MAT 122 Test 4 – Spring, 2017 Name________________

200 Points NO GRAPHING CALCULATORS!!!

10-point deduction for each of the following: Faint writing, Lack of margin, Problems out of order.

1. We convert $(x, y) = (-4, 2)$ to polar coordinates, (r, θ).

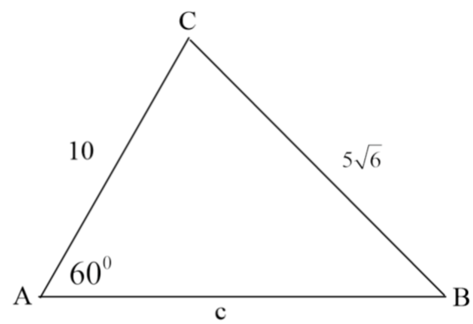
   a. (10 pts) Assume $r > 0$ and $\theta \in [0, 360^0]$. Find the *exact* polar coordinates of the point. This may require leaving your answer with an 'arctan' in it. Use degrees for angle measures.

   b. (10 pts) Approximate your answer in part a, with 4-decimal-place accuracy.

2. (10 pts) Convert $(r, \theta) = \left(7, \ \frac{5\pi}{4}\right)$ to rectangular coordinates. Give an exact answer and a decimal answer, accurate to 4 decimal places.

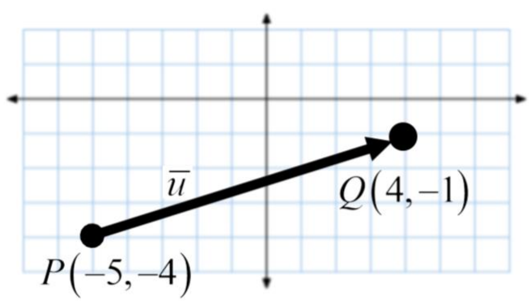
3. (10 pts) Sketch the graph of $r = 7\sin\theta$.

4. (20 pts) Solve the triangle in the figure. Assume lengths are in miles. Round your final answers to 2 places



Bonus 1. (10 pts) Give the *exact* value of side c.

5. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\bar{u}$:



   a. (10 pts) Express the vector $\bar{u} = \overrightarrow{PQ}$ in component form.

   b. (10 pts) Compute the magnitude of $\bar{u}$. Leave your answer in simplified radical form.

   c. (10 pts) Express $\bar{u}$ as a linear combination of the canonical (standard) unit vectors $\bar{i}$ and $\bar{j}$.

   d. (10 pts) Find the direction angle of $\bar{u}$. Use degrees, rounded to 4 places.

6. Let $f(x) = 2x^3 - 19x^2 + 62x - 70$.

   a. (10 pts) Use synthetic division to show that $x = 3 + i$ is a solution of the equation $f(x) = 0$.

b. (10 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

7. (10 pts) Express $z=-3-6i$ in trigonometric form.

8. Let $z=16\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)$.

a. (10 pts) Express z in standard form.

b. (10 pts) Find the principal 4th root of z, i.e., find $\sqrt[4]{z}$. Leave z in trigonometric form for this.

c. (10 pts) Now, find the other *three* 4th roots of z, in trigonometric form.

d. (10 pts) Finally, let $w=2\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)$, and find the trigonometric form of the product $z\cdot w$.

9. (10 pts) Find $\sin(2u),\cos(2u)$ and $\tan(2u)$, given that $\cos(u)=\frac{3}{7}$ and $\sin(u)<0$.

10. (10 pts) Find $\sin\left(\frac{u}{2}\right),\cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u)=\frac{3}{7}$ and $\sin(u)<0$.

11. (10 pts) Build a *cosine** function that achieves its maximum height of y = 100 meters at time x = 7 seconds and its minimum height of $y=16$ meters at x = 27 seconds.

*Last semester, I used a *sine* function, in here, which made it a little trickier to use a high and a low to build. But cosine? Much easier.

Bonus Section

Bonus 2. (10 pts) Find all solutions of the equation $2\sin(2x)-1=0$ in the interval $[0,2\pi)$.

Bonus 3. (10 pts) Sketch the graph of $f(\theta)=11\sin\left(\frac{\pi}{14}\theta-\frac{26\pi}{7}\right)+4$.

Bonus 4. Consider the triangle described by the following (See figure):

Angle $A=60^0$, side b = 18 and side a = 16.

a. (5 pts) Prove that there are two triangles fitting this description.

b. (5 pts) Find both possible values of angle B.