I never mess with 5'1.5

$f(x) = \sin(x)$

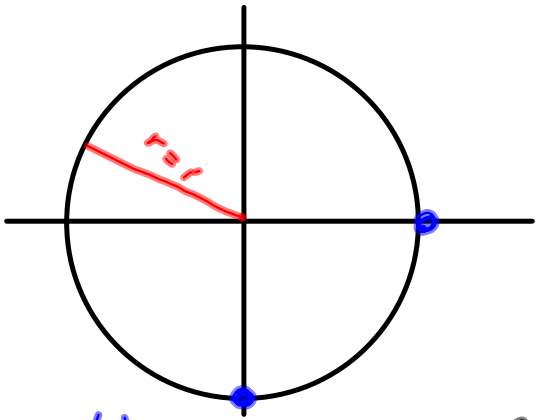
$g(x) = a\sin(bx+c) + d$

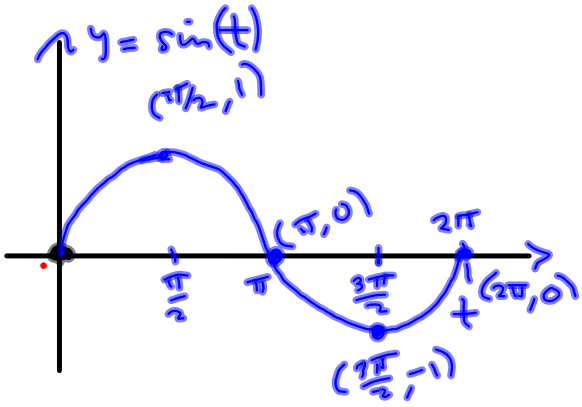
$= a\sin\left(b\left(x+\frac{c}{b}\right)\right) + d$

Relabel:

$g(x) = a\sin(b(x+c)) + d$

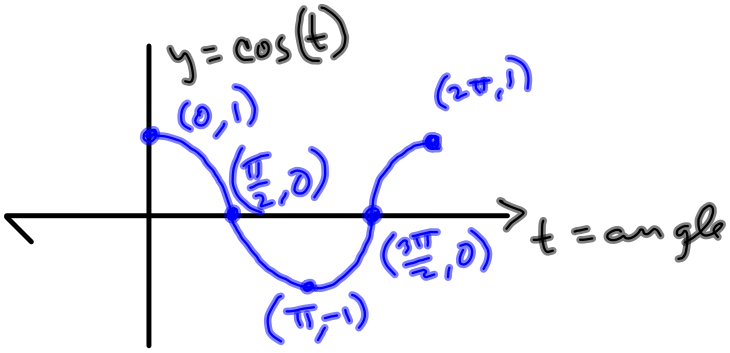
This is what we work with.





sin(t) tracks the y-values

Likewise, cos(t) tracks the x-values.



$f(x)$ = Basic Function $\quad y = f(x)$

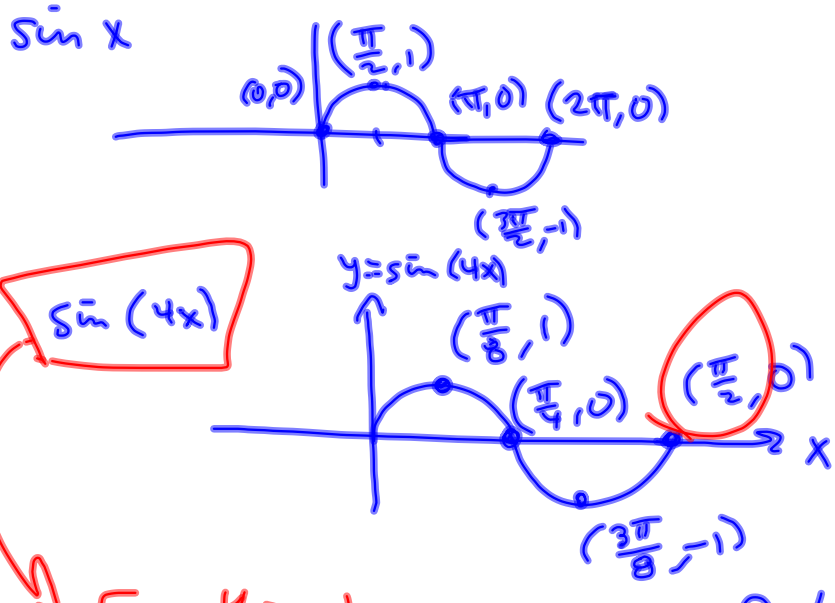
Let (x, y) be on its graph. $\quad y = 3f(x)$

Then $3f(x)$ has $(x, 3y)$ — Vertical Stretch

$f(3x)$ has $\left(\frac{1}{3}x, y\right)$ — Horizontal Stretch

$f(x) + 3$ has $(x, y+3)$ — Vertical Shift up 3

$f(x+3)$ has $(x-3, y)$ — Left shift left 3



Everything happens 4 times faster.
Shrinks towards y-axis by factor of $\frac{1}{4}$.

