MAT 122 Test 1 – Spring, 2014 Name KEY

100 Points Covers Chapter 1 NO GRAPHING CALCULATORS!!!

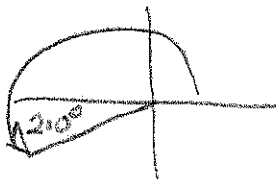
1. (10 pts) Find two angles, between -2π and 2π (i.e., 0^0 and 360^0) that are coterminal with $\frac{91\pi}{6}$. Give exact answers in degrees and radians.

$\frac{91\pi}{6} \cdot \frac{180}{\pi} = (91)(30) = 2730^0$

$\frac{2730}{360} = 7.\text{something}$

$360 \times 7 = 2520$

$2730 - 2520 = 210$



$210^0, -150^0$

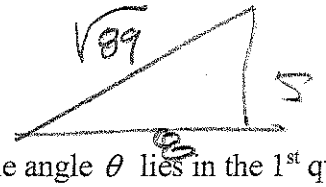
$\frac{7\pi}{6}, -\frac{5\pi}{6}$

$360 - 210 = 150$

2. (5 pts) Find the arc length on a circle of radius $r = 5$ that is intercepted by an angle of 2730^0.

$s = r\theta = (5)\left(\frac{91\pi}{6}\right) \approx 238.2374$

$25 + 64 = 89$



3. Suppose you know that $\tan(\theta) = \frac{5}{8}$.

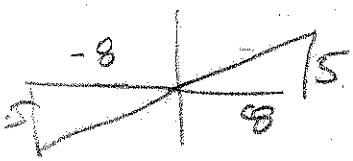
a. (5 pts) Assume the terminal side of the angle θ lies in the 1st quadrant. Find the other five trigonometric functions of θ.

$\sin\theta = \frac{5}{\sqrt{89}}$ $\csc\theta = \frac{\sqrt{89}}{5}$

$\cos\theta = \frac{8}{\sqrt{89}}$ $\sec\theta = \frac{\sqrt{89}}{8}$

$\tan\theta = \frac{5}{8}$ $\cot\theta = \frac{8}{5}$

b. (5 pts) Suppose θ is any angle between 0 and 2π. Draw *two* pictures that satisfy the condition $\tan(\theta) = \frac{5}{8}$. Give two solutions, in degrees, to the equation $\tan(\theta) = \frac{5}{8}$..



$\arctan\left(\frac{5}{8}\right) \approx 32.0054^0$

$\arctan\left(\frac{5}{8}\right) + 180^0 \approx 212.0054^0$

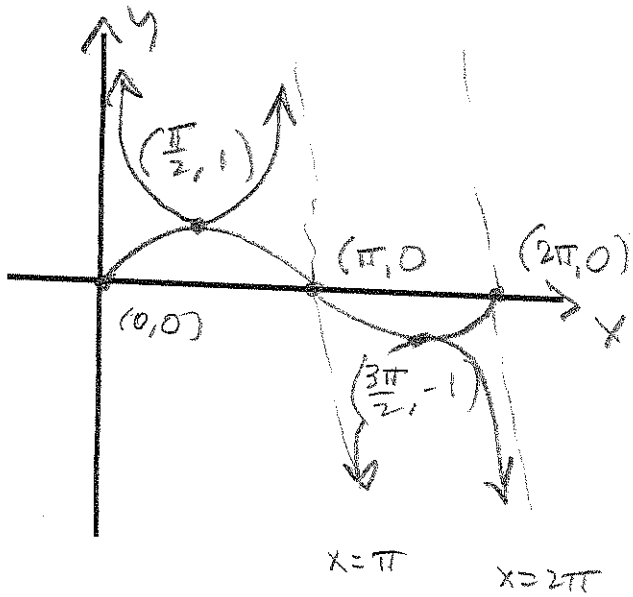
c. (5 pts) Give *all* solutions to the equation $\tan(\theta) = \frac{5}{8}$, in degrees, rounded to four decimal places.

