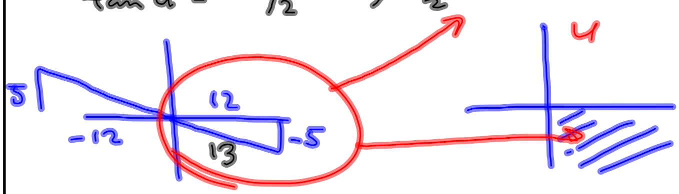
§2.5 #39

(39) Ramiro says I got it wrong.

$\tan u = -\frac{5}{12}$, $\frac{3\pi}{2} < u < 2\pi$



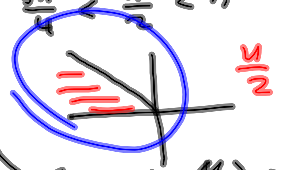
$\cos^2 u = \frac{1+\cos(2u)}{2}$

$\sin^2 u = \frac{1-\cos(2u)}{2}$

$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$

$\frac{3\pi}{2} < u < 2\pi \Longrightarrow$

$\frac{3\pi}{4} < \frac{u}{2} < \pi$



$= +\sqrt{\frac{1-\cos u}{2}}$

so $\sin\left(\frac{u}{2}\right) > 0$

$= \sqrt{\frac{1-\frac{12}{13}}{2}}$

$= \sqrt{\frac{\frac{1}{13}}{2}} = \sqrt{\frac{1}{26}}$

$5^2 + 12^2 = 169$

$= \frac{\sqrt{26}}{26}$ or $\frac{1}{\sqrt{26}} = \sin\left(\frac{u}{2}\right)$

$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\cos u}{2}}$ b/c we're in QII.

$-\sqrt{\frac{\frac{13+12}{13}}{2}} = \sqrt{\frac{25}{26}} = -\frac{5}{\sqrt{26}}$

My solutions got the sign wrong.

$\tan\left(\frac{u}{2}\right) = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \frac{\frac{1}{\sqrt{26}}}{-\frac{5}{\sqrt{26}}} = -\frac{1}{5}$

This was also wrong.

(13) $\tan(2x) - \cot(x) = 0$

$$\frac{2\tan(x)}{1-\tan^2 x} - \cot(x) = 0$$

$\tan^2 x + 1 = \sec^2 x$
So where'd you get $\sec^2 x = 1 - \tan^2 x$?

