§2.3 Questions

#37 Seamus says I missed an extraneous solution

$$\csc x + \cot x = \frac{1}{\sin x} + \frac{\cos x}{\sin x} = 1$$

$$\frac{1+\cos x}{\sin x} - \frac{\sin x}{\sin x} = 0$$

Squaring Both sides was the trick, here

$$\frac{1 + \cos x - \sin x}{\sin x} = 0$$

$$1 + \cos x - \sin x = 0$$

$$1 + \cos x = \sin x$$

$$(1+\cos x)^2 = \sin^2 x$$

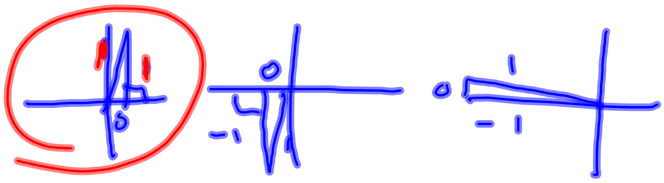
$\underline{1} + \cos^2 x + 2\cos x = \cancel{\sin^2 x} = \underline{1} - \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$2\cos x = 0$ $\qquad$ $\cos x + 1 = 0$

$\cos x = 0$ $\qquad$ $\cos x = -1$



$\frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = \pi$

Check:

$\csc x + \cot x = 1$

$\csc\frac{\pi}{2} + \cot\frac{\pi}{2}$ $\qquad$ $\csc\frac{3\pi}{2} + \cot\frac{3\pi}{2} = 1$

$1 + 0 = 1$ ✓ $\qquad$ $-1 + 0 = 1$ No

$\boxed{\frac{\pi}{2}\text{ GOOD}}$ $\qquad$ $\boxed{\frac{3\pi}{2}\text{ Bad}}$

$\csc\pi + \cot\pi = 1$

$\frac{1}{0}$ $\nexists$ $\qquad$ $\boxed{\pi\text{ Bad}}$

$\boxed{x \in \{\frac{\pi}{2}\}}$

$x = \frac{\pi}{2}$

§2.3 #67

$\cos^{-1}(x) = \arccos(x)$

$2\sin^2 x + 5\cos x = 4$

$2(1-\cos^2 x) + 5\cos x = 4$

$2 - 2\cos^2 x + 5\cos x - 4 = 0$

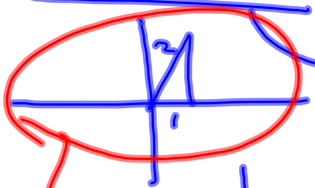
$-2\cos^2 x + 5\cos x - 2 = 0$

$2u^2 - 5u + 2 = 0$

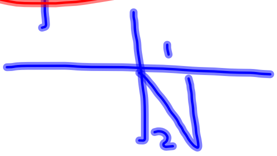
$(2u-1)(u-2) = 0$

$2\cos x - 1 = 0$ $\qquad$ $\cos x - 2 = 0$

$\cos x = \frac{1}{2}$ $\qquad$ Nevah



$\arccos(\frac{1}{2}) = x$



$\cos^2 x = (\cos x)$

$\cos^{-1}(x) \neq \frac{1}{\cos x}$

You have to know from the context when $\cos^{-1}(x)$ is $\arccos x$ or $\frac{1}{\cos x}$

I avoid it, entirely.

Inverse Func or Reciprocal ?! Ambiguity.

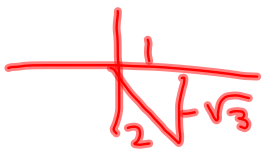
$\arccos(\cos x) = x$ if x is in the right spot.

The only picture for $\arccos(x) = \frac{1}{2}$

Calculator says $60°$

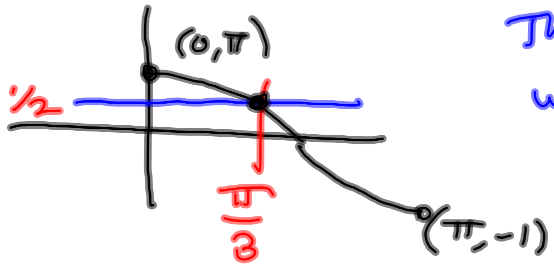
You see it's $\frac{\pi}{3}$

But you need to know there's another solution

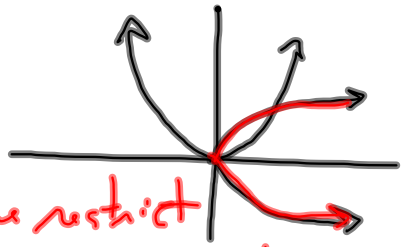


$300°$ or $\frac{5\pi}{3}$ is the other solution in $[0, 2\pi]$

Restricted Cosine:



There's only one place where this guy is $\frac{1}{2}$



$y=\pm\sqrt{x}$ is not a function

we restrict $f(x)= y=x^2$ to $x\geq 0$ to get $f^{-1}(x)=\sqrt{x}$ is an inverse function.

