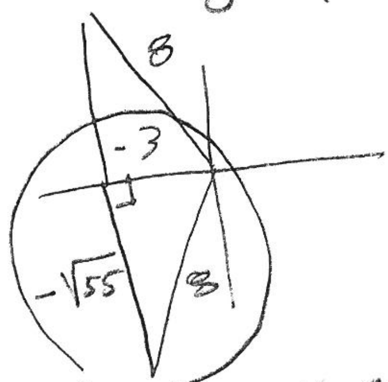
122 TEST 2

① 5pts

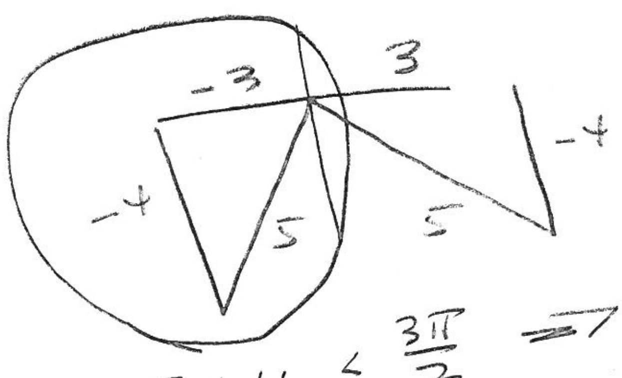
② $\cos u = -\frac{3}{8}$ & $\sin u < 0$



$8^2 - (-3)^2 = 64 - 9$
$= 55$

$$\boxed{\begin{array}{ll} \sin u = -\frac{\sqrt{55}}{8} & \csc u = -\frac{8}{\sqrt{55}} \\ \cos u = -\frac{3}{8} & \sec u = -\frac{8}{3} \\ \tan u = \frac{\sqrt{55}}{3} & \cot u = \frac{3}{\sqrt{55}} \end{array}}$$

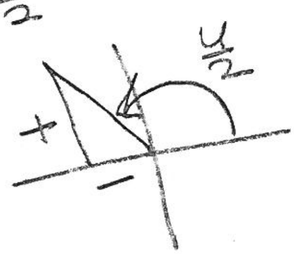
③ $\sin u = -\frac{4}{5}$ & $\cos u < 0$



$\Rightarrow$ $\boxed{\cos u = -\frac{3}{5}}$

So, $\pi < u < \frac{3\pi}{2} \Rightarrow$
$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$

Puts $\frac{u}{2}$ in QII



$\Rightarrow \sin \frac{u}{2} > 0$
& $\cos \frac{u}{2} < 0$

122 E2

3 cont'd

So, $\sin\left(\frac{u}{2}\right) = +\sqrt{\frac{1-\cos u}{2}}$

$= \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}$

$= \sqrt{\frac{1+\frac{3}{5}}{2}}$

$= \sqrt{\frac{\frac{8}{5}}{2}}$

$= \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}} = \sin\left(\frac{u}{2}\right)}$

$= \frac{2\sqrt{5}}{5}$ (optional)

$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}}$

$= -\sqrt{\frac{1-\frac{3}{5}}{2}}$

$= -\sqrt{\frac{\frac{2}{5}}{2}}$

$= -\sqrt{\frac{1}{5}} = \boxed{-\frac{1}{\sqrt{5}} = \cos\left(\frac{u}{2}\right)}$

$= -\frac{\sqrt{5}}{5}$ (optional)

$\rightarrow \boxed{\tan\left(\frac{u}{2}\right) = -2}$

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(*) (a) $3\sec^4 x - 16\sec^2 x + 16 = 0$

