MAT 122 Test 1 – Spring, 2014 Name_____

100 Points Covers Chapter 1 NO GRAPHING CALCULATORS!!!

1. (10 pts) Find two angles, between $-2\pi$ and $2\pi$ (i.e., $0^0$ and $360^0$) that are coterminal with $\frac{43\pi}{4}$. Give exact answers in degrees and radians.

$\frac{43\pi}{4} \cdot \frac{180}{\pi} = 45 \cdot 43 = 1935$

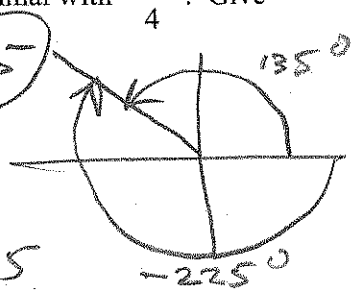
$\frac{1935}{360} = 5.375$

$5 \cdot 360 = 1800$

$1935 - 1800 = 135$

$(135)\left(\frac{\pi}{180}\right) = \frac{3\pi}{4} = 135^0$

$(-225)\left(\frac{\pi}{180}\right) = \frac{-5\pi}{4} = -225^0$



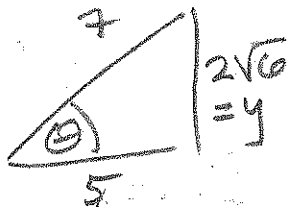

2. (5 pts) Find the arc length on a circle of radius $r = 7$ that is intercepted by an angle of $1935^0$.

$(1935^0)\left(\frac{\pi \text{ rad}}{180^0}\right)(7) = \frac{301\pi}{4} \approx 236.40484472$

$\approx 236.4048$

3. Suppose you know that $\cos(\theta) = \frac{5}{7}$.

a. (5 pts) Assume the terminal side of the angle $\theta$ lies in the 1st quadrant. Find the other five trigonometric functions of $\theta$.




$y^2 + 5^2 = 49$

$y^2 = 49 - 25 = 24$

$y = \sqrt{24} = 2\sqrt{6}$

$\sin(\theta) = \frac{2\sqrt{6}}{7}$

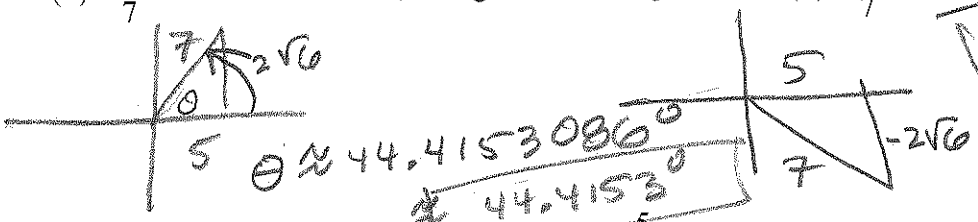
$\tan(\theta) = \frac{2\sqrt{6}}{5}$

$\csc(\theta) = \frac{7}{2\sqrt{6}}$ OR $\frac{7\sqrt{6}}{12}$

$\sec\theta = \frac{7}{5}$

$\cot\theta = \frac{5}{2\sqrt{6}}$ OR $\frac{5\sqrt{6}}{12}$

b. (5 pts) Suppose $\theta$ is any angle between 0 and $2\pi$. Draw *two* pictures that satisfy the condition $\cos(\theta) = \frac{5}{7}$. Give two solutions, in degrees, to the equation $\cos(\theta) = \frac{5}{7}$..