Find inverse of $y=x^2$

$$y^2=x$$

$$\sqrt{y^2}=\sqrt{x}$$

$$|y|=\sqrt{x}$$

$$y=\pm\sqrt{x}$$ Not one y-value for one x-value.

Now if you restrict domain of $y=x^2$ to $x\geq 0$, then $|x|=x$

$$|y|=y$$

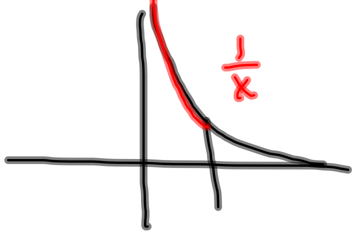
$$\mathcal{D}(x^2)=[0,\infty)=\mathcal{R}(\sqrt{x})$$

$$\mathcal{R}(x^2)=[0,\infty)=\mathcal{D}(\sqrt{x})$$

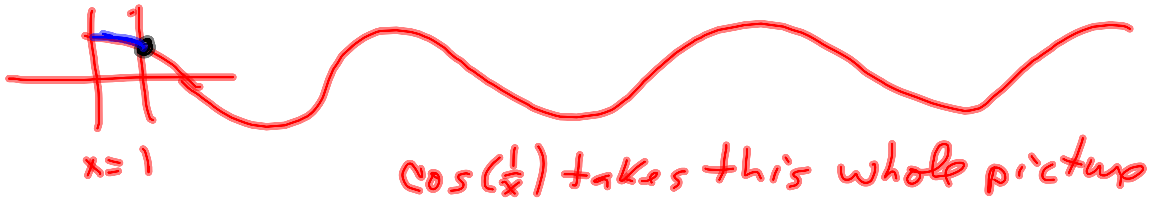
$\cos\left(\frac{1}{x}\right)$

as x goes from 1 to zero,

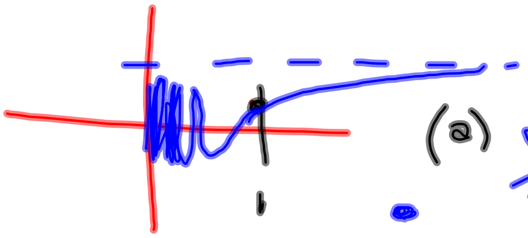
$\frac{1}{x}$ goes from 1 to ∞



We're feeding $\frac{1}{x}$ to cosine. So from $x=1$ to $x=0$ it runs through every input from $1/x=1$ to $1/x=\infty$



$\cos\left(\frac{1}{x}\right)$ takes this whole picture and crams it into the space between 0 & 1.



88. §2.3

(a) $\mathcal{D} = (0,\infty)$
$\mathcal{R} = [-1,1]$

(b) Symmetry: about y-axis
$\left(\cos\left(-\frac{1}{x}\right) = \cos\left(\frac{1}{x}\right)\right)$
It's <u>even</u>

(c) As $x \to 0$, $\cos\left(\frac{1}{x}\right)$ oscillates between -1 & $+1$ infinitely often

(d) $\cos\left(\frac{1}{x}\right)=0$ has an infinite # of solutions

~~2.3~~ If you get it in by tomorrow noon, it won't be late

§2.4 $\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$1^{\underline{st}}$ one covers the $2^{\underline{nd}}$:

Sine is odd
cosine is even

$\sin(u-v) = \boxed{\sin(u+(-v))} \rightarrow$ This 1st formula is all you need.

$= \sin u \cos(-v) + \cos(u)\sin(-v)$

$= \sin u \cos v + \cos(u)(-\sin v)$

$= \sin u \cos v - \cos u \sin v$

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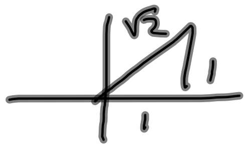
$$\boxed{\begin{aligned}\cos(u+v) &= \cos u\cos v - \sin u \sin v\\ \tan(u+v) &= \frac{\tan u + \tan v}{1-\tan u \tan v}\end{aligned}}$$

For us this will be used to do stuff like $\sin\left(\frac{7\pi}{12}\right)$ without a calculator.

$$\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3} \Longrightarrow$$



$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{2} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}$$

$$= \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$\sin(2x)\cos(3x)$

Example 8

$\sin(x+h)$

$= \sin x \cos h + \cos x \sin h$

comes up when we prove the formula for the derivative of sine.

§2.4 I Wed after questions

Do not turn I & II in together

Separate Assignments.