n^{th} roots of unity.

5^{th} roots of "1" from last time.

evenly dispersed around the unit circle.

$\sqrt[5]{1} = 1$ $\qquad \frac{2\pi}{5} = \frac{360^\circ}{5} = 72^\circ$ apart.

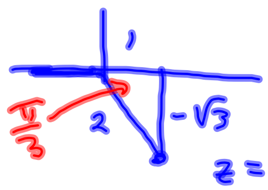
n^{th} roots of $r(\cos\theta + i\sin\theta)$ in general.

$$\sqrt[n]{r}\left(\cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right)$$

$k = 0, 1, \dots, n-1$

Find the square roots of $1-\sqrt{3}\,i$

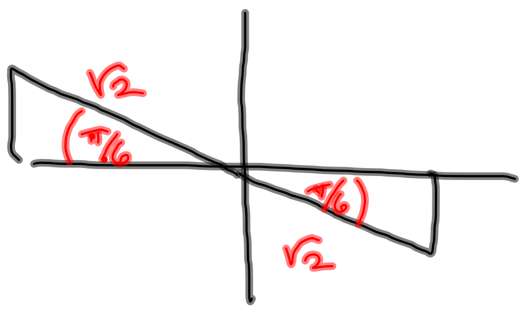
I used $\theta = -60^\circ$



$\theta = \frac{5\pi}{3}$

$z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

we get



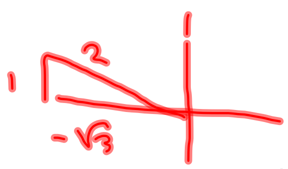
$$\sqrt{2}\left(\cos\frac{\frac{5\pi}{3}}{2} + i\sin\frac{\frac{5\pi}{3}}{2}\right)$$

$$= \boxed{\sqrt{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)}$$

and

$$\sqrt{2}\left(\cos\left(\frac{\frac{5\pi}{3}+2\pi}{2}\right) + i\sin\left(\frac{\frac{5\pi}{3}+2\pi}{2}\right)\right)$$

$$= \boxed{\sqrt{2}\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)}$$



$$\sqrt{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$\sqrt{12} = 2\sqrt{3}$

$$= \frac{-\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i = \frac{1}{2}\left(-\sqrt{6} + \sqrt{2}\,i\right)$$

Is this a square root of z?

$$\left(\frac{1}{2}\left(-\sqrt{6} + \sqrt{2}\,i\right)\right)^2 = \frac{1}{4}\left[6 - 2\sqrt{6}\sqrt{2}\,i - 2\right]$$

Sweet!

$(a+b)^2 = a^2 + 2ab + b^2$

$$= \frac{1}{4}\left[4 - 4\sqrt{3}\,i\right] = 1 - \sqrt{3}\,i$$

$(\sqrt{2}\,i)^2 = 2i^2 = -2$

De Moivre's Formula for powers.

$z = r(\cos\theta + i\sin\theta) \Longrightarrow$

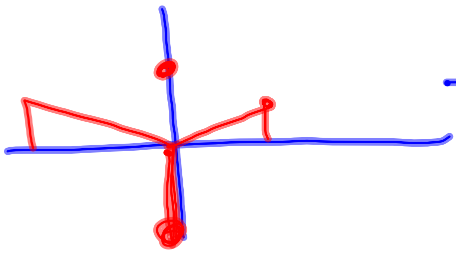
$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

$$\left\{\begin{array}{l} A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \end{array}\right\}$$ other results with DeMoivre

§4.4 is all good.

#s 55-70 Solve $x^3 - i = 0 \Longrightarrow$

$x^3 = i \Longrightarrow$ we want the 3 cube roots of i!

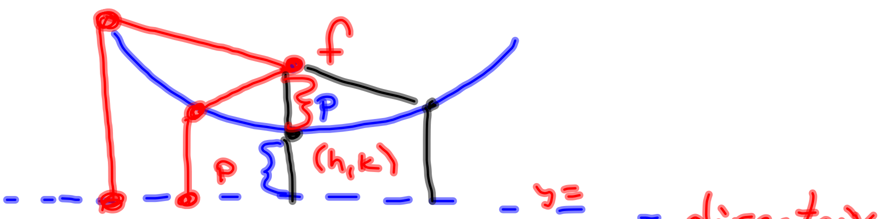


$(-i)^3 = -i^3 = -(i^2)i$

$= i$

§6.2 Parabolas

A parabola is the set of all points equidistant from a point (focus) and a straight line (directrix)



$$(x-h)^2 = 4p(y-k)$$

vertex = (h,k)
focus = $(h,k+p)$
directrix is $y=k-p$

* focus is p units up or down (p negative) from vertex.

#48 Find v, f, directrix for

$x^2-2x+8y+9=0$

$(x-h)^2=4p(y-k)$

$x^2-2x \quad = -8y-9$

$\frac{2}{2}=1 \rightsquigarrow 1^2$

$x^2-x \quad = 5$

$x^2-x+\left(\frac{1}{2}\right)^2=5+\frac{1}{4}$

$x^2-2x+1^2=-8y-9+1$

$(x-1)^2=-8y-8$

$(x-1)^2=-8(y+1)$

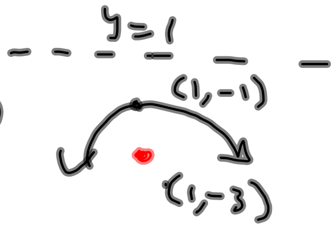
$(x-1)^2=4(-2)(y+1)$

$p=-2$

$v=(1,-1)$

$f=(1,-3)$

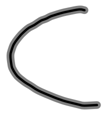
$d: y=1$

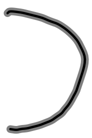


$f=(h,k+p)=(1,-1+(-2))$

$d: y=k-p=-1-(-2)=$

$(y-k)^2 = 4p(x-h)$ same deal.

$p>0$ 

$p<0$ 

#s 61, 62 will help in calculus

Try both B4 next time.

Last Worksheet coming in a couple days.