$\theta \approx 44.41530860^0$

$\theta \approx 44.4153^0$

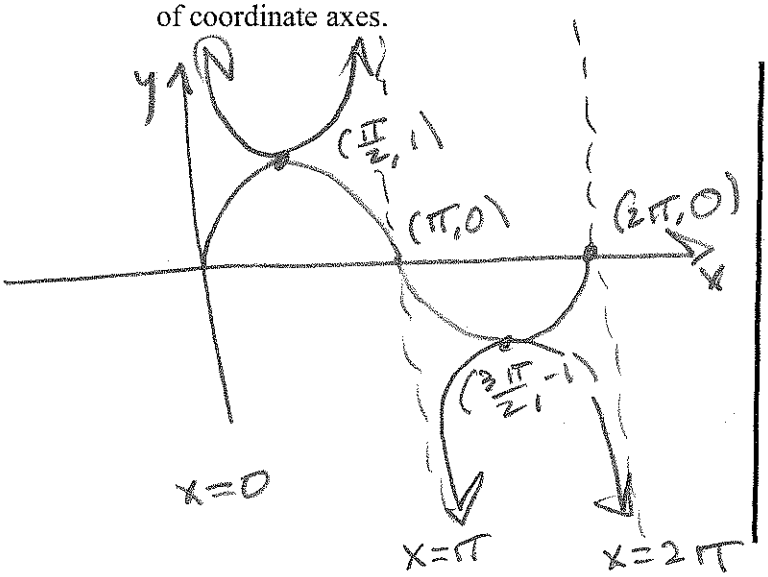
$\theta \approx 44.4153^0$

c. (5 pts) Give *all* solutions to the equation $\cos(\theta) = \frac{5}{7}$, in degrees, rounded to four decimal places.

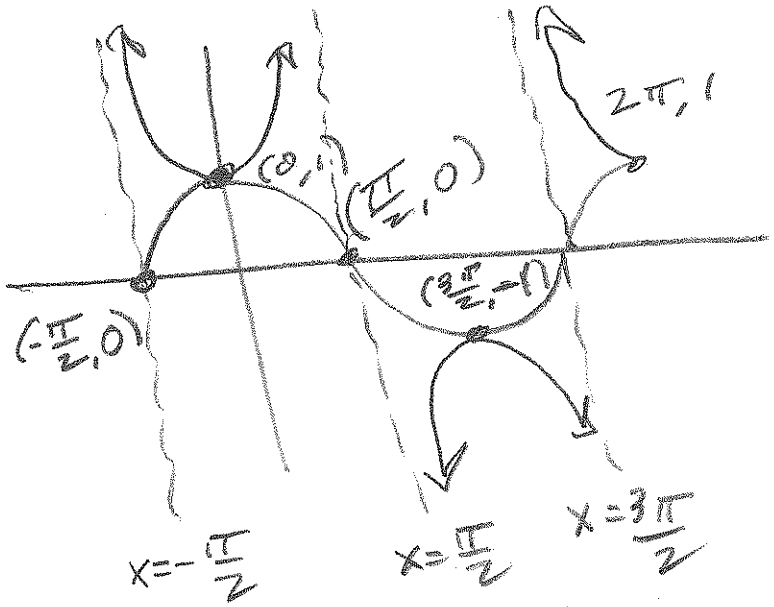
$\{\theta \mid \theta = 44^0 + 360n^0 \text{ or } \theta = -44^0 + 360n^0, n \in \mathbb{Z}\}$

4. (10 pts) Sketch one period of the graphs of …

a. … $y = \sin(x)$ and $y = \csc(x)$ on the same set of coordinate axes.