$3u^4 - 16u^2 + 16 = 0$

$3v^2 - 16v + 16 = 0$

$(3v-4)(v-4) = 0$

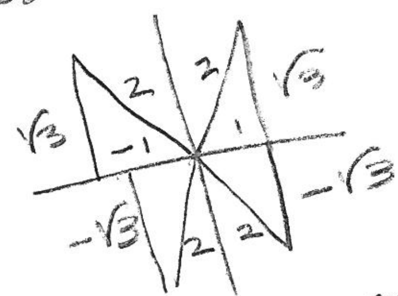
$v = \frac{4}{3}, 4$

$u^2 = \frac{4}{3}$ OR $u^2 = 4$

$u = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$ OR $u = \pm 2$

$\sec = \pm 2$

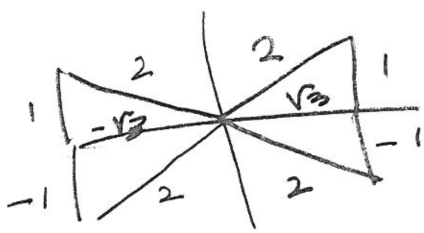
$\cos = \pm\frac{1}{2}$



$u = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\sec x = \pm\frac{2}{\sqrt{3}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

"FACTOR BY GROUPING" is a terrible hint.

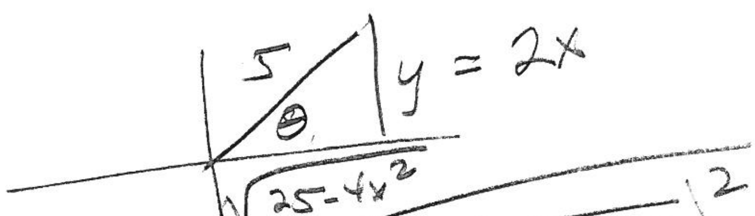
$x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

~~4b~~ 1&2 E 2

*i) Cont'd

$$\boxed{x \in \left\{ \frac{\pi}{6} + n\pi,\ \frac{5\pi}{6} + n\pi,\ \frac{\pi}{3} + n\pi,\ \frac{2\pi}{3} + n\pi \ \middle|\ n \in \mathbb{Z} \right\}}$$

4) $\sin\left(\operatorname{arcsec}\left(\frac{5}{\sqrt{25-4x^2}}\right)\right) = \sin\theta$



$$y = \sqrt{5^2 - \left(\sqrt{25-4x^2}\right)^2}$$

$$= \sqrt{25 - (25-4x^2)}$$

$$= \sqrt{4x^2}$$

$= 2|x|$, technically

we assume $x > 0$

So, $\boxed{\sin\theta = \frac{2x}{5}}$

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MAT 122 E2

(5) $\csc^2 x - 4\csc x = -4$

$u^2 - 4u + 4 = 0$

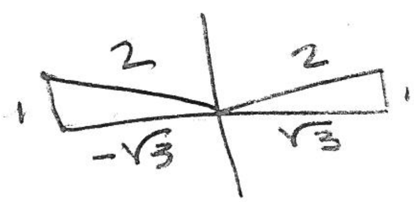
$(u-2)^2 = 0$

$u - 2 = \pm 0 = 0$

$u = 2$

$\csc x = 2$

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\boxed{x \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}}$$

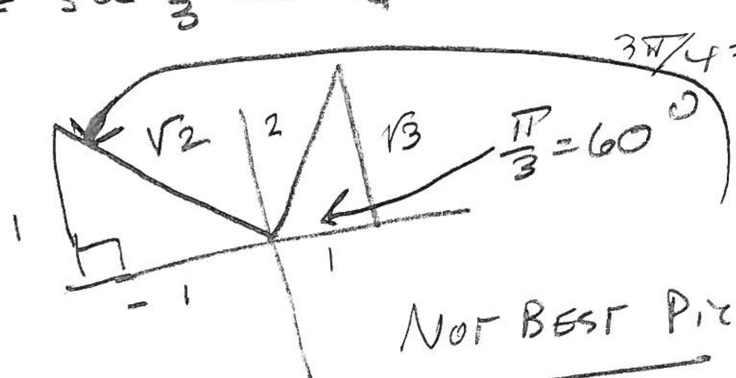
122 E2

(6) $\sin\left(\frac{13\pi}{12}\right) = ?$

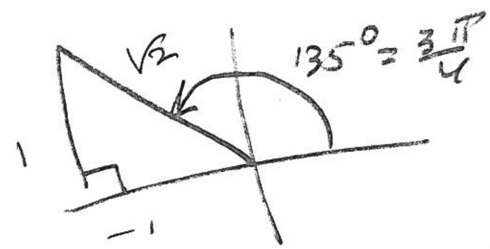
(a) $\frac{13\pi}{12} = \frac{2\pi}{12} + \frac{11\pi}{12} = \frac{4\pi}{12} + \frac{9\pi}{12} = \frac{\pi}{3} + \frac{3\pi}{4} = u + v$

