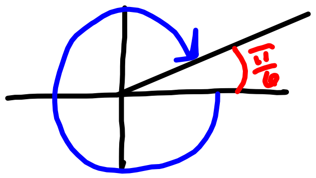
Test Thursday
Bring scientific calculator.

NO GRAPHING CALCULATOR
CELL PHONE
TI-36

1. (10 pts) Find two angles, between -2π and 2π (i.e., 0^0 and 360^0) that are coterminal with $-\frac{23\pi}{6}$. Give exact answers in degrees and radians.

$$\frac{23}{6} = \frac{18+5}{6} = 3 + \frac{5}{6}$$

$$-\frac{23\pi}{6} = -3\pi - \frac{5\pi}{6}$$



$$\boxed{\frac{\pi}{6},\ -\frac{11\pi}{6}}$$

$$-2\pi + \frac{\pi}{6} = \frac{-12\pi + \pi}{6} = -\frac{11\pi}{6}$$

$$\boxed{30^\circ,\ -330^\circ}$$

2. .(5 pts) Arc Length. Suppose we have a kid's wagon wheel of radius $r = 7$ cm. How far does the wagon wheel roll along the ground if it rotates thru an angle of 6000^0? Round to 3 decimal places.

$$s = r\theta = (7)(6000^\circ)\left(\frac{\pi}{180^\circ}\right)$$

$$\frac{\overset{100}{\cancel{600}}}{\underset{3}{\cancel{18}}}$$

$$= \frac{(7)(100)}{3}\pi \text{ cm}$$

$$= \frac{700\pi}{3} \text{ cm}$$

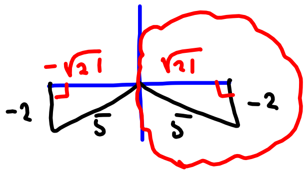
3. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = \frac{2\pi}{3}$ on a circle of radius $r = 12$ cm.

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(12)^2\left(\frac{2\pi}{3}\right) = \frac{\cancel{144}^{48}\,\pi}{\cancel{3}_{1}} = \boxed{48\pi \text{ cm}^2}$

4. Answer the questions about the equation $\csc(\theta) = -\frac{5}{2}$.

a. (5 points) Sketch two triangles that satisfy $\csc(\theta) = -\frac{5}{2}$. $\left(\sin\theta = -\frac{2}{5}\right)$



$2^2 + 5^2 = 4 + 25 = 29$

DOH!

$5^2 - (-2)^2 = 25 - 4 = 21$

b. (5 pts) Suppose that $\cos(\theta) > 0$. Find the other five trigonometric functions of θ.

$\sin\theta = -\frac{2}{5}$ $\csc\theta = -\frac{5}{2}$

$\cos\theta = \sqrt{21}/5$ $\sec\theta = 5/\sqrt{21}$

$\tan\theta = -\frac{2}{\sqrt{21}}$ $\cot\theta = -\sqrt{21}/2$

c. (5 pts) Assume $\boxed{0 \le \theta < 2\pi}$, find θ, in radians *and* degrees, rounded to 3 decimal places.

```
sin(45)
          .7071067812
sin⁻¹(-2/5)
          -23.57817848
Ans*π/180
          -.4115168461
```

$-23.578°$, $-.412$

But this isn't in $[0, 2\pi)$

This puts the answers in $[0, 360°)$ i.e. $[0, 2\pi)$

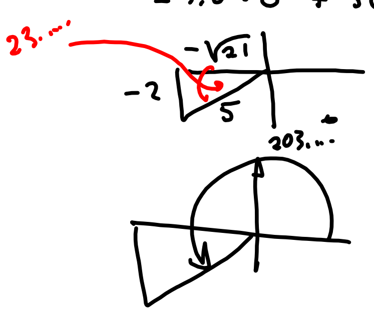
$$\boxed{\begin{gathered} 360° - 23.578 \\ 2\pi - .412 \end{gathered}}$$

is more like it.

d. (5 pts) Give *all* solutions to the equation $\csc(\theta) = -\frac{5}{2}$, in degrees *and* radians, rounded to three (3) decimal places.