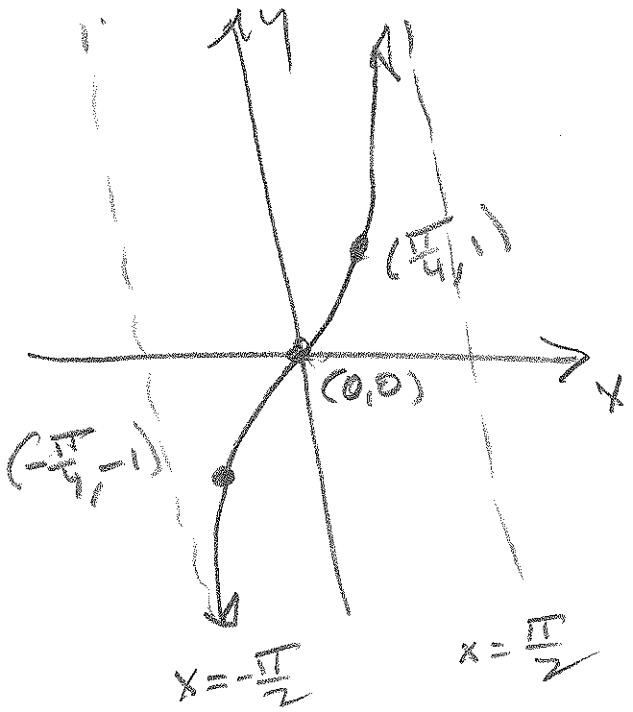



b. … $y = \cos(x)$ and $y = \sec(x)$ on the same set of coordinate axes.