$\boxed{32.0054^0 + 180^0 n}$

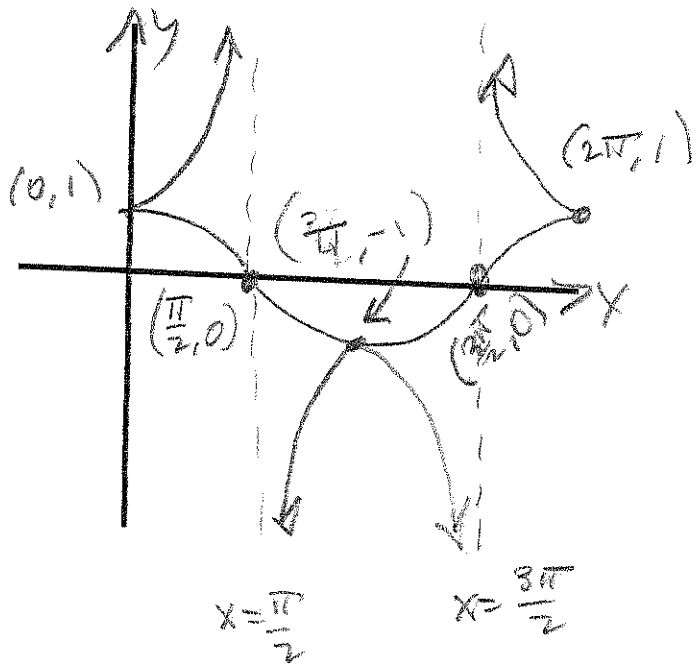
4. (10 pts) Sketch one period of the graphs of …

a. … $y = \sin(x)$ and $y = \csc(x)$ on the same set of coordinate axes.

c.

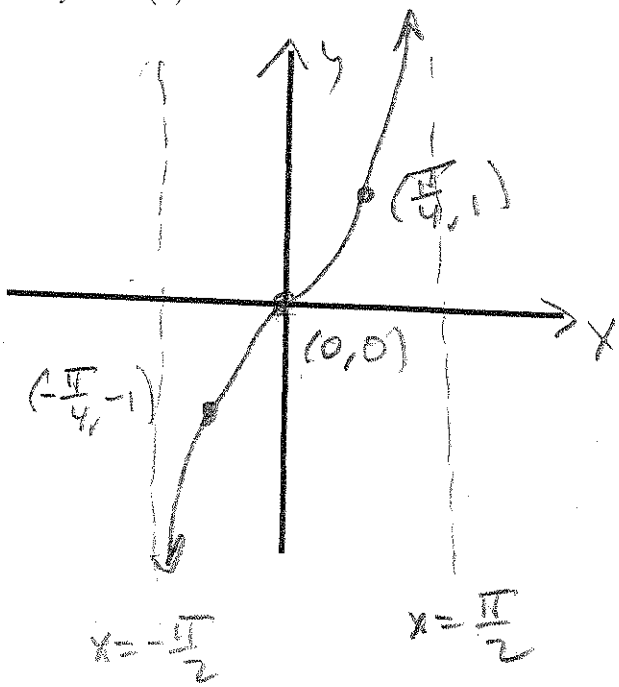


b. … $y = \cos(x)$ and $y = sec(x)$ on the same set of coordinate axes.

