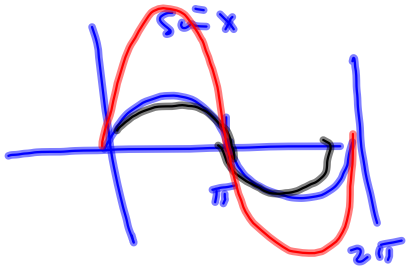
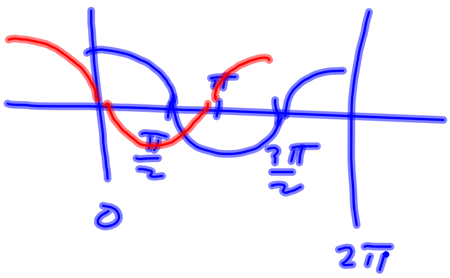
1.6 ~~#69~~ #70

$f(x) = \sin(x) - \cos(x + \frac{\pi}{2})$



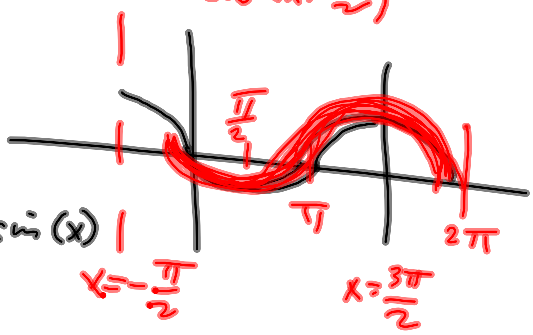
cos x



$\sin(x) - \cos(x + \frac{\pi}{2})$
Basically is

$2\sin(x)$

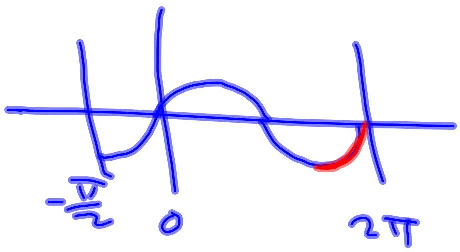
$\cos(x + \frac{\pi}{2})$



$\sin(x) - \cos(x + \frac{\pi}{2}) = 2\sin(x)$

$\boxed{-\sin x = \cos(x + \frac{\pi}{2})}$

$-\cos(x + \frac{\pi}{2})$



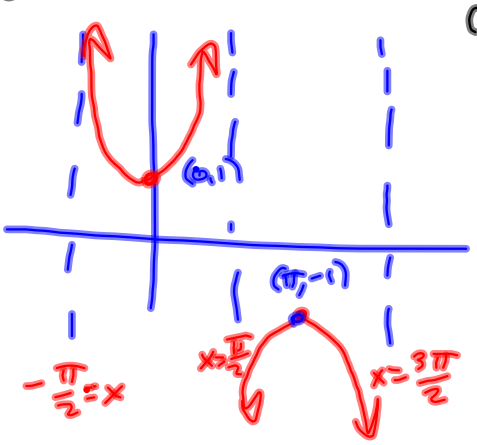
$x - y = 2x$
$-x = y$

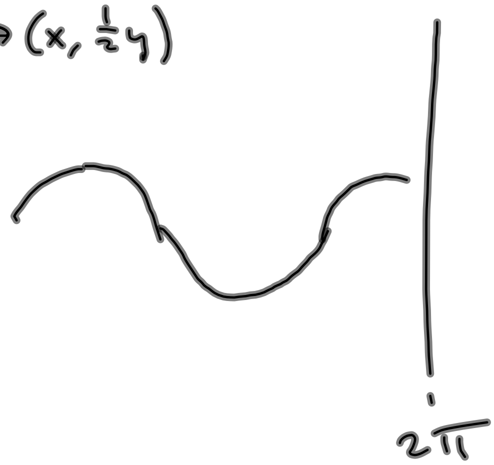
23

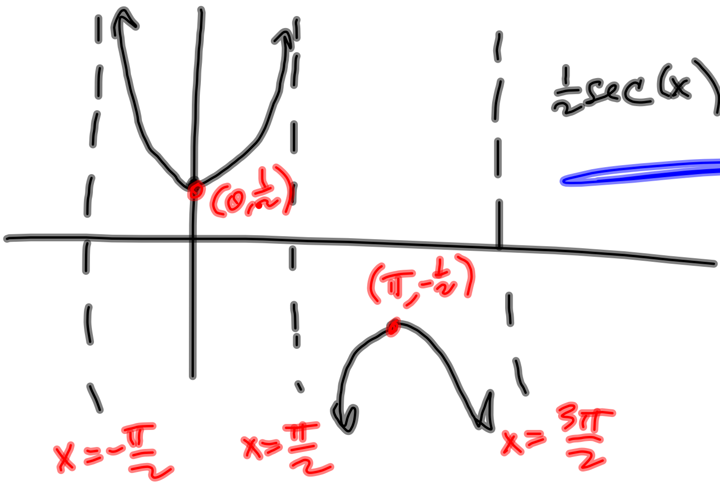
$y = \frac{1}{2}\sec(\pi x)$

$y = \sec(x) \longmapsto \frac{1}{2}y = \frac{1}{2}\sec(x) \longmapsto \frac{1}{2}\sec(\pi x)$