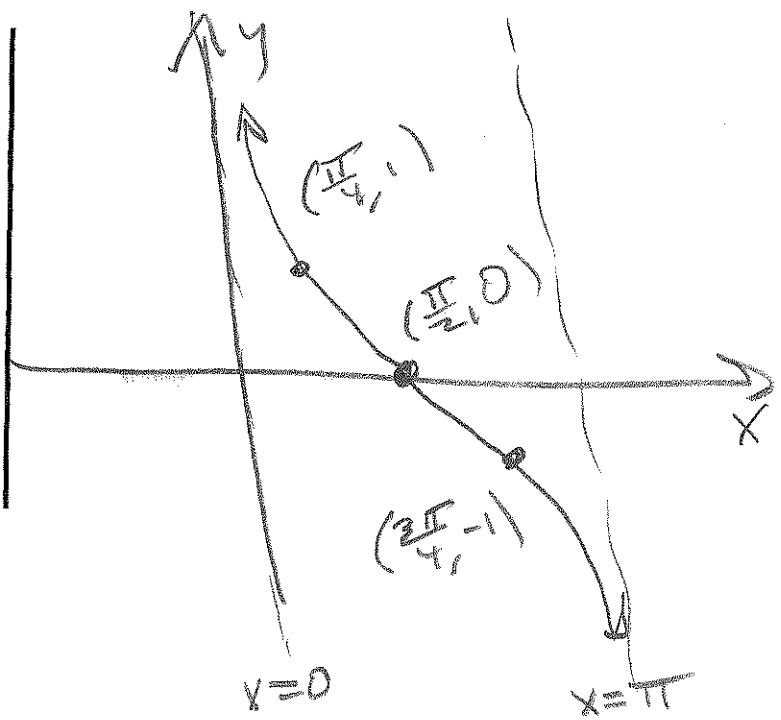



5. (10 pts) Sketch the graph of one period of …

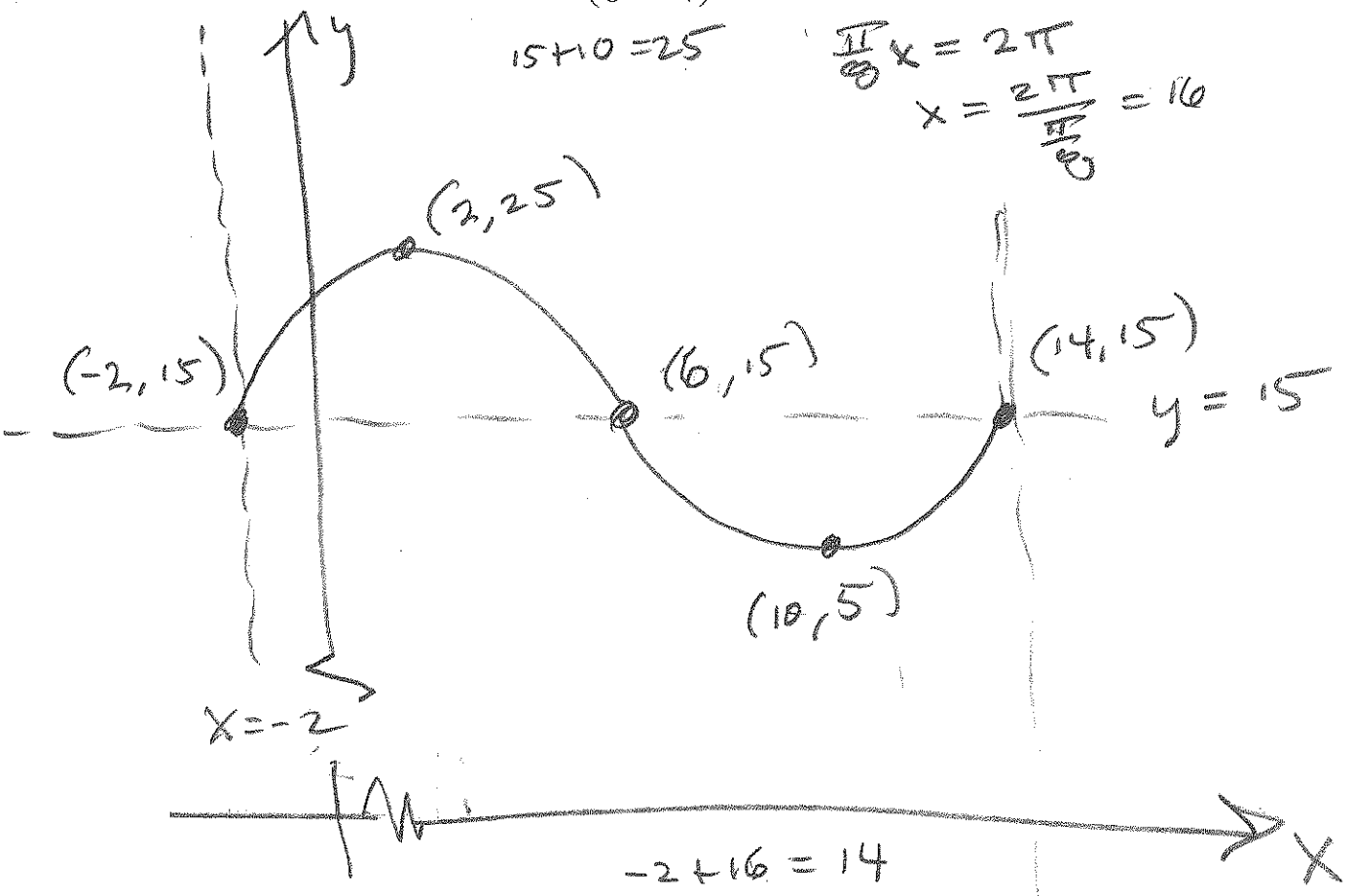
a. … $y = \tan(x)$




b. … $y = \cot(x)$



6. (10 pts) Sketch the graph of $f(x)=10\sin\left(\frac{\pi}{8}x-\frac{\pi}{4}\right)+15$ $= 10\sin\left(\frac{\pi}{8}(x-2)\right)+15$

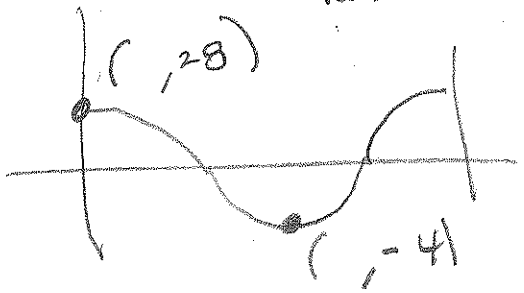



7. (10 pts) Build a cosine function that achieves its maximum height of $y = 28$ meters at time $x = 5$ seconds and its minimum height of $y = -4$ meters at $x = 25$ seconds.