Period of $\sin x$ is 2π

Period of $\sin 4x$ is $\frac{2\pi}{4} = \frac{\pi}{2}$

.. .. $\sin(bx)$ is $\frac{2\pi}{b}$

$g(x) = \sin\left(\frac{\pi}{6}x\right)$

$(x, y) \longmapsto \left(\frac{6}{\pi}x, y\right)$

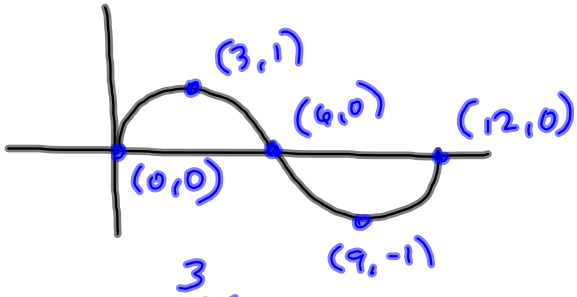
$(0,0) \to (0,0)$

$\left(\frac{\pi}{2}, 1\right) \to (3,1)$

$(\pi, 0) \to (6,0)$

$\left(\frac{3\pi}{2}, -1\right) \to (9,-1)$

$(2\pi, 0) \to (12,0)$



$\frac{3}{\frac{\pi}{6}} \cdot \frac{3\pi}{2} = 9$

$g(x) = \sin(x - 2\pi)$ is same graph.

Here's how to break 'em down.

$$y = \frac{3}{5}\sin\left(\frac{x}{3} - \frac{\pi}{6}\right)$$

$$\frac{\frac{\pi}{6}}{\frac{1}{3}} = \frac{3\pi}{6} = \frac{\pi}{2}$$

$$= \frac{3}{5}\sin\left(\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right)$$

$(x,y) \longmapsto (x+\frac{\pi}{2}, y)$

$(x,y) \longmapsto (x, \frac{3}{5}y)$
Vertical shrink

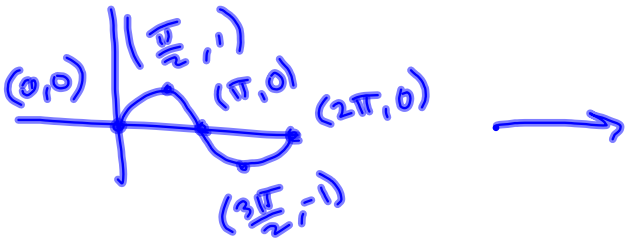
$(x,y) \longmapsto (3x,y)$
Horizontal Stretch

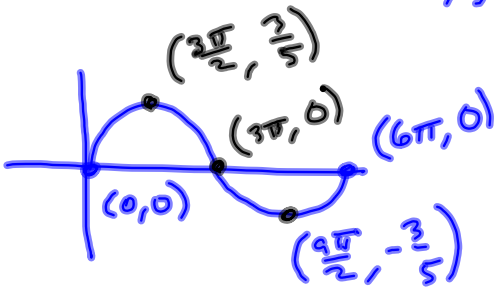
1. Vertical & Horizontal Stretch
2. Horizontal & Vertical Shifts*

* Rigid Transformations.

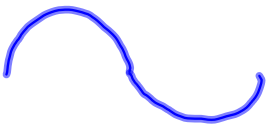
$\sin x \longmapsto \frac{3}{5}\sin\left(\frac{1}{3}x\right) \longmapsto \frac{3}{5}\sin\left(\frac{1}{3}\left(x-\frac{\pi}{2}\right)\right)$

$(x,y) \longmapsto (3x, \frac{3}{5}y)$ $\qquad (x,y) \longmapsto (x+\frac{\pi}{2}, y)$

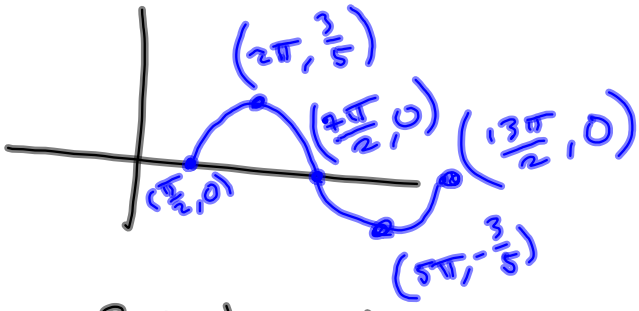


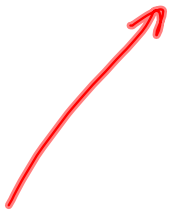


$f(x) = \sin x$



$\frac{3}{5}f\left(\frac{1}{3}x\right) = \frac{3}{5}\sin\left(\frac{1}{3}x\right)$





$\frac{3}{5}f\left(\frac{1}{3}\left(x-\frac{\pi}{2}\right)\right)$

Model Tides:

High: 30 ft @ 12am = 0 hr

Low: 5 ft @ 12pm = 12 hr

Build a trig function that models this

Amplitude = A : is $A \sin x$

$= 12.5$ ft

Period = Wavelength = $T = 24$ hrs

Mid-height = $\frac{35}{2} = 17.5$

$A = 12.5$: $12.5 \cos x$ ← starts @ high Point at $x = 0$

$T = 24$

$\cos(bx) \Rightarrow T =$

when does bx get to 2π?

Want $bx = 2\pi$ when $x = 24$

$24b = 2\pi$

$b = \frac{2\pi}{24} = \frac{\pi}{12}$

$$12.5 \cos\left(\frac{\pi}{12} x\right)$$

want midpt to be 17.5

$g(x) = 12.5 \cos\left(\frac{\pi}{12}x\right) + 17.5$

I lied. High Tide is 3am.
Low Tide is 3pm :

$h(x) = 12.5 \cos\left(\frac{\pi}{12}(x-3)\right) + 17.5$

§1.5 #s 1-21, 39-45, 61, 65, 83, 85, 87

#87 T = period = $\frac{2\pi}{b}$

f = frequency = $\frac{1}{T}$