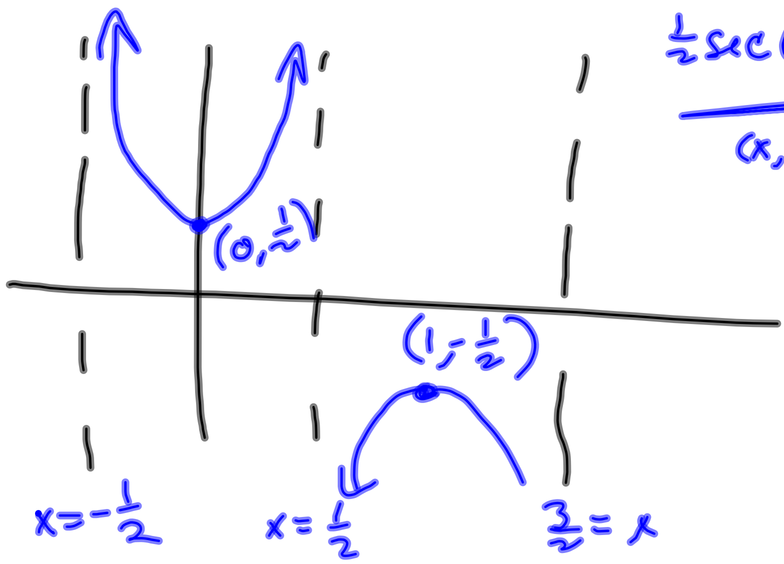
$(x,y) \longmapsto (x, \frac{1}{2}y)$







$\frac{1}{2}\sec(x) = \frac{1}{2}f(x)$



$\frac{1}{2}\sec(\pi x) = \frac{1}{2}f(\pi x)$

$(x,y) \longmapsto (\frac{1}{\pi}x, y)$

$f(x)$

$af(x)$

$(x,y) \longmapsto (x, ay)$

$f(bx)$

$(x,y) \longmapsto (\frac{1}{b}x, y)$

$f(x)+d$

$(x,y) \longmapsto (x, y+d)$

$f(x+c)$

$(x,y) \longmapsto (x-c, y)$

Feb 19-6:10 PM

§1.7 Inverse Functions

$f^{-1}(f(x)) = x$

$f(x) = 5^x \Longrightarrow f^{-1}(x) =$

$5^x = 17$

$\log_5(5^x) = \log_5(17)$

$x = \log_5(7)$

$\log_5(37)$

$x \longrightarrow 5^x \longrightarrow \log_5(5^x) = x$

$f(x) = 5^x$

$f^{-1}(x) = \log_5(x)$

$f^{-1}(f(x)) = \log_5(5^x) = x$

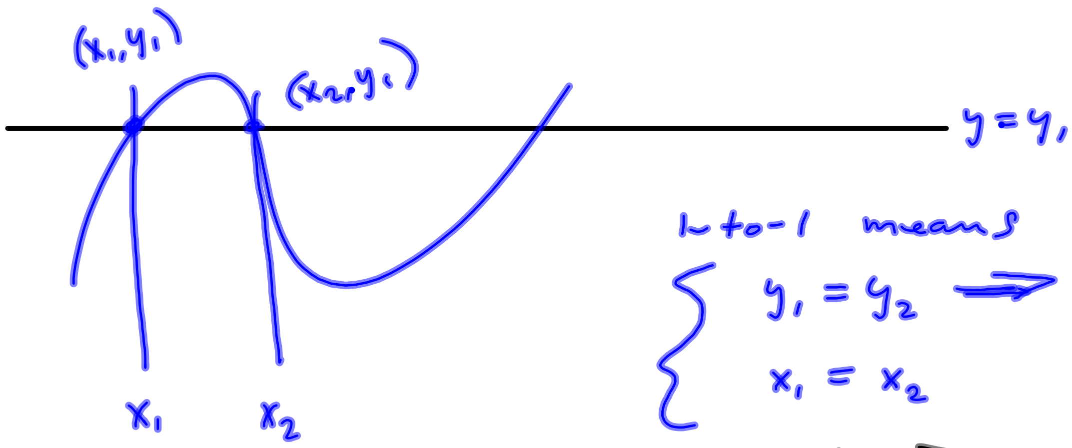
$\log_5(25) = 2$

$\log_5(5^2) = 2$

Feb 19-6:14 PM

