, Left off on coterminal angles.

Bonus question: (counts as 1 homework)
Thrown on top of homework.

Find two angles, α & β, that are coterminal with $\theta = \frac{163\pi}{6}$. One positive, one negative; both between -2π & $+2\pi$.
Modding out. Finding $\theta \bmod 2\pi$.

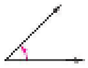
Try checking out Brooks/Cole or cengage.com for eBook. Publisher likely has free trial period.

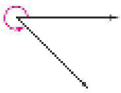
Vocabulary: **Fill in the blanks.**

1. Two angles that have the same initial and terminal sides are ________.
2. One ________ is the measure of a central angle that intercepts an arc equal to the radius of the circle.
3. Two positive angles that have a sum of $\pi/2$ are ________ angles, whereas two positive angles that have a sum of π are ________ angles.
4. The angle measure that is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about an angle's vertex is one ________.
5. The ________ speed of a particle is the ratio of the arc length to the time traveled, and the ________ speed of a particle is the ratio of the central angle to the time traveled.
6. The area A of a sector of a circle with radius r and central angle θ, where θ is measured in radians, is given by the formula ________.

Skills and Applications

Estimating an Angle In Exercises 7–10, estimate the angle to the nearest one-half radian.

7. 

8. 

9. 

10. 

Determining Quadrants In Exercises 11 and 12, determine the quadrant in which each angle lies.

11. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$ **12.** (a) $-\frac{\pi}{6}$ (b) $-\frac{11\pi}{9}$

Sketching Angles In Exercises 13 and 14, sketch each angle in standard position.

13. (a) $\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$ **14.** (a) $\frac{5\pi}{2}$ (b) 4

Finding Coterminal Angles In Exercises 15 and 16, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.

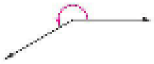
15. (a) $\frac{\pi}{6}$ (b) $\frac{7\pi}{6}$ **16.** (a) $\frac{2\pi}{3}$ (b) $-\frac{9\pi}{4}$

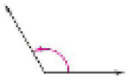
Complementary and Supplementary Angles In Exercises 17–20, find (if possible) the complement and the supplement of each angle.

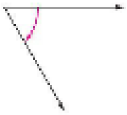
17. (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ **18.** (a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$

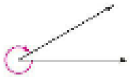
19. (a) 1 (b) 2 **20.** (a) 3 (b) 1.5

Estimating an Angle In Exercises 21–24, estimate the number of degrees in the angle.

21. 

22. 

23. 

24. 

Determining Quadrants In Exercises 25 and 26, determine the quadrant in which each angle lies.

25. (a) $130°$ (b) $8.3°$

26. (a) $-132° 50'$ (b) $-3.4°$

Sketching Angles In Exercises 27 and 28, sketch each angle in standard position.

27. (a) $270°$ (b) $120°$

28. (a) $-135°$ (b) $-750°$

Finding Coterminal Angles In Exercises 29 and 30, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

29. (a) $45°$ (b) $-36°$

30. (a) $120°$ (b) $-420°$

Complementary and Supplementary Angles In Exercises 31–34, find (if possible) the complement and the supplement of each angle.

31. (a) $18°$ (b) $85°$ **32.** (a) $46°$ (b) $93°$

33. (a) $150°$ (b) $79°$ **34.** (a) $130°$ (b) $170°$

Converting from Degrees to Radians In Exercises 35 and 36, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

35. (a) $120°$ (b) $-20°$

36. (a) $-60°$ (b) $144°$

Converting from Radians to Degrees In Exercises 37 and 38, rewrite each angle in degree measure. (Do not use a calculator.)

37. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$

38. (a) $-\frac{7\pi}{12}$ (b) $\frac{5\pi}{4}$

Converting from Degrees to Radians In Exercises 39–42, convert the angle measure from degrees to radians. Round to three decimal places.

39. $45°$ **40.** $-48.27°$

41. $0.54°$ **42.** $345°$

Converting from Radians to Degrees In Exercises 43–46, convert the angle measure from radians to degrees. Round to three decimal places.

43. $\frac{5\pi}{11}$ **44.** $\frac{15\pi}{8}$

45. -4.2π **46.** -0.57

Converting to Decimal Degree Form In Exercises 47 and 48, convert each angle measure to decimal degree form without using a calculator. Then check your answers using a calculator.

47. (a) $54° 45'$ (b) $-128° 30'$

48. (a) $-135° 36''$ (b) $-408° 16' 20''$

Converting to D°M′S″ Form In Exercises 49 and 50, convert each angle measure to degrees, minutes, and seconds without using a calculator. Then check your answers using a calculator.

49. (a) $240.6°$ (b) $-145.8°$

50. (a) $-345.12°$ (b) $-3.58°$

Finding Arc Length In Exercises 51 and 52, find the length of the arc on a circle of radius r intercepted by a central angle θ.

51. $r = 15$ inches, $\theta = 120°$

52. $r = 3$ meters, $\theta = 150°$

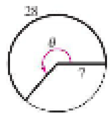
Finding the Central Angle In Exercises 53 and 54, find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.

