MAT 122 Test 2 – Spring, 2014 Name KEY
100 Points Covers Chapter 1 NO GRAPHING CALCULATORS!!!

1. (10 pts) Find the values of all six trigonometric functions, given $\sec(\theta)=4$ and $\sin(\theta)<0$.

2. Consider the equation $2\sin^2(x)-\sin(x)-1=0$.
   a. (10 pts) Find all solutions x, in radians *and* degrees, to the equation in the interval $[0,2\pi]$.
   b. (10 pts) Find all real solutions x, in radians *and* degrees.

   It may be easier for you to use degrees to solve and then convert to radians at the end.

3. Consider the equation $2\sin^2(2x)-\sin(2x)-1=0$. (Use your answer from #2, right or wrong.)
   a. (10 pts) Find all solutions x to the equation in the interval $[0,2\pi]$. (Do degrees and radians in final answer.)

   b. (5 pts) Find all real solutions x, in degrees *and* radians.

4. (10 pts) Re-write $\tan\left(\sec^{-1}\left(\frac{3}{x}\right)\right)$ as an algebraic expression.

5. (5 pts) Square both sides of $\sin(x)+1=\cos(x)$ and solve. Find all solutions in $[0,2\pi]$. Give answer in degrees and radians.

6. Find the *exact* value of $\cos\left(\frac{17\pi}{12}\right)$ in two ways: (Hint: If degrees are easier for you, *use degrees*.)
   a. (10 pts) Use a Sum identity.
   b. (10 pts) Use a Half-Angle identity

7. (5 pts) Find the exact value of $\cos(\arcsin(x)+\arccos(x))$. (Hint: Use Sum identity.)

8. (10 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\sin(u)=\frac{1}{5}$ and $\cos(u)<0$.

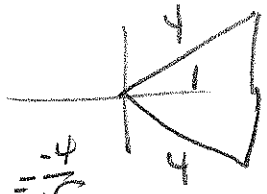
9. (5 pts) Find the arc length on a circle of radius $r = 6$ that is intercepted by an angle of 900^0.

(10 pts) Bonus: Answer *one* of the following, for 10 points:

1. Build a cosine function that achieves its maximum height of $y = 15$ meters at time $x = 3$ seconds and its minimum height of $y = -3$ meters at $x = 27$ seconds.
2. What is the area of the sector intercepted by an arc of 50^0 in a circle of radius 11? Round to 4 decimal places.

122 TEST 2

(1) (10 pts) $\sec\theta = 4$

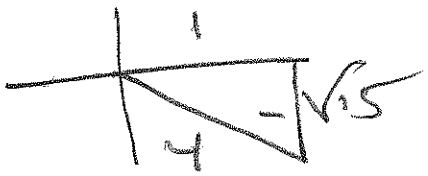


$\sin\theta = -\frac{\sqrt{15}}{4}$ $\csc\theta = -\frac{4}{\sqrt{15}}$

$\cos\theta = \frac{1}{4}$ $\sec\theta = 4$

$\tan\theta = -\frac{\sqrt{15}}{1}$ $\cot\theta = -\frac{1}{\sqrt{15}}$

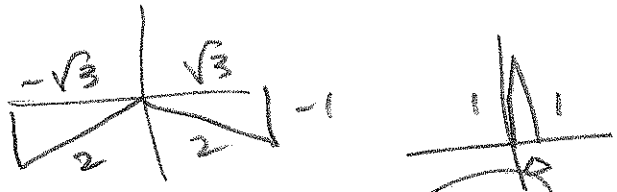
$\sin\theta < 0$



$\sqrt{16-1} = \sqrt{15}$

(2) $2u^2 - u - 1 = 0$

$(2u+1)(u-1)$



$\sin\theta = -\frac{1}{2}$ $\sin\theta = 1$

(a) $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, 210^\circ, 330^\circ, 90^\circ$ (10 pts)

(b) $\theta = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, \frac{\pi}{2} + 2n\pi$ (10 pts)

$\theta = 210^\circ + 360^\circ n, 330^\circ + 360^\circ n, 90^\circ + 360^\circ n$

(3) (a) $2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, 210^\circ, 330^\circ, 90^\circ$ (10 pts)

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{\pi}{4}, 105^\circ, 165^\circ, 45^\circ$