$f^{-1}(x)$ is a function if

$f(x)$ is $\underline{\text{1-to-1}}$.   Andrew



1-to-1 means

$\left\{ \begin{array}{l} y_1 = y_2 \Longrightarrow \\ x_1 = x_2 \end{array} \right.$

$\Longleftrightarrow$

$\left\{ \begin{array}{l} x_1 \neq x_2 \Longrightarrow \\ y_1 \neq y_2 \end{array} \right.$

Not 1-to-1, b/c
$y_1 = y_2$, but $x_1 \neq x_2$

$A \Longrightarrow B$ is equivalent to
Not $B \Longrightarrow$ Not A.

How does this apply to Trig?

We want to know the angle, given its trig value.

We want to solve

$$\sin\theta = \frac{\sqrt{3}}{2}$$

If only we had an inverse sine...

CAUTION: $\sin^{-1}(x)$ means the angle whose sine is x, NOT $\frac{1}{\sin(x)}$

Another way to say it:

$$\sin^{-1}(x) = \arcsin(x)$$

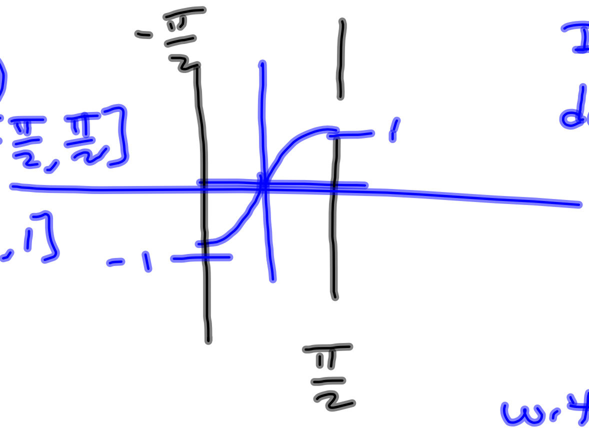
Weird Thing: Sine is NOT 1-to-1.