$-23.578° + 360°n \quad \forall n \in \mathbb{Z}$

```
          -23.57817848
Ans*π/180
          -.4115168461
sin⁻¹(-2/5)
          -23.57817848
-180+Ans
          -203.5781785
█
```



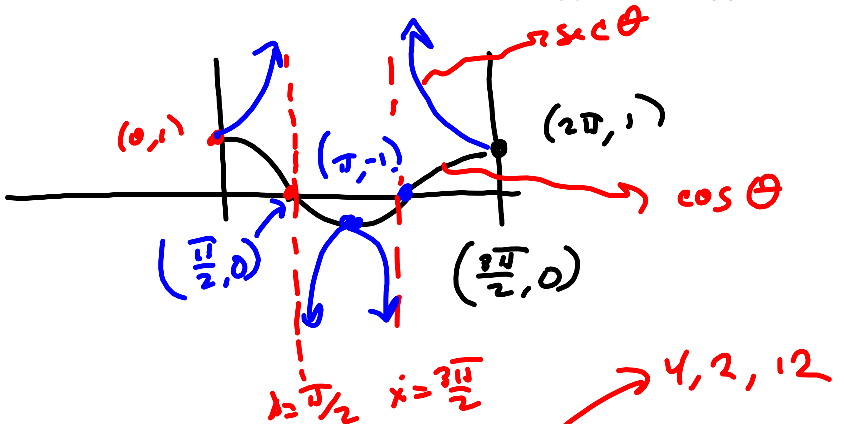
$203.578° + 360°n, \forall n \in \mathbb{Z}$

$-.412 + 2\pi n, \forall n \in \mathbb{Z}$

$3.553 + 2\pi n, \forall n \in \mathbb{Z}$

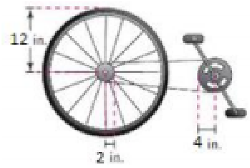
```
          -.4115168461
sin⁻¹(-2/5)
          -23.57817848
-180+Ans
          -203.5781785
Ans*π/180
          -3.5531095
█
```

5. (10 pts) Sketch one period of the graphs of $y = \cos(x)$ and $y = \sec(x)$ on the same set of coordinate axes.



→ 4, 2, 12

6. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 3 inches and 15 inches, respectively. A cyclist is pedaling at a rate of 1.4 revolutions per second. Find the speed of the bicycle in feet per second. Then convert that to miles per hour. Round final answers to 1 decimal place.



$$\underbrace{\underbrace{\left(\frac{1.4\ \cancel{\text{rev front}}}{s}\right)\left(\frac{4\ \text{rev back}}{2\ \cancel{\text{rev front}}}\right)}_{\frac{\text{rev}}{s}\text{ on back}}\left(\frac{2\pi \cdot 12\ \text{in}}{1\ \text{rev back}}\right)}_{\text{in}/s}\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$\approx \qquad \frac{\text{ft}}{s}$

$(1.4)(2)(2\pi)\left(\frac{60\ \text{mi/hr}}{88\ \text{ft/s}}\right) \approx \qquad \frac{\text{mi}}{\text{hr}}$

7. (10 pts) Sketch the graph of $f(x)=13\cos\left(\frac{\pi}{20}x+\frac{7\pi}{20}\right)+12$.

Amplitude midline

$\frac{\pi}{20}(x+7)$ start @ $x=-7$

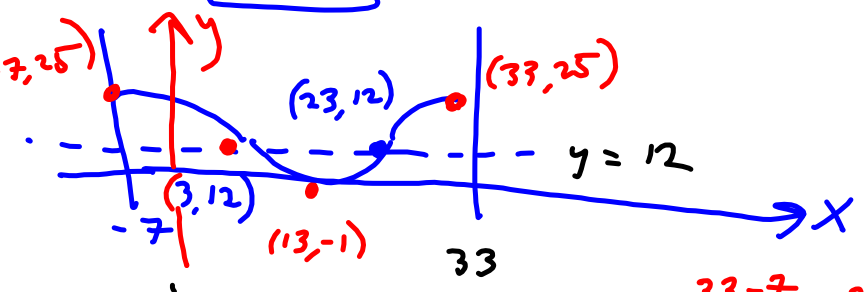
left 7

$\frac{\frac{7\pi}{20}}{\frac{\pi}{20}}=7$

Period

$\frac{\pi}{20}x=2\pi$ when?