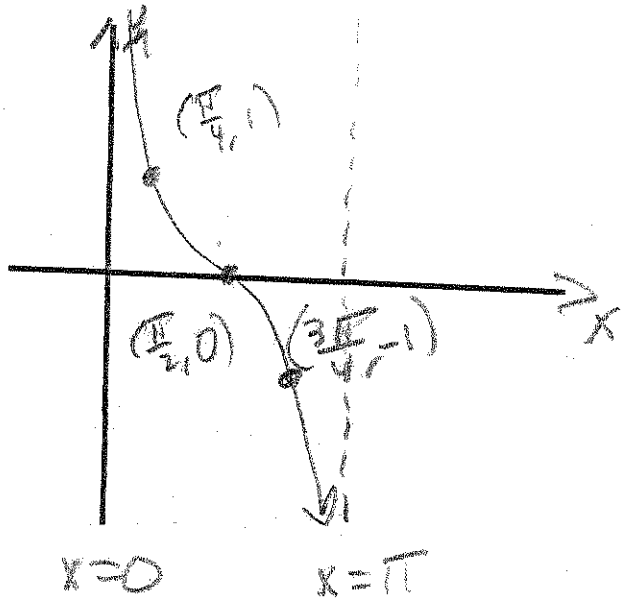


5. (10 pts) Sketch the graph of one period of …

a. … $y = \tan(x)$



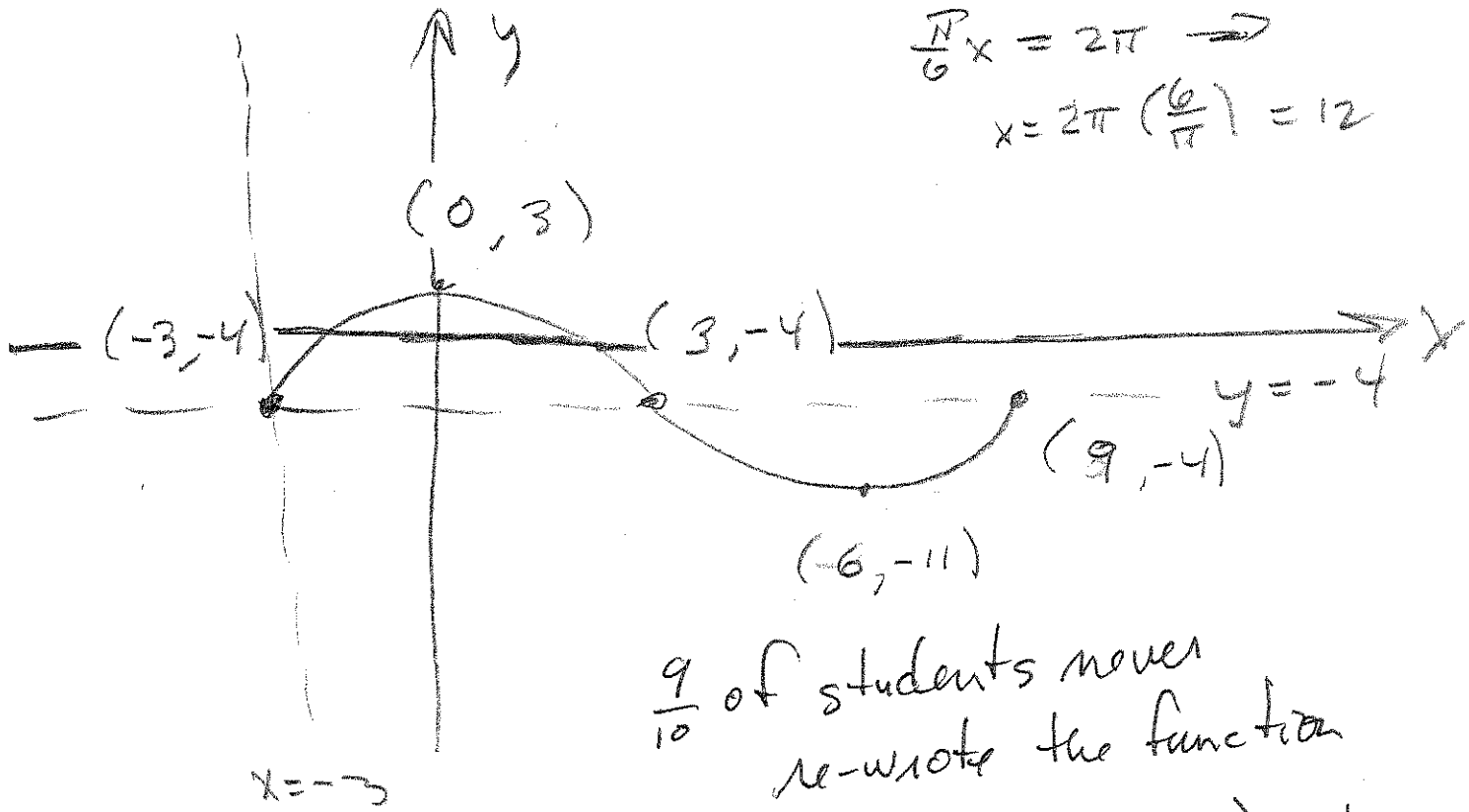
b. … $y = \cot(x)$



6. (10 pts) Sketch the graph of $f(x) = 7\sin\left(\frac{\pi}{6}x + \frac{\pi}{2}\right) - 4$ $= 7\sin\left(\frac{\pi}{6}(x+3)\right) - 4$