But it IS 1-to-1 on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin(x)$

$D = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$R = [-1, 1]$



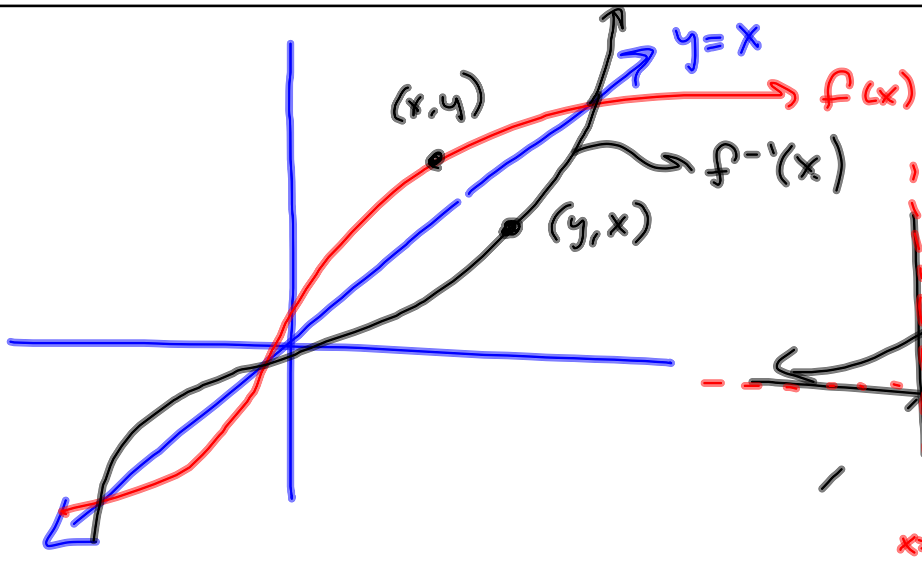
If we restrict the domain of sine to $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, it's 1-to-1, and so Arcsine is a function with