$x=\frac{(2\pi)(20)}{\pi}=\boxed{40=T}$



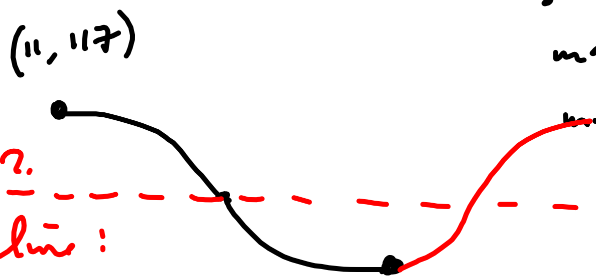
$\frac{33-7}{2}=\frac{26}{2}=13$

8. (10 pts) Write the cosine function that achieves its maximum height of $y=117$ feet at time $t=11$ seconds and its minimum height of $y=3$ feet at $t=39$ seconds.

starts @ $t=11$

max $117=y$ @ $t=11$

min $y=3$ @ $t=39$



y=?

midline:

$\frac{117+3}{2}=\frac{120}{2}=60$ (39,3)

$T=?$

$\begin{array}{r}39\\-11\\\hline 28\end{array}$ → $\boxed{T=56}$

$bx=2\pi$ when $x=56$

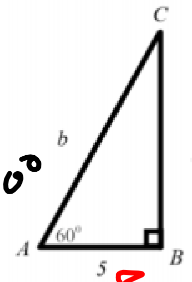
$56b=2\pi$

$b=\frac{2\pi}{56}=\frac{\pi}{28}$

$57\cos\left(\frac{\pi}{28}(x-11)\right)+60$

Amplitude: $\frac{117-3}{2}=\frac{114}{2}=57$

9. (5 pts) Solve the triangle. That means, find all lengths and angles. Exact answers required.



$A = 60°$ $\quad$ $a = 5\sqrt{3}$

$B = 90°$ $\quad$ $b = 10$

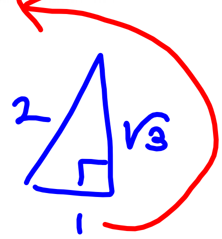
$C = 30°$ $\quad$ $c = 5$

$\frac{a}{5} = \tan 60°$

$a = 5\tan 60°$

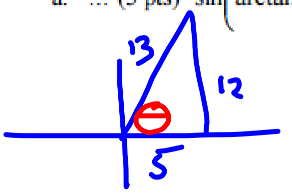
$\frac{b}{5} = \sec 60°$

$b = 5\sec 60°$



10. Find the exact value of…

a. … (5 pts) $\sin\left(\arctan\left(\frac{12}{5}\right)\right)$. $= \sin\theta = \frac{12}{13}$

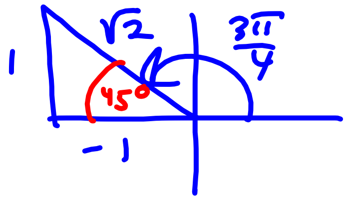


$12^2 + 5^2 =$

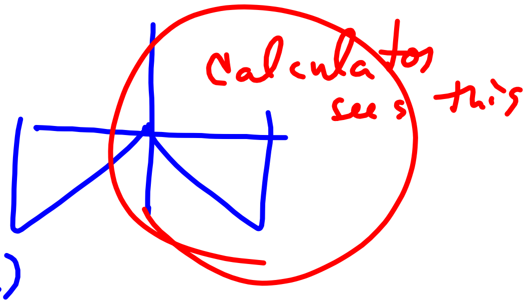
$144 + 25 = 169$

$\sqrt{169} = 13$

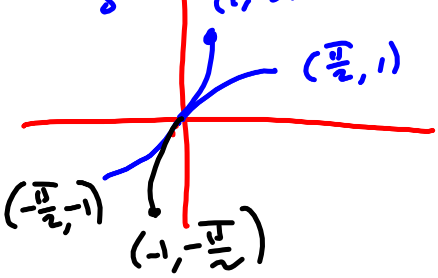
b. … (5 pts) $\arcsin\left(\cos\left(\frac{3\pi}{4}\right)\right)$ $= \arcsin\left(-\frac{1}{\sqrt{2}}\right)$



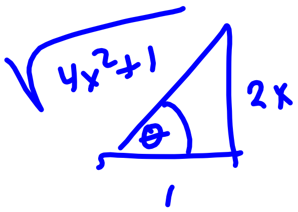
$= -\frac{\pi}{4}$, because that's all $\sin^{-1}(x)$ sees for negatives



Restricted sine



11. (5 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to $\cos(\arctan(2x))$

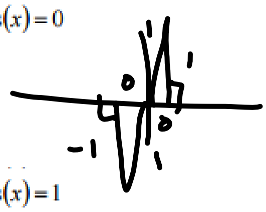


$$= \frac{1}{\sqrt{4x^2+1}}$$

Bonus: Answer *two* of the following, for *up to* 10 points:

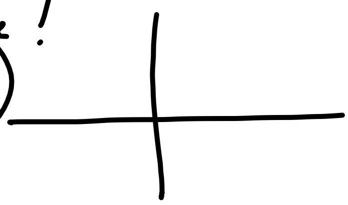
12. (5 pts) Sketch the picture(s) corresponding to the following information, *if possible*. If it is *not* possible, briefly explain why.

a. $\cos(x) = 0$

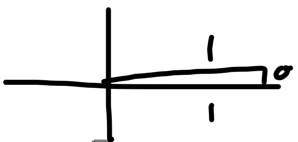


unpossible!

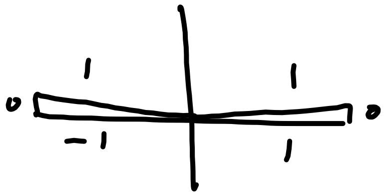
d. $\cos(x) = \frac{5}{\sqrt{7}}$



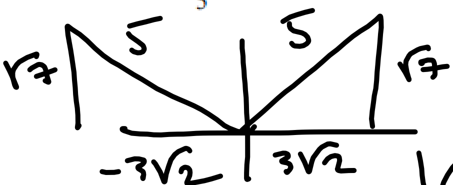
b. $\cos(x) = 1$



e. $\sin(x) = 0$

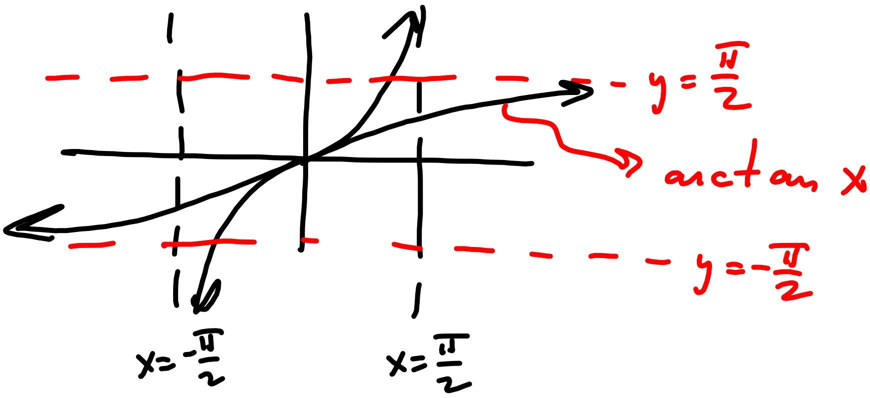


c. $\sin(x) = \frac{\sqrt{7}}{5}$

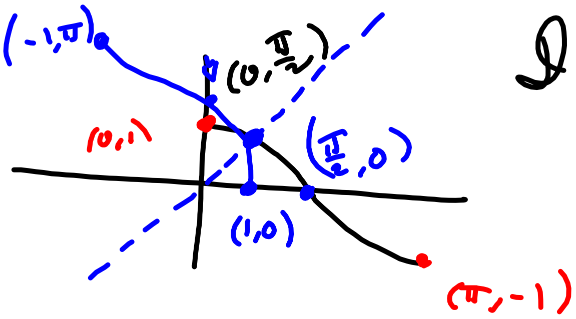


$$\sqrt{5^2 - \sqrt{7}^2} = \sqrt{25-7} = \sqrt{18}$$
$$= 3\sqrt{2}$$

13. (5 pts) Sketch the graph of one period of $y = \tan(x)$ (restricted to make it 1-to-1) *and* $y = \arctan(x)$ on the same set of coordinate axes. I want to see the function and its inverse in the same picture. Label key points as ordered pairs (ALWAYS). State the domain and range of the restricted sine function and its inverse.



14. (5 pts) Sketch the graph of one period of $y = \cos(x)$ (restricted to make it 1-to-1) *and* $y = \arccos(x)$ on the same set of coordinate axes. I want to see the function and its inverse in the same picture. Label key points as ordered pairs (ALWAYS). State the domain and range of the restricted cosine function and its inverse.



$\mathcal{D}$(cosine) = $[0, \pi]$
= $\mathcal{R}$(arccosine)

$\mathcal{R}$(cosine) = $[-1, 1]$
= $\mathcal{D}$(arccosine)

15. Explain, in your own words, how to reason to the arc-length and area-of-a-sector formulas.