

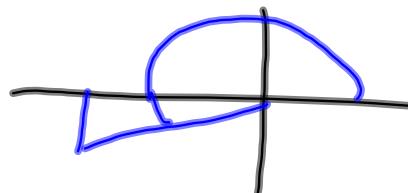
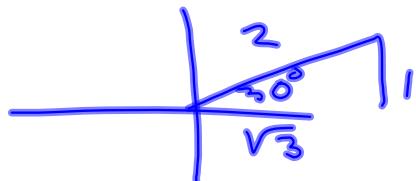
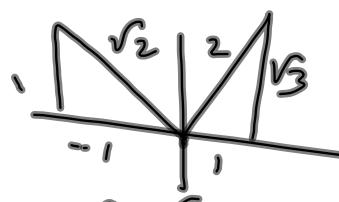
$$\sin\left(\frac{13\pi}{12}\right) = \sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right)$$

$$= \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{3\pi}{4}$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) = \frac{1 - \sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\sin\frac{13\pi}{12} = \sin\left(-\frac{\frac{13\pi}{6}}{2}\right) = -\sqrt{\frac{1 - \cos\left(\frac{13\pi}{6}\right)}{2}}$$

$$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$



$$\sin\left(\frac{7 \cdot \text{Pi}}{12}\right)$$

$$\sin\left(\frac{5}{12} \pi\right)$$

$$\frac{(\sqrt{2} - \sqrt{6})}{4}$$

$$\frac{1}{4} \sqrt{2} - \frac{1}{4} \sqrt{6}$$

evalf(%)

$$-0.2588190453$$

$$-\sqrt{\frac{\left(1 - \frac{\sqrt{3}}{2}\right)}{2}}$$

$$\frac{1}{4} \sqrt{2} - \frac{1}{4} \sqrt{6}$$

evalf(%)

$$-0.2588190453$$

$$-\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{-\frac{\frac{2 - \sqrt{3}}{2}}{2}}$$

$$= -\sqrt{\frac{2 - \sqrt{3}}{4}} = -\sqrt{\frac{1}{2} - \frac{\sqrt{3}}{4}}$$

$$= -\sqrt{\frac{2 - \sqrt{3}}{4}} = -\frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$= -\frac{1}{2} \sqrt{2 - \sqrt{3}} = -\frac{1}{2} \sqrt{\frac{(2 - \sqrt{3})(2 + \sqrt{3})}{(2 + \sqrt{3})}}$$

$$= -\frac{1}{2} \sqrt{\frac{4 - 3}{2 + \sqrt{3}}} = -\frac{1}{2} \sqrt{\frac{1}{2 + \sqrt{3}}}$$

$$= -\frac{1}{2} \frac{1}{\sqrt{2 + \sqrt{3}}}$$

~~Reciprocal Identities~~

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

~~Quotient Identities~~

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

~~Pythagorean Identities~~

$$\sin^2 u + \cos^2 u = 1$$

~~$$1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$~~

~~Cofunction Identities~~

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \quad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \quad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

~~Double-Angle Formulas~~

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

~~Power-Reducing Formulas~~

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

~~Sum-to-Product Formulas~~

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

~~Even/Odd Identities~~

~~$\sin(-u) = -\sin u$
 $\cot(-u) = -\cot u$
 $\cos(-u) = \cos u$
 $\sec(-u) = \sec u$
 $\tan(-u) = -\tan u$
 $\csc(-u) = -\csc u$~~

Sum and Difference Formulas

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\begin{aligned}\sin u \sin v &= \frac{1}{2}[\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2}[\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2}[\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2}[\sin(u+v) - \sin(u-v)]\end{aligned}$$

From the Power-Reducing Formulas,

we have the half-angle formulas:

$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1-\cos(u)}{2}}$, $\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1+\cos(u)}{2}}$, with the \pm being controlled by the quadrant in which $\frac{u}{2}$ lies, and therefore $\frac{u}{2}$, reside. $\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin(u)}{1+\cos(u)}$ may bear discussing in class. But basically, this last formula's sign will follow the sign of sine.