53. $r = 80$ kilometers, $s = 150$ kilometers

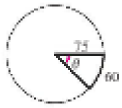
54. $r = 14$ feet, $s = 8$ feet

Finding an Angle In Exercises 55 and 56, use the given arc length and radius to find the angle θ (in radians).

55.



56.



Area of a Sector of a Circle In Exercises 57 and 58, find the area of the sector of a circle of radius r and central angle θ.

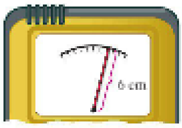
57. $r = 12$ millimeters, $\theta = \frac{\pi}{4}$

58. $r = 2.5$ feet, $\theta = 225°$

59. Distance Between Cities Find the distance between Dallas, Texas, whose latitude is $32° 47' 39''$ N, and Omaha, Nebraska, whose latitude is $41° 15' 50''$ N. Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (Omaha is due north of Dallas).

60. Difference in Latitudes Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Lynchburg, Virginia, and Myrtle Beach, South Carolina, where Lynchburg is about 400 kilometers due north of Myrtle Beach?

61. Instrumentation The pointer on a voltmeter is 6 centimeters in length (see figure). Find the number of degrees through which the pointer rotates when it moves 2.5 centimeters on the scale.



62. Linear Speed A satellite in a circular orbit 1250 kilometers above Earth makes one complete revolution every 110 minutes. Assuming that Earth is a sphere of radius 6378 kilometers, what is the linear speed (in kilometers per minute) of the satellite?

63. Angular and Linear Speeds The circular blade on a saw rotates at 5000 revolutions per minute.

(a) Find the angular speed of the blade in radians per minute.

(b) The blade has a diameter of $7\frac{1}{4}$ inches. Find the linear speed of a blade tip.

64. Angular and Linear Speeds A carousel with a 50-foot diameter makes 4 revolutions per minute.

(a) Find the angular speed of the carousel in radians per minute.

(b) Find the linear speed (in feet per minute) of the platform rim of the carousel.

65. Angular and Linear Speeds A DVD is approximately 12 centimeters in diameter. The drive motor of the DVD player rotates between 200 and 500 revolutions per minute, depending on what track is being read.

(a) Find an interval for the angular speed of the DVD as it rotates.

(b) Find an interval for the linear speed of a point on the outermost track as the DVD rotates.

66. Angular Speed A car is moving at a rate of 65 miles per hour, and the diameter of its wheels is 2 feet.

(a) Find the number of revolutions per minute the wheels are rotating.

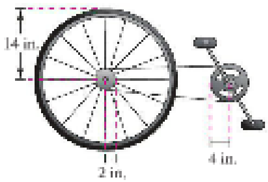
(b) Find the angular speed of the wheels in radians per minute.

67. Linear and Angular Speeds A computerized spin balance machine rotates a 25-inch-diameter tire at 480 revolutions per minute.

(a) Find the road speed (in miles per hour) at which the tire is being balanced.

(b) At what rate should the spin balance machine be set so that the tire is being tested for 55 miles per hour?

68. Speed of a Bicycle The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second.



(a) Find the speed of the bicycle in feet per second and miles per hour.

(b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

(c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).

69. Area A sprinkler on a golf green sprays water over a distance of 15 meters and rotates through an angle of 140°. Draw a diagram that shows the region that the sprinkler can irrigate. Find the area of the region.

70. Area A car's rear windshield wiper rotates 125°. The total length of the wiper mechanism is 25 inches and wipes the windshield over a distance of 14 inches. Find the area covered by the wiper.

Exploration

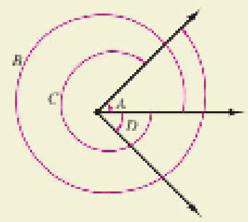
True or False? **In Exercises 71–73, determine whether the statement is true or false. Justify your answer.**

71. A measurement of 4 radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

72. The difference between the measures of two coterminal angles is always a multiple of 360° when expressed in degrees and is always a multiple of 2π radians when expressed in radians.

73. An angle that measures $-1260°$ lies in Quadrant III.

74. HOW DO YOU SEE IT? Determine which angles in the figure are coterminal angles with angle A. Explain your reasoning.



75. Think About It A fan motor turns at a given angular speed. How does the speed of the tips of the blades change when a fan of greater diameter is on the motor? Explain.

76. Think About It Is a degree or a radian the greater unit of measure? Explain.

77. Writing When the radius of a circle increases and the magnitude of a central angle is constant, how does the length of the intercepted arc change? Explain your reasoning.

78. Proof Prove that the area of a circular sector of radius r with central angle θ is

$$A = \frac{1}{2}\theta r^2$$

where θ is measured in radians.

Jan 17-8:28 AM

Recall coterminal

θ is coterminal with any

$\alpha = \theta + 2n\pi$, where $n \in \mathbb{Z}$.

