

S'2.3 Homework will be  
posted later this morning.

S'2.4 by noon? I'll have it up

S'2.3 due Monday

Chapter 1 Re-take

1 week from Wednesday.

$$\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x \sqrt{\sin x}$$

↳ we must assume, here, that  $x$  is such that it makes  $\sin x \geq 0$ .

Recall  $\frac{a+bi}{c+di} \cdot \frac{c-di}{c-di}$  Conjugate trick

#s 24, 30, 31, 32, 40, 41, 42 involve this sort of trick.

$$(24) \quad \frac{\sec \theta - 1}{1 - \cos \theta} = \frac{\sec \theta - 1}{1 - \cos \theta} \cdot \frac{1 + \cos \theta}{1 + \cos \theta}$$

$$= \frac{\sec \theta + \sec \theta \cos \theta - 1 - \cos \theta}{1 - \cos^2 \theta} = \frac{\sec \theta + 1 - 1 - \cos \theta}{\sin^2 \theta}$$

$$= \frac{\sec \theta - \cos \theta}{\sin^2 \theta} = \frac{\frac{1}{\cos \theta} - \cos \theta}{\sin^2 \theta}$$

$$= \frac{\frac{1}{\cos \theta} - \frac{\cos \theta}{1} \cdot \frac{\cos \theta}{\cos \theta}}{\sin^2 \theta} = \frac{\frac{1 - \cos^2 \theta}{\cos \theta}}{\sin^2 \theta}$$

$$= \frac{\frac{\sin^2 \theta}{\cos \theta}}{\frac{\sin^2 \theta}{1}} = \frac{\sin^2 \theta}{\cos \theta} \cdot \frac{1}{\sin^2 \theta} = \frac{1}{\cos \theta} = \sec \theta$$

24 re-done

$$\left(\frac{\sec\theta - 1}{1 - \cos\theta}\right)\left(\frac{\sec\theta + 1}{\sec\theta + 1}\right) = \frac{\sec^2\theta - 1}{\sec\theta + 1 - \cos\theta \sec\theta - \cos\theta}$$

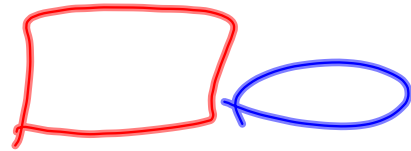
$$= \frac{\tan^2\theta}{\sec\theta + 1 - 1 - \cos\theta} = \frac{\tan^2\theta}{\sec\theta - \cos\theta} = \frac{\tan^2\theta}{\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}}$$

$$= \frac{\frac{\tan^2\theta}{\sin^2\theta}}{\frac{\cos\theta}{\sin^2\theta}} = \frac{\tan^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos\theta} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\sin^2\theta \cos\theta}$$

Mishandled  
the  
fraction.

$$= \frac{\cancel{\sin^2\theta}}{\cos^2\theta \cancel{\sin^2\theta} \cos\theta}$$

Ryan's  
a big  
help.

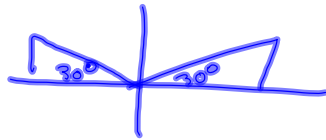
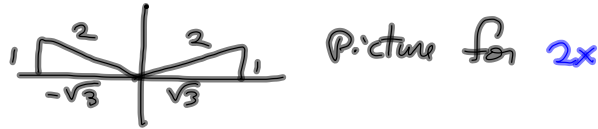


$$\frac{\tan^2\theta}{\left(\frac{\sin^2\theta}{\cos\theta}\right)} = \frac{\tan^2\theta}{\cancel{\tan\theta} \cdot \sin\theta}$$

$$= \frac{\tan\theta}{\sin\theta} = \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\frac{\sin\theta}{1}} = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta$$

$$\sin(2x) = \frac{1}{2}$$

If you can capture all solutions in  $[0, 2\pi]$ , you can find ALL solutions.



