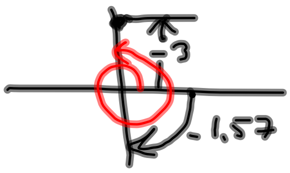
§6.7, 6.8 Assignments on your cheat sheet

§4.3 #s 5-27, 35-39, 49, 51, ~~55, 57, 59, 61, 63, 65~~
55-65 odds

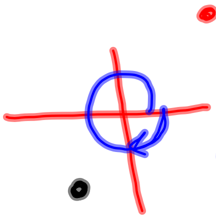
§4.4 #s 5-11, 21-33, 41, 43, 47, 53, 55-61

6.7 #s 15-18 Plot & find 2 more representations.

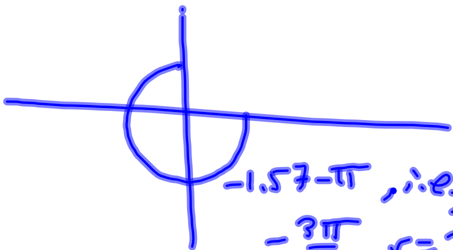
17 $(-3, -1.57) = (3, 1.57) = (3, 1.57 \pm 2\pi)$
$\approx (3, 7.85)$



Not $\in (-2\pi, 2\pi)$
Bad.



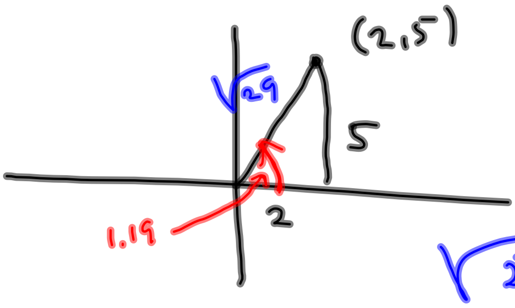
$(-3, 2\pi - 1.57)$



$-1.57-\pi$, i.e.,
$-\frac{3\pi}{2}$, $r=3$

$(3, -1.57-\pi) \approx (3, -4.71)$

$$\begin{array}{r} -3.14 \\ \hline -4.71 \end{array}$$



convert to polar form

$\sqrt{3^2 - x^2} \neq 3 - x$

$\sqrt{2^2+5^2}$

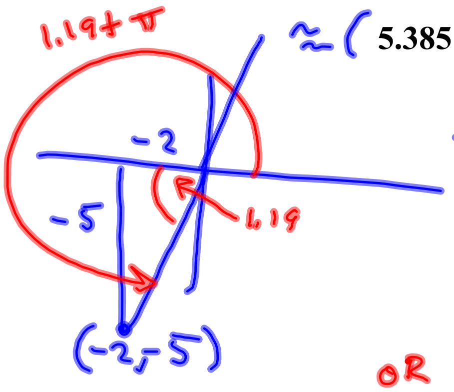
$\sqrt{29}$

$\tan^{-1}\left(\frac{5}{2}\right) \approx$ 1.190289950

$(r, \theta) \approx (\sqrt{29}, 1.190289950)$

$\approx ($5.385164807, 68.19859051$^\circ)$

$$\begin{array}{r} 1.19 \\ 3.14 \\ \hline 4.33 \end{array}$$



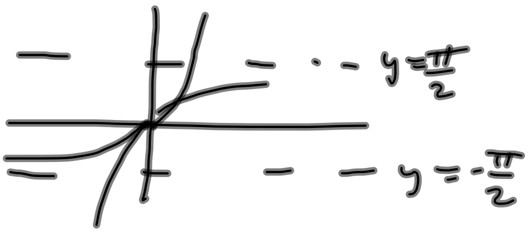
$\tan^{-1}\left(\frac{5}{2}\right) \approx 1.190289950$

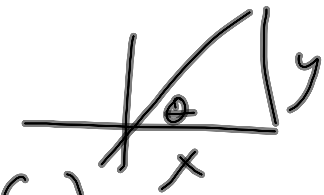
$(r, \theta) \approx (5.39, 4.33)$

$\approx (\sqrt{29}, 1.19 + \pi)$

OR Khasim's evil scheme:

$(-\sqrt{29}, 1.19)$





$\tan^{-1}(\omega) = \arctan(\omega)$

= angle whose tangent is ω

$f(x) = \sqrt{x}$

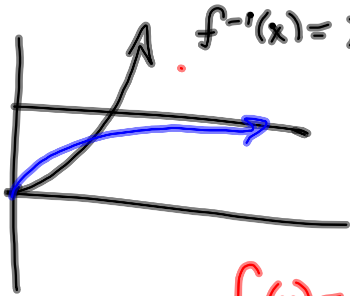
$\frac{1}{f(x)} \neq f^{-1}(x) = x^2$

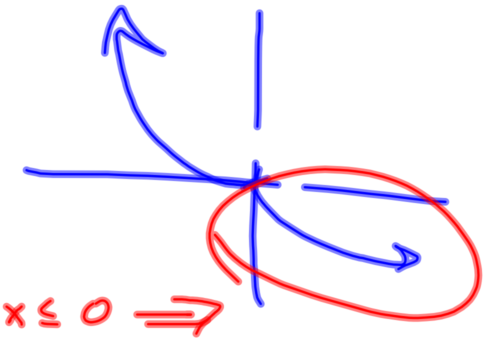
Inverse wrt function composition

NOT arithmetic.

