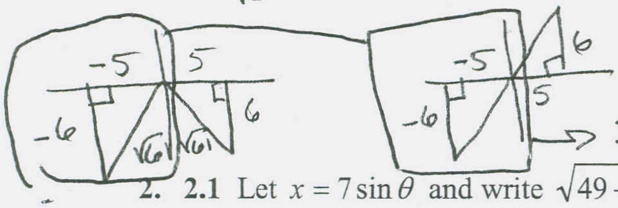
MAT 122 4pts each Quiz 4 Name KEY

March 7^{th}, 2012 24 poss. 2.1 – 2.4

1. **2.1** If $\sin(x) = -\frac{6}{\sqrt{61}}$ and $\tan(x) = \frac{6}{5}$, what is ~~$\sin(x) = -\frac{6}{\sqrt{61}}$~~? $\cos(x) = ?$

$\sin x = -\frac{6}{\sqrt{61}}$ $\tan x = \frac{6}{5}$

3rd Quadrant



Satisfies both

$\therefore$ $\boxed{\cos x = -\frac{5}{\sqrt{61}}}$

2. **2.1** Let $x = 7\sin\theta$ and write $\sqrt{49 - x^2}$ as a trigonometric function of θ. Assume $0 \le \theta < \frac{\pi}{2}$.

Bonus: What if the restriction on θ were $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$? Would that change your answer? If so, what would your answer be?

$\sqrt{49-x^2} = \sqrt{49-49\sin^2 x}$

$= \sqrt{49(1-\sin^2\theta)}$

$= 7\sqrt{\cos^2\theta}$

$= 7|\cos\theta|$

$= 7\cos\theta$, since $\cos\theta \ge 0$ in QI

In QII & QIII, $\cos\theta \le 0$

$\Rightarrow 7|\cos\theta| = \boxed{-7\cos\theta}$

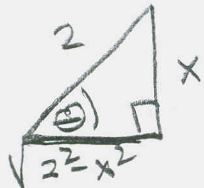
3. **2.2** Verify the identity $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{|\sin\theta|}$

$$\sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} = \frac{\sqrt{(1-\cos\theta)^2}}{\sqrt{1-\cos^2\theta}} = \frac{|1-\cos\theta|}{\sqrt{\sin^2\theta}}$$

$$= \frac{1-\cos\theta}{|\sin\theta|} \quad \text{since } 1-\cos\theta \ge 0$$

4. **2.2** Use a drawing to verify the identity $\cot\left(\sin^{-1}\left(\frac{x}{2}\right)\right) = \frac{\sqrt{4-x^2}}{x}$. Hint: Let $\theta = \sin^{-1}\left(\frac{x}{2}\right)$.

$\theta = \sin^{-1}\left(\frac{x}{2}\right)$



$\cot\theta = \frac{\sqrt{4-x^2}}{x}$

5. **2.3** Find all solutions θ, such that $0 \le \theta \le 2\pi$. Then find *all* solutions.

$\sin^2\theta = 3\cos^2\theta$

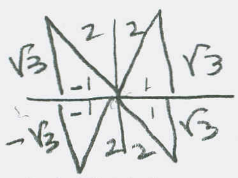
$\sin^2\theta = 3(1-\sin^2\theta)$

$\sin^2\theta = 3 - 3\sin^2\theta$

$4\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$



$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

ALL SOL'NS:

$\theta = \frac{\pi}{3} + n\pi, \ n \in \mathbb{Z}$

$\theta = \frac{2\pi}{3} + n\pi, \ n \in \mathbb{Z}$

ALTERNATE:

$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

Same picture.

6. **2.4** Find the exact values of sine, cosine, and tangent for $\theta = \frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$. Simplify as much as you can without a calculator.

$\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{4}$

$\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$

$= \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}} = \sin\left(\frac{7\pi}{12}\right)}$

$\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4} = \frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$

$= \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}} = \cos\left(\frac{7\pi}{12}\right)}$

$$\frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{1-\sqrt{3}}{2\sqrt{2}}} = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\left(\frac{2\sqrt{2}}{1-\sqrt{3}}\right)$$

$\tan\left(\frac{7\pi}{12}\right) = \frac{\sin\left(\frac{7\pi}{12}\right)}{\cos\left(\frac{7\pi}{12}\right)} = \boxed{\frac{\sqrt{3}+1}{1-\sqrt{3}} = \tan\frac{7\pi}{12}}$