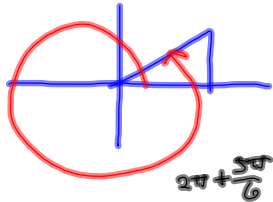
$$2x = 30^\circ = \frac{\pi}{6}$$

OR

$$2x = 150^\circ = \frac{5\pi}{6}$$

So  $x = 15^\circ = \frac{\pi}{12}$  OR  $x = 75^\circ = \frac{5\pi}{12}$

Wait! There's More!



$$2x = \frac{13\pi}{6} = 390^\circ$$

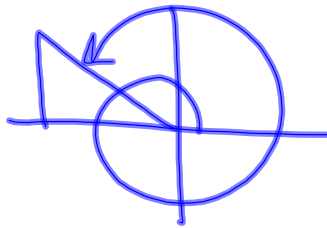
$$\Rightarrow x = \frac{13\pi}{12} = 195^\circ$$

OR

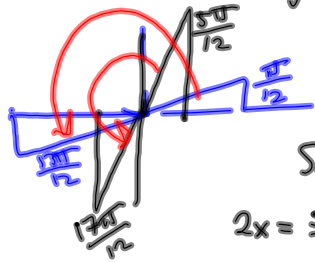
Not used to looking for these solutions on a 2<sup>nd</sup> time around the circle

$$2x = \frac{17\pi}{6} = 510^\circ$$

$$x = \frac{17\pi}{12} = 255^\circ$$



Put it all together =



$$\frac{13\pi}{12} = \frac{18\pi}{12} - \frac{\pi}{12}$$

$$= \frac{2\pi}{12} + \frac{\pi}{12}$$

Solutions for  $2x$ :

$$2x = \frac{5\pi}{6} + 2n\pi$$

OR

$$2x = \frac{\pi}{6} + 2n\pi$$

$$x = \frac{5\pi}{12} + n\pi$$

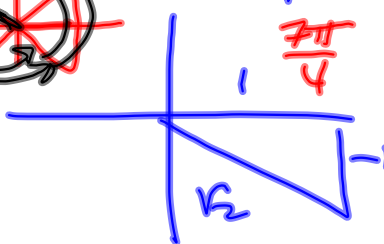
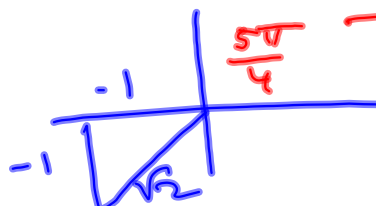
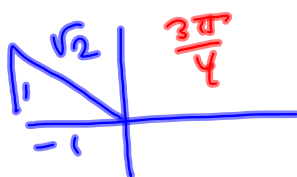
$$x = \frac{\pi}{12} + n\pi$$

} These they are.

$$\sin^2(2x) = \frac{1}{2}$$

$$\sin(2x) = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

2x pictures



Four pictures.

Pictures for x?



$$x = \frac{\pi}{4} + \frac{n\pi}{2}$$

$$x = \frac{3\pi}{4} + \frac{n\pi}{2}$$

$$n \in \mathbb{Z}$$

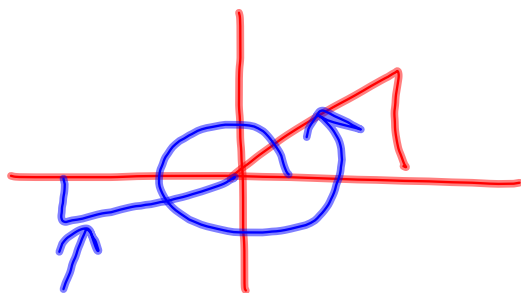
n

Solutions for 2x :

$$2x = \frac{\pi}{4} + n\pi$$

$$2x = \frac{3\pi}{4} + n\pi$$

Just divide  
by 2.



$\frac{9\pi}{8}$  we get by  
dividing  $\frac{9\pi}{4}$  by 2.

$$2x = \frac{\pi}{4}$$

$$2x = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$$

Divide by 2

$$\frac{9\pi}{8}$$

For  $2x$ , you're going to have some  
 $x$ 's from the 2<sup>nd</sup> time around the circle.