$\frac{28+(-4)}{2} = 12 = mid$ → $16\cos\left(\frac{\pi}{10}(x-5)\right)+12$

$\frac{28-(-4)}{2} = \frac{32}{2} = 16 = Amplit.$

Period:




$\frac{1}{2}$ a period from $x=5$ to $x=25$

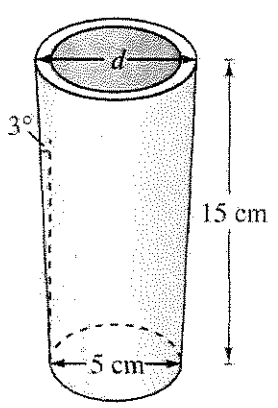
$\frac{25-5}{2} = \frac{20}{2} = 10$ is $\frac{1}{2}$-period so period is 20

$bx = 2\pi$ when $x=20$

$20b = 2\pi$

$b = \frac{\pi}{10}$

8. (10 pts) A tapered shaft has a diameter of 5 centimeters at the small end and is 15 centimeters long (See figure.). The taper is $3^0$. Find the diameter $d$ of the large end of the shaft.





d = diameter @ the top, in centimeters

$\frac{x}{15} = \tan 3^0$

$x = 15\tan 3^0 \approx .7861166892$ is how much the radius grew

$d = 5 + 2(x) \approx 5 + 2(.7861166892)$

$\approx 5 + 1.5722333378$

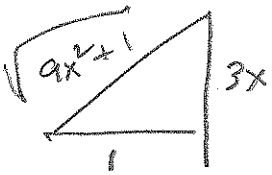
$= 6.5722333378$

$\approx$ 6.5722 cm $\approx d$

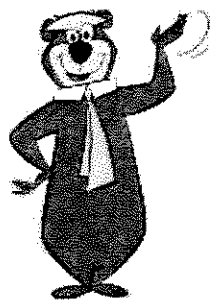
9. (5 pts) Find the exact value of $\csc\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$.

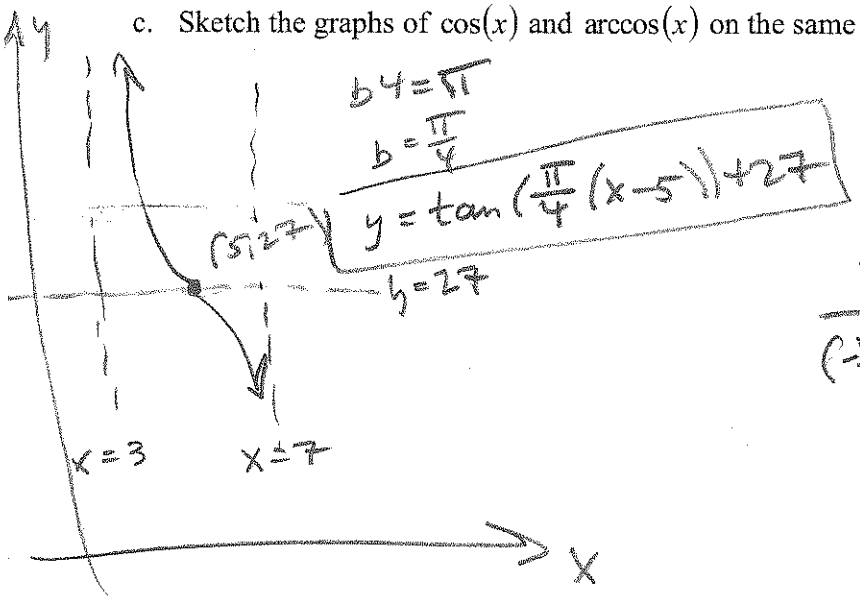

$= 2$

10. (5 pts) Write an algebraic expression that is equivalent to $\sec(\arctan(3x))$ $= \sqrt{9x^2+1}$




11. (10 pts) Bonus: Answer *one* of the following, for 10 points:
    a. Build a tangent function with vertical asymptotes at $x = 3$ and $x = 7$ that passes through the points $(4,59), (5,27)$, and $(6,-5)$:
    b. Sketch the graphs of $\sin(x)$ and $\sin^{-1}(x)$ on the same axes.
    c. Sketch the graphs of $\cos(x)$ and $\arccos(x)$ on the same axes.






$b4 = \pi$

$b = \frac{\pi}{4}$

$y = \tan\left(\frac{\pi}{4}(x-5)\right) + 27$

$y = 27$