$$\begin{aligned}\sin^2 \theta &= \frac{1-\cos(2\theta)}{2} \\ \cos^2 \theta &= \frac{1+\cos(2\theta)}{2}\end{aligned}\quad \left.\right\} \text{from Double-angle, which is from angle sum formula}$$

$$\sin(u+v) = \sin u \cos v + \sin v \cos u$$

$$\sin(u+u) = \sin u \cos u + \sin u \cos u$$

$$\boxed{\sin(2u) = 2 \sin u \cos u}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u+u) = \cos u \cos u - \sin u \sin u$$

$$\begin{aligned}\cos(2u) &= \cos^2 u - \sin^2 u \\ &= \cos^2 u - (1 - \cos^2 u) \\ &= 2\cos^2 u - 1 \\ &= 2(1 - \sin^2 u) - 1 \\ &= 2 - 2\sin^2 u - 1 \\ &= 1 - 2\sin^2 u\end{aligned}$$

$$1 - 2\sin^2 u = \cos(2u)$$

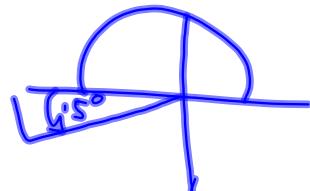
$$-2\sin^2 u = \cos(2u) - 1$$

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$

$$\sin u = \pm \sqrt{\frac{1 - \cos(2u)}{2}}$$

You decide on \pm by where u lives.

$$\begin{aligned}\sin \frac{13\pi}{12} &= \sin \left(\frac{13\pi}{12}, \frac{180^\circ}{\pi}\right) \\ &= \sin(195^\circ)\end{aligned}$$



$$\begin{aligned}
 &= \sin(135^\circ + 60^\circ) \\
 &= \sin 135^\circ \cos 60^\circ + \sin 60^\circ \cos 135^\circ
 \end{aligned}$$

etc.

Angle Sum Method.

Half-angle

$$\sin(195^\circ) = \sin\left(\frac{390^\circ}{2}\right) = -\sqrt{\frac{1-\cos 390^\circ}{2}}$$

$$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

$$\stackrel{!}{=} \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

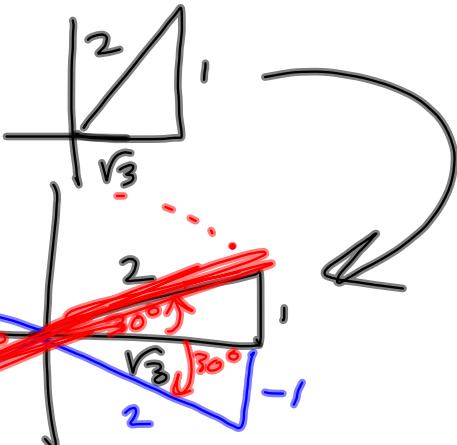
$$3 \sec^2(5x) - 4 = 0$$

$$\sec^2(5x) = \frac{4}{3}$$

$$\sec(5x) = \pm \frac{2}{\sqrt{3}}$$

$$\frac{1}{\cos(5x)} = \pm \frac{2}{\sqrt{3}} = \pm \frac{2\sqrt{3}}{3}$$

$$\cos(5x) = \pm \frac{\sqrt{3}}{2}$$



$$5x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

$$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$150^\circ + 180^\circ n$$

$$5x = 30^\circ + 360^\circ n, \quad \left[\begin{array}{l} 150^\circ + 360^\circ n \\ 210^\circ + 360^\circ n \\ 330^\circ + 360^\circ n \end{array} \right] \quad 30^\circ + 180^\circ n$$

$$\Rightarrow x = 6^\circ + 72^\circ n$$

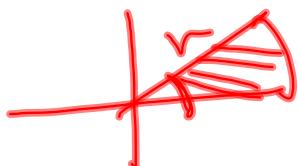
$$30^\circ + 72^\circ n$$

$$42^\circ + 72^\circ n$$

$$66^\circ + 72^\circ n$$

$$S = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



$$\pi r^2 = \frac{1}{2}(2\pi)r^2$$

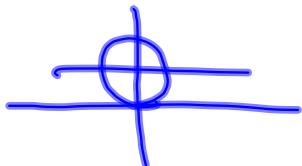
$$r = \sin(3\theta) ?$$



$$(y-1)^2 = 1-x^2$$

$$y = \pm \sqrt{1-x^2} + 1$$

$$x^2 + (y-1)^2 = 1$$



$$r = \sin \theta$$