$\sqrt{x^2} = x$, if $x \ge 0$

$f^{-1}(x) = x$



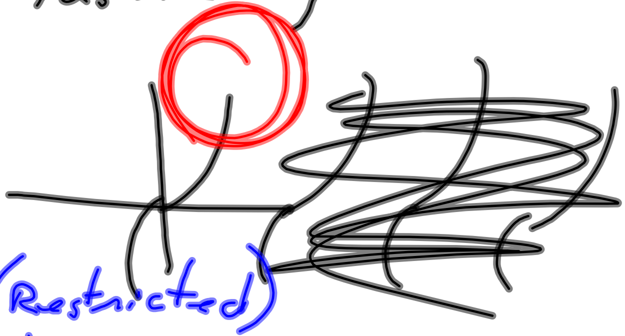


$f(x) = x^2$, $x \le 0 \Rightarrow$

$f^{-1}(x) = -\sqrt{x}$

$f(x) = x^2$ w/o restricting $x \Rightarrow$

$\nexists$ an f^{-1}



tan: $R = (-\infty, \infty)$

$D = (-\frac{\pi}{2}, \frac{\pi}{2})$ (Restricted)

$\tan^{-1}$: $R = (-\frac{\pi}{2}, \frac{\pi}{2})$

$D = (-\infty, \infty)$

Bigger
smaller $\rightarrow \infty$

Go from Rect. to Polar.

$$(x^2+y^2)^2 = x^2-y^2$$

$$r = \sqrt{x^2+y^2}$$

$$\tan\theta = \frac{y}{x}$$

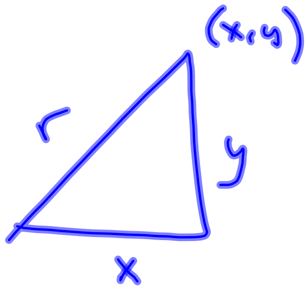
$$(r^2)^2 = r^2\cos^2\theta - r^2\sin^2\theta$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$(r^2)^2 = r^2(\cos^2\theta - \sin^2\theta)$$

$$r^2 = \cos^2\theta - \sin^2\theta \quad \text{OK}$$



$$r^2 = \cos(2\theta)$$ This is easier to graph in the sequel

$$\cos^2\theta - (1-\cos^2\theta)$$

$$= 2\cos^2\theta - 1$$

$$= 2\left(\frac{1+\cos 2\theta}{2}\right) - 1 = 1+\cos(2\theta) - 1$$

$$= \cos(2\theta)$$

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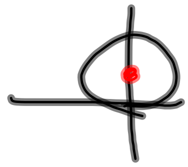


$$r = 4\sin\theta$$

$$\sqrt{x^2+y^2} = 4\frac{y}{r} = 4\frac{y}{\sqrt{x^2+y^2}}$$

$$x^2+y^2 = 4y$$

$$x^2+y^2-4y+2^2 = 0+4$$

$$x^2+(y-2)^2 = 4 = 2^2$$

circle of radius $r=2$

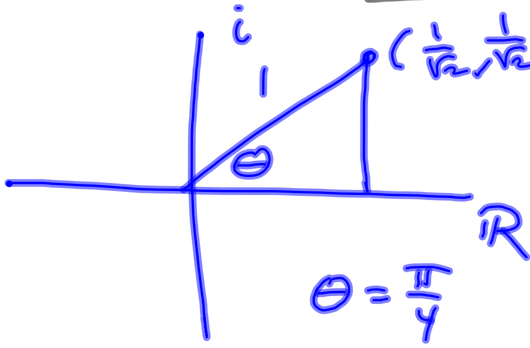
centered @ $(0,2)$

$$(x-h)^2+(y-k)^2=r^2$$

(h,k) = center.

§4.3, 4.4 $(x,y) = (r\cos\theta, r\sin\theta)$

Punchline <u>DeMoivre's</u> Theorem.