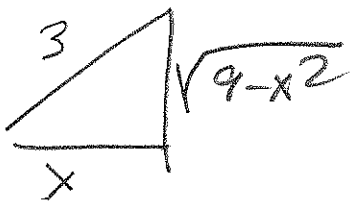
(b) $\theta = \frac{7\pi}{12} + n\pi, \frac{11\pi}{12} + n\pi, \frac{\pi}{4} + n\pi$ (5 pts)

$105^\circ + 180^\circ n, 165^\circ + 180^\circ n, 45^\circ + 180^\circ n$

122 TEST 2

(4) $\tan\left(\sec^{-1}\left(\frac{3}{x}\right)\right) = \boxed{\frac{\sqrt{9-x^2}}{x}}$

10 pts

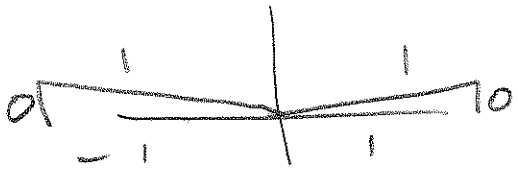


(5) $\sin x + 1 = \cos x$

$\sin^2 x + 2\sin x + 1 = \cos^2 x = 1 - \sin^2 x$

$2\sin^2 x + 2\sin x = 0$

$2\sin x(\sin x + 1) = 0$



$\sin x = 0$

$x = 0, \pi, 2\pi$

$0, 180^\circ$

$\sin x = -1$

$x = \frac{3\pi}{2}$,

270°

5 pts

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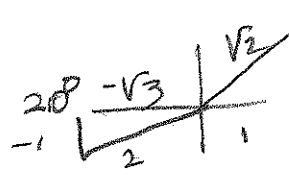
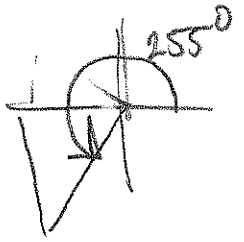
(6) $\cos\left(\frac{17\pi}{12}\right) = \cos(255^\circ)$





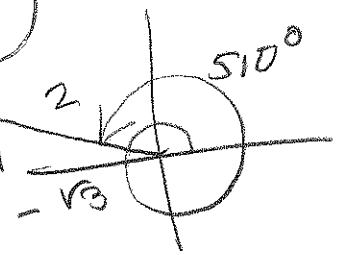
(a) $\cos\left(\frac{7\pi}{6} + \frac{\pi}{4}\right) = \cos(210^\circ + 45^\circ)$

$= \cos\frac{7\pi}{6}\cos\frac{\pi}{4} - \sin\frac{7\pi}{6}\sin\frac{\pi}{4} = \cos 210^\circ \cos 45^\circ - \sin 210^\circ \sin 45^\circ$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{-\sqrt{3}+1}{2\sqrt{2}}}$ OR $\frac{\sqrt{2}-\sqrt{6}}{4}$

10 pts

(b) $\cos\frac{17\pi}{12} = \cos\left(\frac{\frac{17\pi}{6}}{2}\right) = \cos\left(\frac{510^\circ}{2}\right)$



$= -\sqrt{\frac{1+\cos\frac{17\pi}{6}}{2}} = -\sqrt{\frac{1+\cos 510^\circ}{2}} = -\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}$

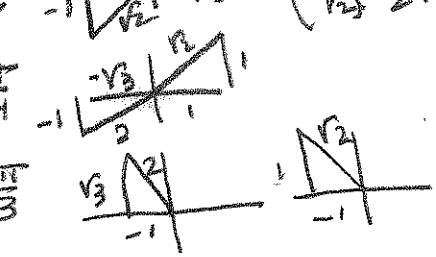
$= \boxed{-\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}}$ OR $-\sqrt{\frac{2-\sqrt{3}}{4}}$ OR $-\frac{\sqrt{2-\sqrt{3}}}{2}$

10 pts

$\frac{17\pi}{12} = \frac{15\pi}{12} + \frac{2\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$

$= \frac{14\pi}{12} + \frac{3\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$

$= \frac{9\pi}{12} + \frac{8\pi}{12} = \frac{3\pi}{4} + \frac{2\pi}{3}$



$\left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{-\sqrt{6}+\sqrt{2}}{4}$