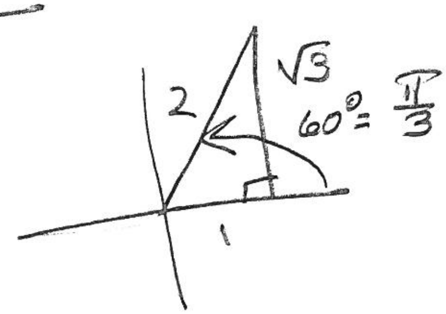
$\sin(u+v) = \sin u \cos v + \sin v \cos u$

$= \sin\frac{\pi}{3}\cos\frac{3\pi}{4} + \sin\frac{3\pi}{4} \cdot \cos\frac{\pi}{3}$



Not Best Pic.





$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \boxed{\frac{-\sqrt{3}+1}{2\sqrt{2}}}$ OR $\frac{\sqrt{2}-\sqrt{6}}{4}$

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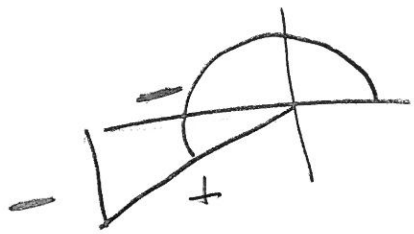
6b $\dfrac{13\pi}{12} = \dfrac{1}{2}\left(\dfrac{13\pi}{6}\right) = \dfrac{\frac{13\pi}{6}}{2} = \dfrac{u}{2}$

$\dfrac{13\pi}{12} \in Q\ \text{III}$

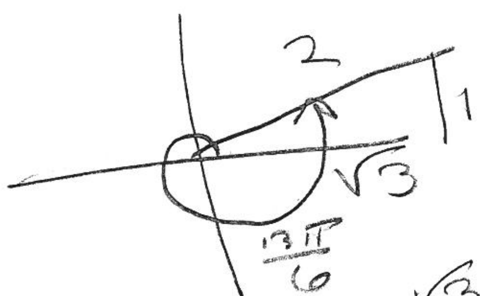


So $\sin \dfrac{13\pi}{12}$ is negative

$\sin \dfrac{13\pi}{12} = \sin\left(\dfrac{u}{2}\right) = -\sqrt{\dfrac{1-\cos u}{2}}$

$\dfrac{13\pi}{6} = \dfrac{12\pi}{6} + \dfrac{\pi}{6} = 2\pi + \dfrac{\pi}{6}$

$\dfrac{13\pi}{6} = 390^\circ$



$\cos u = \dfrac{\sqrt{3}}{2}$

So, $\sin \dfrac{u}{2}$

$= -\sqrt{\dfrac{1-\cos u}{2}}$

$= -\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}}$

$= -\sqrt{\dfrac{2-\sqrt{3}}{4}} = \boxed{-\dfrac{\sqrt{2-\sqrt{3}}}{2} = \sin\left(\dfrac{13\pi}{12}\right)}$