$$\frac{2\tan(x)}{\sec^2 x} - \cot(x) = 0$$

Newp

$$\frac{2\tan x - (1-\tan^2 x)\cot(x)}{1-\tan^2 x} = 0$$

$$2\tan x - \cot x + \tan^2 x \cot x = 0$$

$$2\tan x - \cot x + \tan x = 0$$

$$3\tan x - \frac{1}{\tan x} = 0$$

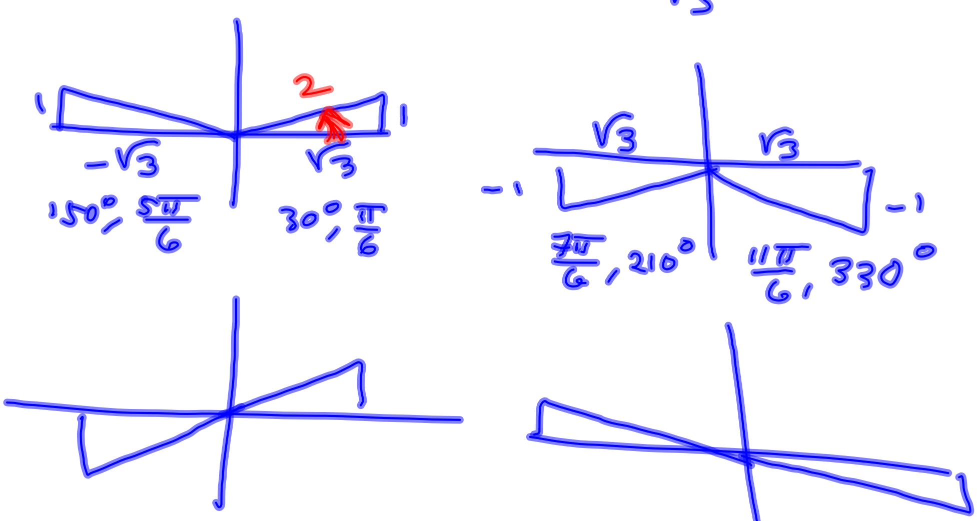
$$\frac{3\tan^2 x - 1}{\tan x} = 0$$

$$3\tan^2 x = 1$$

Right here, it appears I lost a solution by throwing away the denominator.
See Below

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$$

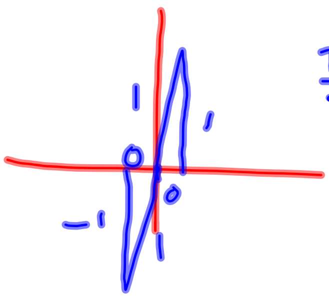


$$\frac{3\tan^2 x - 1}{\tan x} = 0$$

$$(\cot x)(3\tan^2 x - 1) = 0$$

$\cot x = 0$ OR $3\tan^2 x - 1 = 0$

Not done — Done



$\frac{\pi}{2}, \frac{3\pi}{2}$ are the other 2 solutions.

Teacher needs to ponder this.

$\sin(5\theta)\sin(3\theta)$

(41) $|\sin(3x)| = \boxed{\sqrt{\dfrac{1-\cos(6x)}{2}}}$

(43) $-\sqrt{\dfrac{1-\cos 8x}{1+\cos 8x}}$

$= -|\tan(4x)|$

$$\tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)}$$

$$\sqrt{\tan^2 u} = \sqrt{\frac{1-\cos 2u}{1+\cos 2u}}$$

$$|\tan u| = \sqrt{\frac{1-\cos(2u)}{1+\cos(2u)}}$$

$$\tan u = \pm\sqrt{\frac{1-\cos(2u)}{1+\cos(2u)}}$$

$$\frac{\sin^2\left(\frac{u}{2}\right)}{\cos^2\left(\frac{u}{2}\right)} = \frac{\frac{1-\cos u}{2}}{\frac{1+\cos u}{2}} = \frac{1-\cos u}{1+\cos u}$$

(45) $\sin\frac{x}{2} + \cos x = 0$

$\cos(2x)$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$\cos x$
$\Rightarrow = 1 - 2\sin^2(\frac{x}{2})$

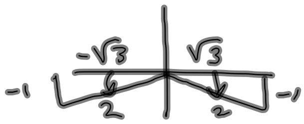
$\sin(\frac{x}{2}) + 1 - 2\sin^2(\frac{x}{2}) = 0$

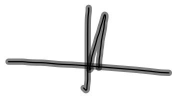
$-2\sin^2(\frac{x}{2}) + \sin(\frac{x}{2}) + 1 = 0$

$2\sin^2(\frac{x}{2}) - \sin(\frac{x}{2}) - 1 = 0$

$(2\sin u + 1)(\sin u - 1) = 0$

$\sin u = -\frac{1}{2}$ $\qquad\qquad$ $\sin u = 1$





$u = \frac{\pi}{2} = \frac{x}{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6} = u = \frac{x}{2}$

$2u = \boxed{\pi = x}$

appears to be the only solution

$x = 2u = \frac{7\pi}{3}, \frac{11\pi}{3}$ → not in $[0, 2\pi)$

$\sin u \sin v = \frac{1}{2}(\cos(u-v) - \cos(u+v))$

$\sin 5\theta \sin 3\theta = \frac{1}{2}(\cos 2\theta - \cos(8\theta))$

can be useful in calculus

$\int \sin 5\theta \sin 3\theta \, d\theta$ is hard!

But $\int \frac{1}{2}(\cos 2\theta - \cos 8\theta)\, d\theta$

$= \frac{1}{2}\int \cos 2\theta \, d\theta - \frac{1}{2}\int \cos 8\theta \, d\theta$ is EZ