122 TEST 2

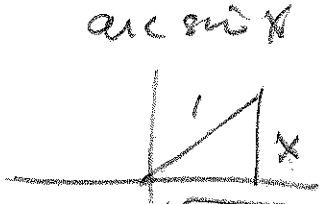
(7) $\cos(\arcsin(x) + \arccos(x))$

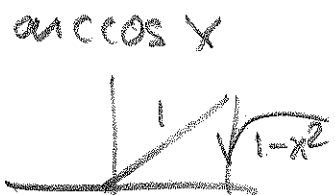
$= \cos(\arcsin x)\cos(\arccos x) - \sin(\arcsin x)\sin(\arccos x)$

$= (\sqrt{1-x^2})x - x\sqrt{1-x^2}$

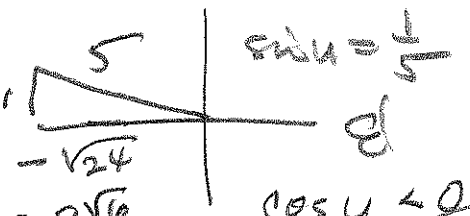
$= 0$!

5pts





(8) $\sin u = \frac{1}{5}$, $\cos u < 0$



$\sin(2u) = 2\sin u \cos u$

$= 2\left(\frac{1}{5}\right)\left(-\frac{2\sqrt{6}}{5}\right) = \boxed{-\frac{4\sqrt{6}}{25} = \sin(2u)}$ 10pts

$\cos(2u) = 1 - 2\sin^2 u = 1 - 2\left(\frac{1}{5}\right)^2 = 1 - \frac{2}{25} = \boxed{\frac{23}{25} = \cos(2u)}$

$\tan(2u) = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(-\frac{1}{\sqrt{24}}\right)}{1-\left(-\frac{1}{\sqrt{24}}\right)^2} = \frac{-\frac{2}{2\sqrt{6}}}{1-\frac{1}{24}}$

$= \frac{-\frac{2}{\sqrt{24}}}{\frac{23}{24}} = -\frac{2}{\sqrt{24}} \cdot \frac{24}{23} = \boxed{-\frac{48}{46\sqrt{6}} = \tan(2u)} = -\frac{24}{23\sqrt{6}} = \frac{-4\sqrt{6}}{23}$

$= \frac{-48\sqrt{24}}{(23)(24)} = -\frac{2\sqrt{24}}{23} = \tan(2u)$

You could also get $\tan(2u)$ by $\frac{\sin(2u)}{\cos(2u)}$

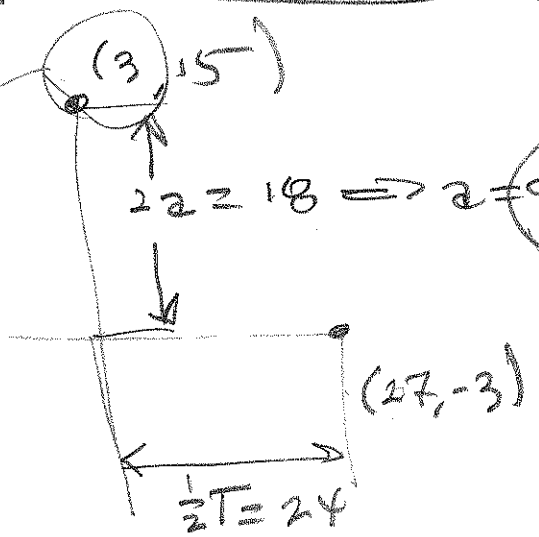
122 TEST 2

(9) $s = r\theta = (6)(900^\circ)\left(\frac{\pi}{180^\circ}\right) = 30\pi$

$\approx$ 94.24777961

5pts

(B1)



T = 48

$bx = 2\pi$ when $x = 48$

$48b = 2\pi$

$b = \frac{\pi}{24}$

10pts

Start @ x = 3 (Right shift 3)

c = 3

$a\cos(b(x-c)) + d$

midline

$y = \frac{15 + (-3)}{2} = \frac{12}{2} = 6$

$y = 12 = d$

$9\cos\left(\frac{\pi}{24}(x-3)\right) + 6$

52.79620987

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B2 $A = \frac{1}{2} r^2 \theta = \frac{1}{2}(11)^2 (50^\circ)\left(\frac{\pi}{180}\right)$

$= \boxed{\frac{605}{36}\pi} = \boxed{16.805\overline{5}\,\pi} \approx \boxed{52.79620987}$

10 pts

Area of circle:
$\pi r^2 = \pi(11)^2 = 121\pi$
≈ 380.132741
$\left(\frac{50}{360}\right)(380.132741)$
≈ 52.79620987

$\frac{50\pi}{180} = \frac{5\pi}{18}$