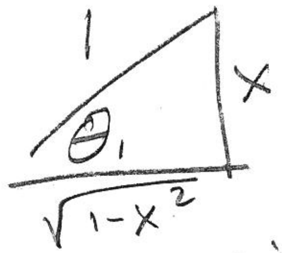
$\approx -.258819045$

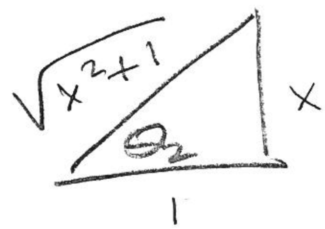
122 E2

7 $\cos(\arcsin(x) + \arctan(x)) = \cos\theta$

$= \cos(\theta_1 + \theta_2)$

$= \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$





$\theta_1 = \arcsin(x)$ $\qquad$ $\theta_2 = \arctan(x)$

So,

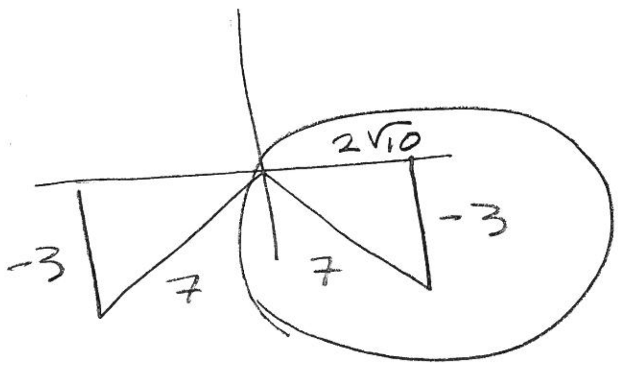
$$= \left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) - (x)\left(\frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{\sqrt{1-x^2} - x^2}{\sqrt{x^2+1}}$$

122 E2

8 $\sin u = -\frac{3}{7}$ & $\cos u > 0$

Find $\sin(2u)$, $\cos(2u)$ & $\tan(2u)$



$\cos u = \frac{2\sqrt{10}}{7}$

$49 - 9 = 40$

$\sqrt{40} = 2\sqrt{10}$

$\sin(2u) = 2\sin u \cos u = 2\left(-\frac{3}{7}\right)\left(\frac{2\sqrt{10}}{7}\right)$

$= \boxed{-\frac{12\sqrt{10}}{49} = \sin(2u)}$

$\cos(2u) = 2\cos^2 u - 1 = 2\left(\frac{2\sqrt{10}}{7}\right)^2 - 1$

$= 2\left(\frac{4(10)}{49}\right) - \frac{49}{49}$

$= \frac{80-49}{49} = \boxed{\frac{31}{49} = \cos(2u)}$

$\Rightarrow \tan(2u) = \left(-\frac{12\sqrt{10}}{49}\right)\left(\frac{49}{31}\right) = \boxed{-\frac{12\sqrt{10}}{31} = \tan(2u)}$

Check: $\cos(2u) = 1 - 2\sin^2 u = 1 - 2\left(-\frac{3}{7}\right)^2$

$= 1 - 2\left(\frac{9}{49}\right) = \frac{49-18}{49} = \frac{31}{49}$ ✓

122 E2

B1

$d = \text{diam} = 30 \text{ cm}$

$D = \text{distance} = 300 \text{ m} = (300 \text{ m})\left(\frac{100 \text{ cm}}{1 \text{ m}}\right) = 30{,}000 \text{ cm}$

$= 30{,}000 \text{ cm}$

$\Rightarrow r = 15 \text{ cm}$

$\text{Arc Length} = r\theta = 15\theta = 30{,}000 = \text{DISTANCE}$

$\theta = 2000 \text{ radians}$

$\Rightarrow \theta = 20 \text{ radians}$

$= (2000 \text{ radians})\left(\frac{1 \text{ rot}}{2\pi \text{ rad}}\right)$

$(2000 \text{ radians})\left(\frac{1 \text{ rotation}}{2\pi \text{ radians}}\right) = \frac{1000}{\pi} \text{ rotations}$

≈ 318.3098862 rotations

≈ 318 rotations

$= \frac{1000}{\pi} \text{ rotations}$

≈ 318.3098862

OR $\boxed{318 \text{ rotations}}$

Degrees version:

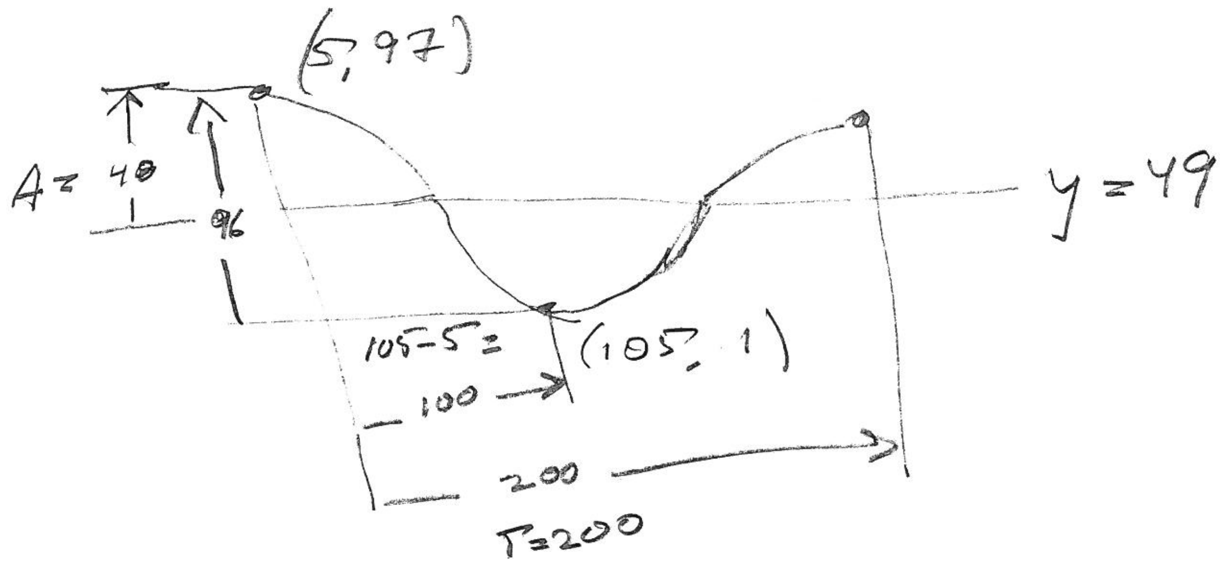
$(20 \text{ radians})\left(\frac{180^\circ}{\pi \text{ radians}}\right) \approx 1145.91559^\circ$

$\& \frac{1145.91559^\circ}{360^\circ} \approx 318.3098862 \text{ rotations}$

OR $\approx \boxed{318 \text{ rotations}}$

122 E2

B2



Midline: $\frac{97+1}{2} = \frac{98}{2} = 49$

$$\boxed{y = 48\cos\left(\frac{\pi}{100}(x-5)\right) + 49}$$

STARTS @ $x = 5$

$bx = 2\pi$, when $x = 200$

$200b = 2\pi$

$b = \frac{\pi}{100}$

122 E2

B3 $A = \frac{1}{2} r^2 \theta = \frac{1}{2}(60)^2(290^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$= \frac{1}{2}(3600)\left(\frac{29\pi}{18}\right)$

$= \frac{1}{2}(200)(29\pi)$

$= \boxed{2900\pi \text{ cm}^2 = \text{Area}}$

B4 $y = 25\sin\left(\frac{3\pi}{8}x + \frac{21\pi}{8}\right) - 12$

Amplitude

$y = -12$ MIDLINE

$\frac{3\pi}{8}\left(x + \frac{\frac{21\pi}{8}}{\frac{3\pi}{8}}\right)$

$\frac{21\pi}{\cancel{8}} \cdot \frac{\cancel{8}}{3\pi} = 7$

$\frac{-\frac{5}{3} - \frac{13}{3}}{2} = -\frac{18}{6} = -3$

$= \frac{3\pi}{8}(x + 7)$

START @ $x = -7$

$\frac{-\frac{13}{3} - \frac{21}{3}}{2} = \frac{-34}{6} = -\frac{17}{3}$

$\frac{3\pi}{8}x = 2\pi$

$x = (2\pi)\left(\frac{8}{3\pi}\right) = \frac{16}{3}$ = Period!?

$-7 + \frac{16}{3} = \frac{-21 + 16}{3} = -\frac{5}{3}$

$\frac{-\frac{21}{3} + -\frac{5}{3}}{2} = -\frac{26}{6} = -\frac{13}{3}$