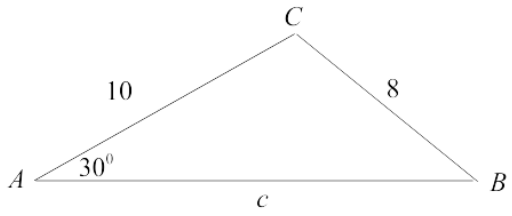
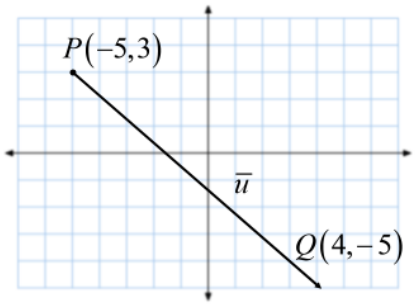
MAT 122 Test 3 – Spring-2017 Name_____
100 Points NO GRAPHING CALCULATORS!!!

1. Consider the triangle in the figure. All lengths are given in centimeters.



   a. (5 pts) Show that there are two possible triangles from the information given.
   b. (5 pts) Assume that *B* is acute. Use the Law of Sines to find the measure of Angle *B*. Round your final answer to 4 decimal places.
   c. (5 pts) Still assuming *B* is acute, use the Law of Cosines to find the length of side c. Round your final answer to 4 decimal places.

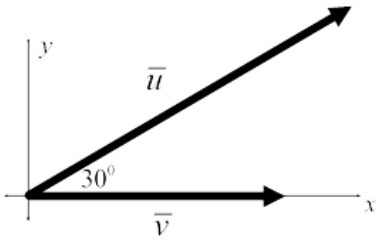
2. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\bar{u}$:



   a. (5 pts) Express the vector $\bar{u} = \overrightarrow{PQ}$ in component form.
   b. (5 pts) Compute the magnitude of $\bar{u}$. Leave your answer in simplified radical form.
   c. (5 pts) Find the direction angle of $\bar{u}$. Use degrees, rounded to 4 places.

3. Let $\bar{u} = \langle -3, 7 \rangle$.
   a. (5 pts) Express $\bar{u}$ as a linear combination of the canonical (standard) unit vectors $\bar{i}$ and $\bar{j}$.
   b. (5 pts) What's another word for the sum of 2 vectors?

4. Forces with magnitudes $\|\bar{u}\| = 90$ **N** and $\|\bar{v}\| = 60$ **N** are acting on a hook, as shown in the figure.



   a. (5 pts) Express $\bar{u}$ and $\bar{v}$ in component form.
   b. (5 pts) Express the resultant force, in component form.
   c. (5 pts) Find the direction angle of the resultant force, in degrees, rounded to 4 decimal places.

5. Let $f(x) = 5x^3 - 23x^2 + 77x - 39$.
   a. (5 pts) Use synthetic division to find $f(2)$.
   b. (5 pts) Use synthetic division to show that $x = 2 + 3i$ is a solution of the equation $f(x) = 0$.
   c. (5 pts) Find the linear factorization of f that is promised to us by the Fundamental Theorem of Algebra (, i.e., the Math gods.)

6. Let $z = -9 - 12i$
   a. (5 pts) Find $z + \bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.
   b. (5 pts) Express z in trigonometric form. Use degrees, and round the angle to the nearest degree.

7. Let $z = 1\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$.

   a. (5 pts) Express z in standard form.
   b. (5 pts) Find the principal 5th root of z, i.e., find $\sqrt[5]{z}$, in trigonometric form.
   c. (5 pts) Now, find *all* the 5th roots of z, in trigonometric form.
   d. (5 pts) Find the trigonometric form of z^3
   e. (5 pts) Finally, let $w = 5\left(\cos\left(\frac{4\pi}{7}\right) + i\sin\left(\frac{4\pi}{7}\right)\right)$, and find the trigonometric form of the product $z \cdot w$.

Answer up to as many as 20 points' worth.

B1 (5 pts) Find the area of the triangle in the 1st problem.

B2 A cannon with a muzzle velocity of 500 meters per second is fired, with an angle of 22.5^0 from the horizontal.

   a. (5 pts) Find the horizontal and vertical components of the shell, as it leaves the muzzle, accurate to the nearest meter.
   b. (5 pts) Use a half-angle formula to find the *exact* value for the answer to the previous.
   c. (5 pts) Using - 9.8 $\frac{m}{s^2}$ for the acceleration due to gravity, and neglecting air friction, predict where and when the bullet will hit the ground, in the gun question.

B3 (5 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\cos(u) = -\frac{2}{5}$ and $\sin(u) > 0$.

Use the 1st two answers to *build* the 3rd. It's *silly* to go back to your cheat sheet and deal with the mess.

B4 (5 pts) Build a **cosine** function that achieves its maximum height of y = 500 meters at time x = 7 seconds and its minimum height of $y = -8$ meters at x = 13 seconds.

B5 (5 pts) Find all solutions of the equation $2\sin^2(3x) - 1 = 0$ in the interval $[0, 2\pi)$.

B6 (5 pts) Sketch the graph of $f(x) = 500\sin\left(\frac{\pi}{14}x - \frac{13\pi}{14}\right) + 300$.