$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

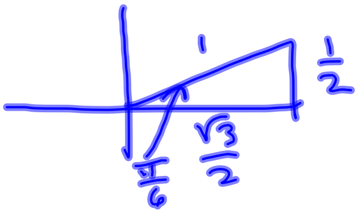
is represented in trigonometric form as $1\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

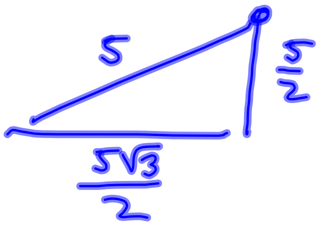
$= r(\cos\theta + i\sin\theta)$

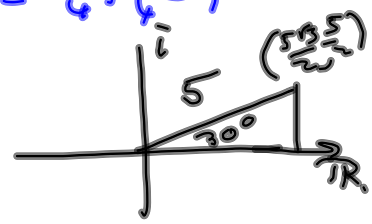
Trig Representation for

$\frac{5\sqrt{3}}{2} + \frac{5}{2} i = 5\left(\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)$

$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$







Find $\left(5\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\right)^2$

$$=\left(\frac{5}{2}\left(\sqrt{3}+1i\right)\right)^2$$

$$=\left(\frac{5}{2}\right)^2\left(\sqrt{3}+i\right)^2$$

$$=\frac{25}{4}\left(\sqrt{3}^2+2\sqrt{3}\,i+i^2\right)$$

$$=\frac{25}{4}\left(3+2i\sqrt{3}-1\right)$$

$$=\frac{25}{4}\left(2+2i\sqrt{3}\right)$$

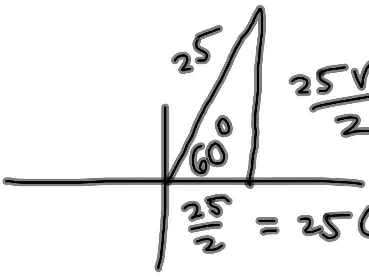
$$=\frac{25}{2}\left(1+i\sqrt{3}\right)=\frac{25}{2}+\frac{25i\sqrt{3}}{2}$$

$(\sin x+1)^2$

$\sin^2 x+2\sin x+1$

$(a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$



$$\frac{25\sqrt{3}}{2}=25\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{25}{2}=25\left(\frac{1}{2}\right)$$

$$\frac{25}{2}(\sqrt{3})$$

$$\frac{25}{2}(1)$$

$$\frac{25}{2}(2)$$

$z = 5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ is trig representation

$= \cancel{5(\cos 30^\circ + i\sin 30^\circ)}$

$z^2 = 25\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$z^3 = 125\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

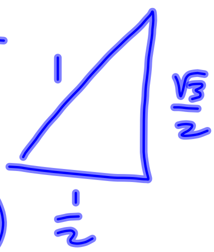
$= 5^3\left(\cos\left(3\left(\frac{\pi}{6}\right)\right) + i\sin\left(3\left(\frac{\pi}{6}\right)\right)\right)$

$z^n = 5^n\left(\cos\left(\frac{n\pi}{6}\right) + i\sin\left(\frac{n\pi}{6}\right)\right)$

DeMoivre's Theorem

$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$

$\left(3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right)\left(5\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right)$



$3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\left(5\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)$

$= 15\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right)$

$\frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$

multiply moduli

Add Arguments.

$$\frac{15}{4}\sqrt{2}+\frac{15}{4}I\sqrt{2}+\frac{15}{4}I\sqrt{6}-\frac{15}{4}\sqrt{6}$$

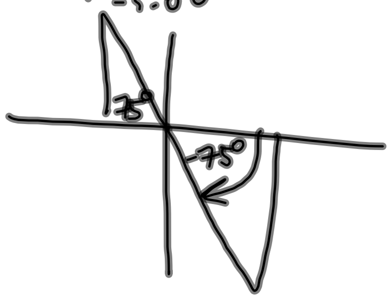
$$= \frac{15}{4}\sqrt{2} - \frac{15}{4}\sqrt{6} + \left(\frac{15}{4}\sqrt{2} + \frac{15}{4}\sqrt{6}\right) i$$

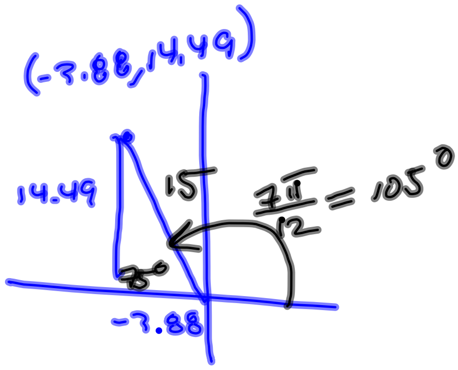
$$= \frac{15}{4}\left[\left(\sqrt{2}-\sqrt{6}\right) + \left(\sqrt{2}+\sqrt{6}\right) i\right]$$ Bleah

$\approx 3.75($

-3.882285678+14.48888739I

$$\tan^{-1}\left(\frac{14.49}{-3.88}\right) \approx -75^\circ$$





180 − 75 = 105

$$z_1 z_2 = r_1(\cos\theta_1 + i\sin\theta_1)\left(r_2(\cos\theta_2 + i\sin\theta_2)\right)$$

$$= r_1 r_2 \left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$$

Punchline.

§6.7 Next time

Read 6.8 & write the symmetry props/tips & try some graphs.

§4.3 Next time.

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