$$
\begin{aligned}
& \tan \left(\frac{\pi}{2}-\theta\right) \tan \theta \\
= & \cot \theta \tan \theta=1
\end{aligned}
$$

Requines Cofunction Identity

$$
\begin{aligned}
& \tan \left(\frac{\pi}{2}-\theta\right)=\tan \left(-\theta+\frac{\pi}{2}\right) \\
& =\frac{\tan \left(-\left(\theta-\frac{\pi}{2}\right)\right)}{\tan \theta=\frac{\sin \theta}{\cos \theta}=\frac{-}{1}=-}=-\tan \left(\theta-\frac{\pi}{2}\right)
\end{aligned}
$$





$$
\begin{aligned}
x= & -\frac{\pi}{2} \quad x=\frac{\pi}{2} \\
& -\tan \theta
\end{aligned}
$$

$$
\begin{gathered}
x=0 \\
-\tan \left(\theta-\frac{x^{x}}{2}\right)^{\prime}=\pi \\
-\cot \theta
\end{gathered}
$$

what if $r=1$ ?
$r$
$\theta$
$x^{2}+y^{2}=r^{2}$
$x^{2}+y^{2}=1$ \& $\sin c x$
since $\sin \theta=\frac{y}{r}=y$
and $\cos \theta=\frac{x}{r}=x$,

$$
\sin ^{2} \theta+\cos ^{2} \theta=1
$$

Pythagorean Identity
$\sqrt{1-\sin ^{2} \theta}$

$$
=\sqrt{\cos ^{2} \theta}
$$

$$
=|\cos \theta|
$$

$$
\frac{\sqrt{x^{2}}}{(3)^{2}}=\frac{|x|}{\sum}
$$

$$
=|\dot{0}|
$$

$$
\begin{aligned}
& \cot ^{2} x+1=\csc ^{2} x \\
& \cot ^{2} x+1=\frac{\cos ^{2} x}{\sin ^{2} x}+1-\frac{\sin ^{2} x}{\sin ^{2} x} \\
& =\frac{\cos ^{2} x+\sin ^{2} x}{\sin ^{2} x}=\frac{1}{\sin ^{2} x}=\csc ^{2} x
\end{aligned}
$$

S2.1 Stuff:

$$
\cos ^{4} x-2 \cos ^{2} x+1
$$

Let $u=\cos x$. This gives

$$
u^{4}-2 u^{2}+1
$$

Let $v=u^{2}$. Then

Factor \& simplify
Goal:
solving tire. equations.

$$
\begin{aligned}
& v^{2}-2 v+1=(v-1)^{2} \\
& \begin{array}{ll}
a=1, b=-2, c=1 & V=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
b^{2}-4 a c=(-2)^{2}-4(1)(1)
\end{array} \\
& b^{2}-4 a c=(-2)^{2}-4(1)(1) \\
& =4-4 \\
& =0 \\
& (v-1)^{2}=\left(u^{2}-1\right)^{2} \\
& =((u-1)(u+1))^{2} \\
& =\frac{2 \pm 0}{2(1)}=\frac{2}{2}=1 \\
& V=1 \text { makes it zero. } \\
& \text { So } v-1 \text { is a factor! } \\
& \text { This says }(v-1)(v-1) \\
& =v^{2}-2 v+1
\end{aligned}
$$

$u^{2}-1$ to factor:

$$
\begin{aligned}
& u^{2}+0 u-1 \\
& 2=1, b=0, c=-1 \\
& \left.b^{2}-4 a c=0^{2}-4 a\right)(-1)=4 \\
& \sqrt{b^{2}-4 a c}=2 \\
& u=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}=\frac{0 \pm 2}{2(1)}=\frac{ \pm 2}{2}= \pm 1
\end{aligned}
$$

This says $u=1$ \& $u=-1$ are enos
This .. $u-1 \& u+1$ are factors
So $u^{2}-1=(u-1)(u+1)$

$$
\begin{aligned}
\left(u^{2}-1\right)^{2} & =((u-1)(u+1))^{2} \\
& =\frac{\left(\begin{array}{ll}
(\cos \theta-1)(\cos \theta+1) \\
1 & \text { of } \\
-1
\end{array}\right)}{}=0
\end{aligned}
$$

$$
\begin{aligned}
& \cos ^{4} \theta-2 \cos ^{2} \theta+1 \\
&=\left(\cos ^{2} \theta-1\right)^{2} \\
&=\left(-\sin ^{2} \theta\right)^{2} \\
&= \sin ^{4} \theta \quad \stackrel{S 6 T}{=} 0 \Rightarrow \\
& \sqrt[4]{\sin ^{4} \theta}=\sqrt[4]{0} \\
&|\sin \theta|=0 \\
& \sin \theta=0
\end{aligned}
$$

Two piss


$$
\theta=0 \text { or } \theta=\pi
$$

That was 2 ways of factoring/ simplifying $\cos ^{4} x-2 \cos ^{2} x+1$

I went on to put this into the context of an equation.