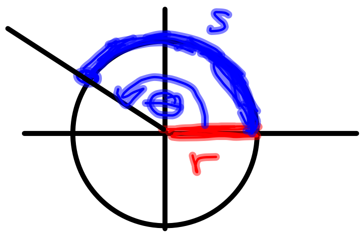
$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\theta + 2\pi$

$\theta + (-17)(2\pi)$

" modding θ out by 2π. "

Radian measure of θ is $\theta = \frac{s}{r}$



$\frac{17\pi}{3}$ to see where it lies, you want to think in multiples of π & 2π.

$\frac{17}{3}\pi =$

$17 = 5(3) + 2$

Remainder

$\frac{17}{3} = 5 + \frac{2}{3}$

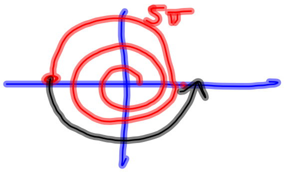
Divisor

Quotient

$\frac{17\pi}{3} = \left(5+\frac{2}{3}\right)\pi = 5\pi + \frac{2\pi}{3}$

Go to 5π

Add $\frac{2\pi}{3}$ to that

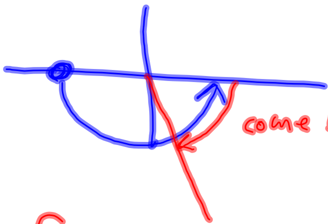


To locate $\frac{2\pi}{3}$, look at nearest integer multiple of π.

$\frac{2\pi}{3}$ is just under π:

$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$ = reference

$\frac{\pi}{3}$ from adding another full π to the 5π we have.

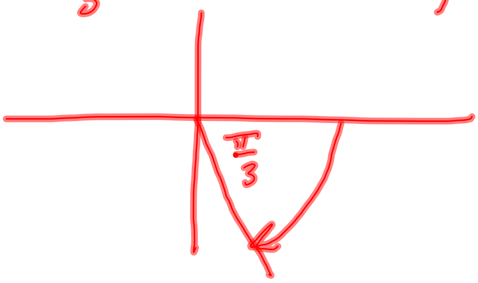


come back $\frac{\pi}{3}$ from the $5\pi + 1\pi$

π is $180°$.

Reference Angle is the acute angle measured from the x-axis.

$\frac{\pi}{3}$ = reference angle.

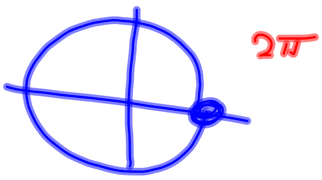


is the picture for $\theta = \frac{17\pi}{3}$.

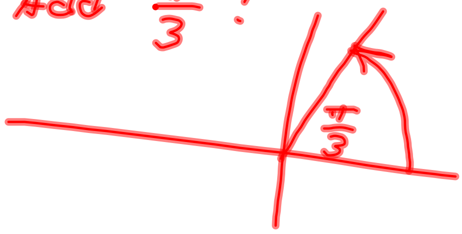
$$\frac{7\pi}{3} = \frac{6\pi}{3} + \frac{1\pi}{3} = 2\pi + \frac{\pi}{3}$$

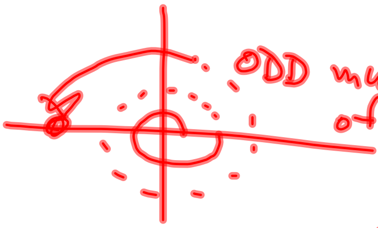
ref. angle.

$$\frac{\pi}{3} = \frac{7\pi}{3} \mod 2\pi$$



Add $\frac{\pi}{3}$:





ODD multiple of π.

even multiple of π

Complementary Angles:

$\alpha, \beta > 0, \ \alpha + \beta = \frac{\pi}{2} \ (90^\circ)$

Nonexample.

$\alpha = \frac{2\pi}{3}$ They ain't one!

$$6\left[\frac{2\pi}{3} + x = \frac{\pi}{2}\right]$$

$$4\pi + 6x = 3\pi$$

$$6x = -\pi$$

$$x = -\frac{\pi}{6} = \beta$$

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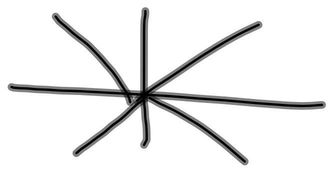
Supplementary Angles:

$$\alpha, \beta > 0, \quad \alpha + \beta = \pi \quad (180^\circ)$$

$$(\beta = \pi - \alpha)$$

Degrees to Radians



$$360^\circ = 2\pi \text{ radians.}$$

$$(360)(1^\circ) = 2\pi \text{ radians}$$

$$1^\circ = \frac{2\pi \text{ radians}}{360} = \frac{\pi}{180} \text{ radians.}$$

$$45^\circ = (45^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right) = \frac{\pi}{4} \text{ radians.}$$

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$$2\pi \text{ rad} = 360^\circ$$

$$(2\pi)(1\text{rad}) = 360^\circ$$

$$1\text{rad} = \frac{360^\circ}{2\pi} = \frac{180}{\pi} \text{ degrees}$$