\cancel{140}}\right)$$

$$\frac{6x^2+5x-6}{5x^2+15x-14}$$

$$= \frac{6}{5} + \frac{-13x+22}{5x^2+15x-140}$$

$$\frac{expand((2\cdot x+3)\cdot(3\cdot x-2))}{expand((5\cdot x-20)\cdot(x+7))} \qquad \frac{6x^2+5x-6}{5x^2+15x-140}$$

Long division: $(3x^3-5x^2+7x-1) \div (x^2+7x)$

$$3x - 26 \quad r \quad 189x - 1$$

$$x^2+7x \;\big)\; 3x^3 - 5x^2 + 7x - 1$$

$$-(3x^3+21x^2)$$

$$-26x^2+7x-1$$

$$-(-26x^2-182x)$$

$$189x - 1$$

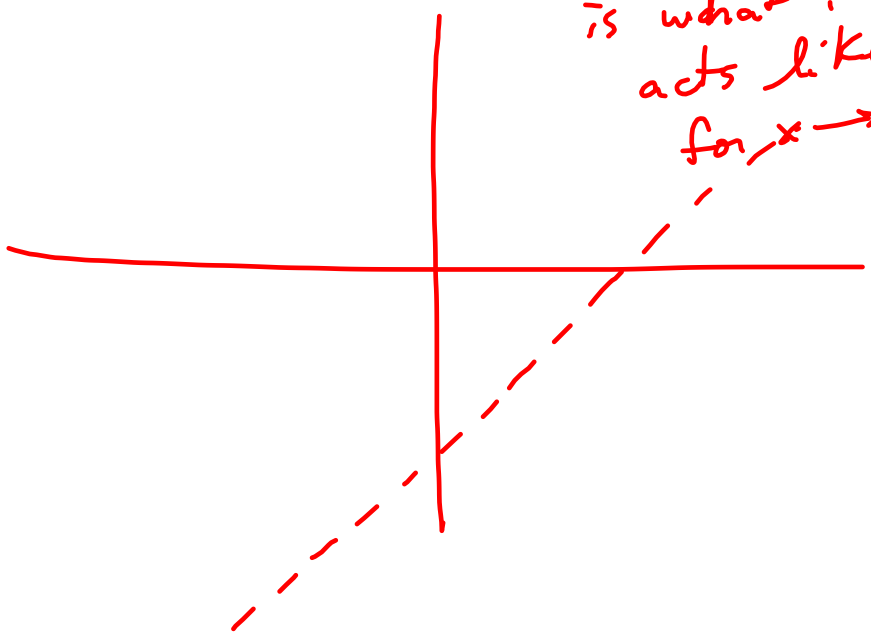
$$26 \times 7 = 182$$

This says $\boxed{\dfrac{3x^3-5x^2+7x-1}{x^2+7x}} = \boxed{3x-26} + \dfrac{189x-1}{x^2+7x}$

$$\frac{28}{3} = 9 + \frac{1}{3}$$

$\to 0$ as $x \to \pm\infty$

$y = 3x - 26$ is what it acts like for $x \to \pm$ BIG



NOT (A OR B)

≡ NOT A AND NOT B

NOT (A AND B)

≡ NOT A OR NOT B

11

Same as previous with a HOLE at x = 2

$$\frac{\text{expand}((2\cdot x+3)\cdot(3\cdot x-2)\cdot(x-2))}{\text{expand}((5\cdot x-20)\cdot(x+7)\cdot(x-2))} \quad \frac{6x^3-7x^2-16x+12}{5x^3+5x^2-170x+280}$$

V.A.: x=4, x=-7

Hole @ x=2

Rational Function with Oblique (Slant) Asymptote.

$$\frac{expand((2\cdot x+3)\cdot(3\cdot x-2)\cdot(x-5))}{expand((5\cdot x-20)\cdot(x+7))} \qquad \frac{6x^3-25x^2-31x+30}{5x^2+15x-140}$$

$$\frac{28}{3} = 9 + \frac{1}{3}$$