$\frac{\pi}{6}x = 2\pi \rightarrow$

$x = 2\pi\left(\frac{6}{\pi}\right) = 12$

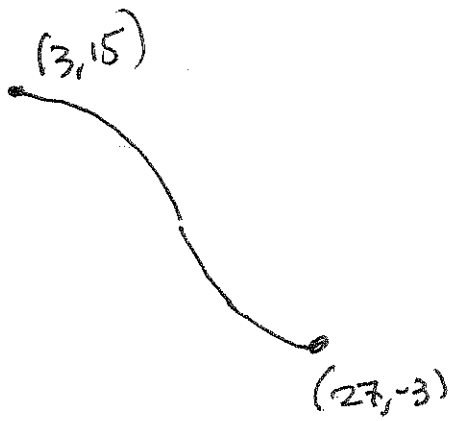


$\frac{9}{10}$ of students never re-wrote the function like $7\sin\left(\frac{\pi}{6}(x+3)\right) - 4$

Why?

7. (10 pts) Build a cosine function that achieves its maximum height of $y = 15$ meters at time $x = 3$ seconds and its minimum height of $y = -3$ meters at $x = 27$ seconds.

$9\cos\left(\frac{\pi}{24}(x-3)\right) + 6$



$(3, 15), (27, -3)$

$T = (24)(2) = 48$

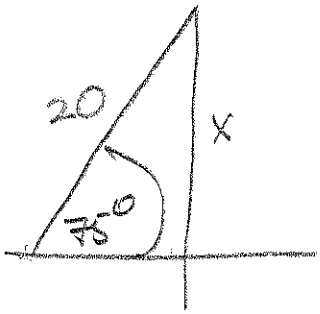
$\frac{15-3}{2} = \frac{12}{2} = 6$ Mid

$\frac{15-(-3)}{2} = 9$ Amp.

$bx = 2\pi$ $x = 48$

$b = \frac{2\pi}{48} = \frac{\pi}{24}$

8. (10 pts) A 20-foot ladder is leaning against a 2-story house. The angle the ladder makes with the ground is 75^0. How high off the ground is the top of the ladder touching the house?



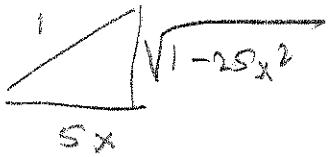
$$\frac{x}{20} = \sin 75^0$$

$$x = 20 \sin 75^0$$

$$\approx 19.31851653$$

9. (10 pts) Find the exact value of $\sec\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$. $= \sqrt{2}$

10. (10 pts) Write an algebraic expression that is equivalent to $\csc(\arccos(5x))$ $= \dfrac{1}{\sqrt{1-25x^2}}$

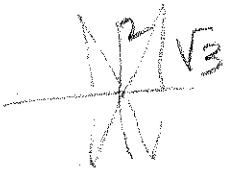


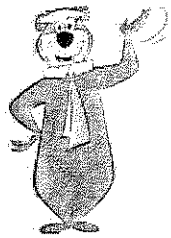
11. (10 pts) Bonus: Answer *one* of the following, for 10 points:

a. Sketch the graphs of $\sin(x)$ and $\sin^{-1}(x)$ on the same axes.

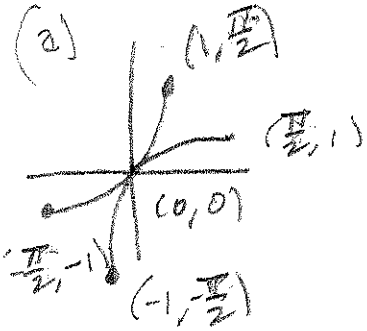
b. Sketch the graphs of $\cos(x)$ and $\arccos(x)$ on the same axes.

c. Find all solutions in $[0,2\pi]$ to the equation $4\sin^2\theta - 3 = 0$

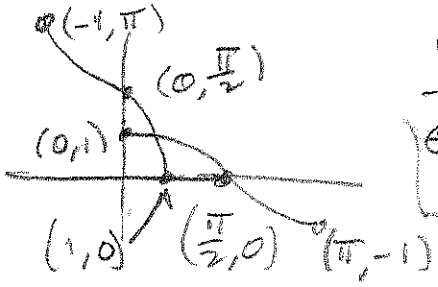




(a)



(b)



(c) $\sin\theta = \pm\frac{\sqrt{3}}{2}$

$$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$