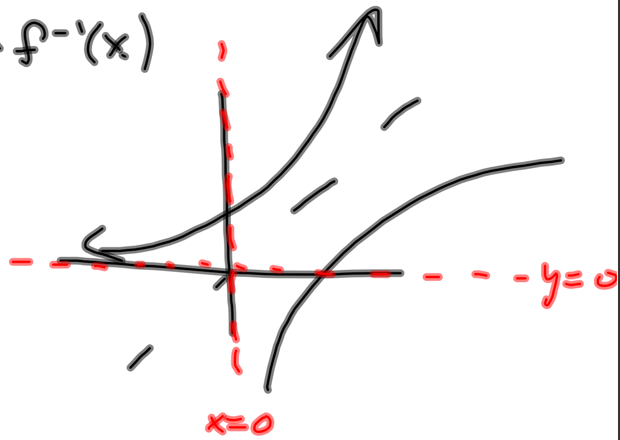
Domain $= [-1, 1]$

Range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$

Range of f = Domain of f^{-1}

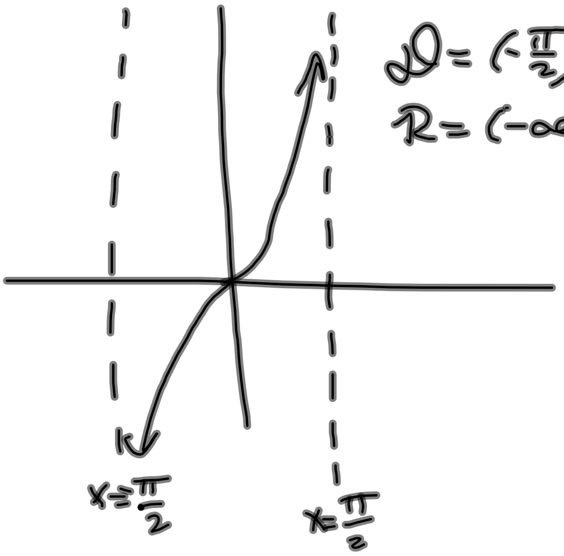
Domain of f = Range of f^{-1}





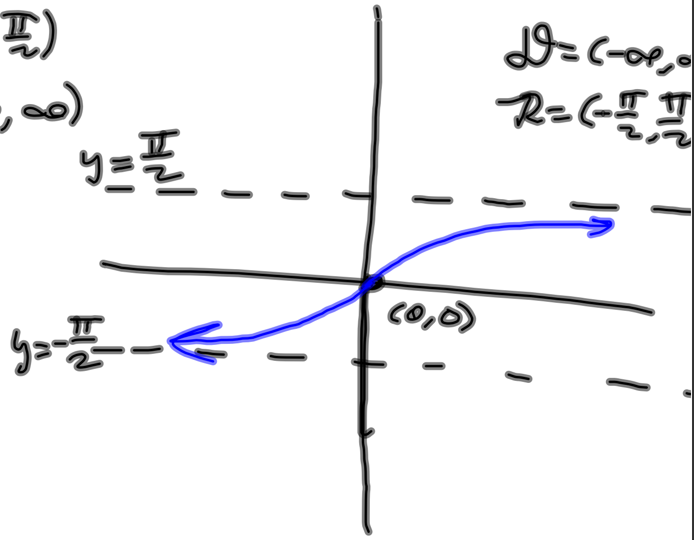
$f(x) = \tan(x)$

$f^{-1}(x) = \arctan(x)$



$\mathcal{D} = (-\frac{\pi}{2}, \frac{\pi}{2})$

$\mathcal{R} = (-\infty, \infty)$



$\mathcal{D} = (-\infty, \infty$

$\mathcal{R} = (-\frac{\pi}{2}, \frac{\pi}{2}$

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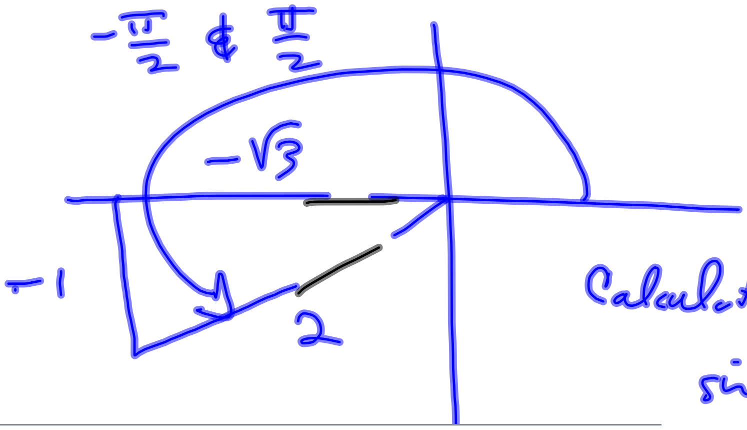
CAUTION

There are other angles than just $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ for sine.

arcsine only spits out angles between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$

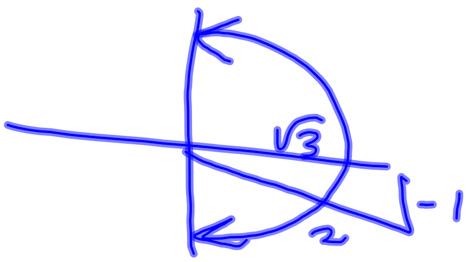


Calculator says $\sin^{-1}$ for arcsine

$\arcsin\left(-\frac{1}{2}\right)$

$-\frac{1}{6}\pi$

All it sees is



You need to know what quadrant you're in, before "trusting" $\cos^{-1}$, $\sin^{-1}$, $\tan^{-1}$ keys.

```
tan(π/2+.01)
   -99.99666664
sin⁻¹(-1/2)
   -.5235987756
Ans/π*180
            -30
```

Read §1.7

write down domains & ranges of those beasts.