See solutions to see where to stop.
$\# 30$ Factor $6 \cos ^{2} x+5 \cos x-6$

$$
\begin{aligned}
& =6 \cos ^{2} x+9 \cos x-4 \cos x-6 \\
& =3 \cos x(2 \cos x+3)-2(2 \cos x+3) \\
& =(2 \cos x+3)(3 \cos x-2) \\
& 6 u^{2}+5 u-6 \\
& a=6, b=5, c=-6 \\
& b^{2}-4 a c=25-4(6)(-6) \\
& \begin{array}{r}
236 \\
\frac{4}{144}
\end{array} \\
& =25+144 \\
& =49 \text { \& } \sqrt{169}=13 \\
& u=\frac{-5 \pm 13}{2(6)}=\frac{-5 \pm 13}{12} \longrightarrow \frac{-18}{12}=-\frac{3}{2}=\frac{2}{3} \\
& 6\left(u-\frac{2}{3}\right)\left(u+\frac{3}{2}\right) \\
& =2.3\left(u-\frac{2}{3}\right)\left(u+\frac{3}{2}\right) \\
& =(3 u-2)(2 u+3) \\
& =(3 \cos x-2)(2 \cos x+3)
\end{aligned}
$$

\#s 53-56 use ting substitution to white as thing func. CALC II
$54 \sqrt{49-x^{2}} \quad$ Khasi

$$
\begin{aligned}
& \text { 54) } \sqrt{49-x^{2}} \quad x=7 \sin \theta \\
& =\sqrt{49-(7 \sin \theta)^{2}}=\sqrt{49-49 \sin ^{2} \theta} \\
& =\sqrt{49\left(1-\sin ^{2} \theta\right)}=\sqrt{49} \sqrt{1-\sin ^{2} \theta} \\
& =7 \sqrt{\cos ^{2} \theta}=7|\cos \theta|
\end{aligned}
$$

Assume $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
If $\quad \frac{\pi}{2} \leq \theta \leq \frac{3 \pi}{2}$


$$
\sqrt{\cos ^{2} \theta}=|\cos \theta|=-\cos \theta
$$

But Book says $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$
So $|\cos \theta|=\cos \theta$
we are here:


Two pics for $\cos \theta=\frac{5}{7}$


$$
\sqrt{24}=2 \sqrt{6}
$$

Two pics for $\sin \theta=\frac{5}{7}$


Two pies for $\tan \theta=\frac{5}{7}$


\#557,58 Do trig substitution.
Solve for $\sin \theta, \cos \theta$, whatever.
(58)

$$
\eta^{-5 \sqrt{3}}=\sqrt{100-x^{2}} \quad x=10 \cos \theta
$$

No way Major problem with this equation.
pretend it's

$$
\begin{aligned}
5 \sqrt{3} & =\sqrt{100-x^{2}} \\
& =\sqrt{100-100 \cos ^{2} \theta} \\
& =10 \sqrt{1-\cos ^{2} \theta}
\end{aligned}
$$

(a) $\cos \theta=$
$S_{2.1}$ Dup wed

$$
\begin{aligned}
& =10 \sqrt{\sin ^{2} \theta} \\
& =10|\sin \theta| \\
-\frac{\pi}{2} & <\theta<\frac{\pi}{2} \text { Does NoT let }
\end{aligned}
$$

us get rid of 1
So let's assume $\sin \theta \geq 0$,

$$
\begin{gathered}
\text { ie., } 0 \leq \theta \leq \pi, \text { then } \\
\begin{array}{ll}
5 \sqrt{3}=10 \sin \theta \\
\sin \theta=\frac{5 \sqrt{3}}{10}=\frac{\sqrt{3}}{2} \\
\theta=\frac{\pi}{3}, \frac{2 \pi}{3} \\
60^{\circ}, 120^{\circ} & \frac{2}{3} / \sqrt{3}
\end{array}
\end{gathered}
$$

