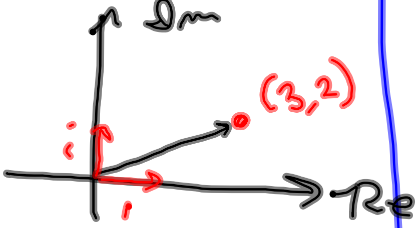
§4.3 Trig Forms of complex #s.

$z = a + bi$, $a, b \in \mathbb{R}$

Complex #s in the complex plane.

$z = 3 + 2i$



Canonical "unit" vectors:

$1, i$

Trig form for $3+2i$ is

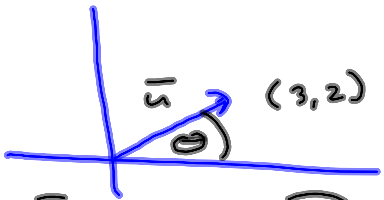
$\sqrt{13}\,(\cos 33.7 + i \sin 33.7^\circ) = \sqrt{13}\,\langle \cos 33.7^\circ, \sin 33.7^\circ \rangle$

$= |z|\,(\cos\theta + i \sin\theta)$

$= r\,(\cos\theta + i \sin\theta)$

Vectors in the plane.

$\bar{u} = 3\bar{i} + 2\bar{j}$

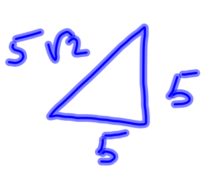


$\left.\begin{array}{l} \bar{i} = \langle 1, 0 \rangle \\ \bar{j} = \langle 0, 1 \rangle \end{array}\right\}$ canonical unit vectors.

$\theta = \arctan\left(\frac{2}{3}\right) \approx 33.7^\circ$

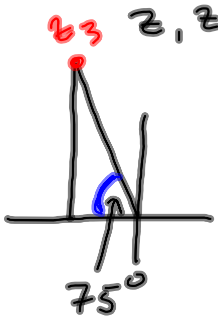
$\|\bar{u}\| = \sqrt{2^2 + 3^2} = \sqrt{13}$

so $\bar{u} \approx \sqrt{13}\,(\cos 33.7^\circ \bar{i} + \sin 33.7^\circ \bar{j})$



$$z_1 = 5\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 5+5i$$

$$z_2 \quad 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2+2\sqrt{3}\,i$$



$$z_1 z_2 = (5+5i)(2+2\sqrt{3}\,i)$$

$$= 10 + 10\sqrt{3}\,i + 10i + 10\sqrt{3}\,i^2$$

$$= 10 + (10\sqrt{3}+10)i - 10\sqrt{3}$$

$$= (10-10\sqrt{3}) + (10\sqrt{3}+10)i = z_3$$

$180^\circ - 75^\circ$
$= 105^\circ$

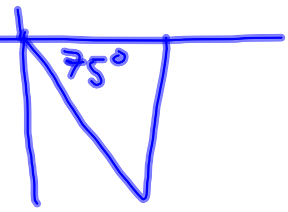
$$\theta' = \arctan\left(\frac{10\sqrt{3}+10}{10-10\sqrt{3}}\right) = \text{reference angle}$$

$$= -75^\circ$$

So

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$z_3 = |z_3|\left(\cos 105^\circ + i\sin 105^\circ\right)$$



Need $|z_3|$

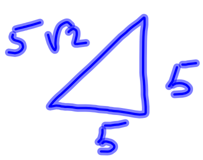
$$= \sqrt{(10-10\sqrt{3})^2 + (10\sqrt{3}+10)^2}$$

$$= \sqrt{100 - 200\sqrt{3} + 100\cdot 3 + 300 + 200\sqrt{3} + 100}$$

$$= \sqrt{800} = 10\sqrt{8} = 20\sqrt{2}$$

$$\text{so} \quad z_3 = \underline{20\sqrt{2}}\left(\cos 105^\circ + i\sin 105^\circ\right)$$

$$= z_1 z_2 = \underline{5\sqrt{2}}\left(\cos 45^\circ + i\sin 45^\circ\right)\cdot \underline{4}\left(\cos 60^\circ + i\sin 60^\circ\right)$$



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$|z|$ = the modulus of $z = r$

θ = the argument of z

$$z_1 z_2 = r_1(\cos\theta_1 + i\sin\theta_1) \cdot r_2(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1 r_2 \left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right)$$

This is cool.

Roots of unity.

Find all 4^{th} roots of 1

$1^4 = 1$

$i^4 = i^2 i^2 = (-1)(-1) = 1$

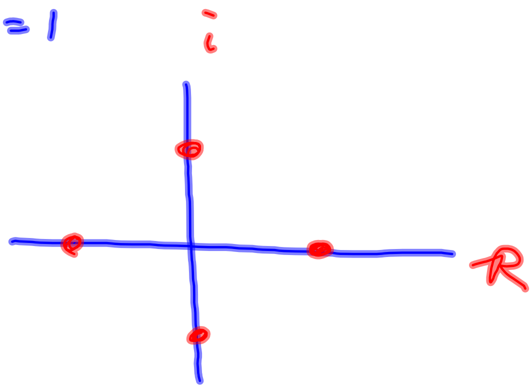
$(-1)^4 = 1$

$(-i)^4 = 1$

$1, i, -i, -1$

what about 5^{th} roots?

$1^5 = 1$



$\frac{360}{5} = 72^\circ$ We know $\cos 72^\circ + i \sin 72^\circ$ lies on the unit circle.

What's $(\cos 72^\circ + i \sin 72^\circ)^5$

$= \cos(5 \cdot 72^\circ) + i \sin(5 \cdot 72^\circ)$

$= \cos 360^\circ + i \sin 360^\circ$

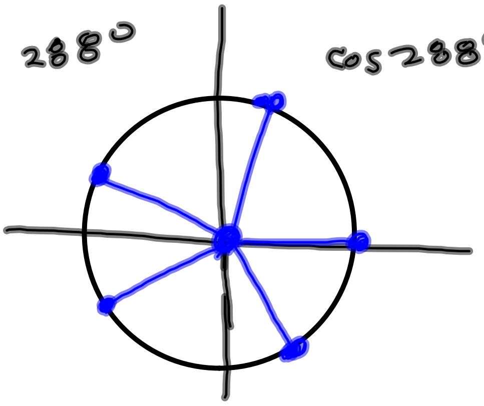
$= 1 + i \cdot 0 = 1$

$\frac{360}{5} = 72$

$72 \cdot 2 = 144^\circ$ $\quad \cos 144^\circ + i \sin 144^\circ$ is another.

$72 \cdot 3 = 216^\circ$ $\quad \cos 216^\circ + i \sin 216^\circ$ is, too.

$72 \cdot 4 = 288^\circ$ $\quad \cos 288^\circ + i \sin 288^\circ$ is, too

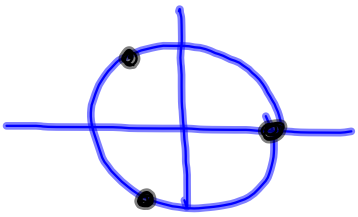


$\sqrt[5]{87}$

$87 = 87 \cdot 1$

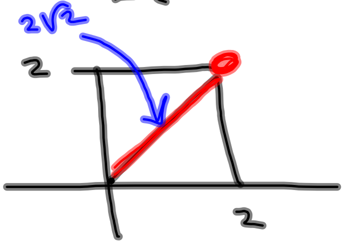
So the 5^{th} roots of 87 are $\sqrt[5]{87}$ (all the 5^{th} roots of unity)

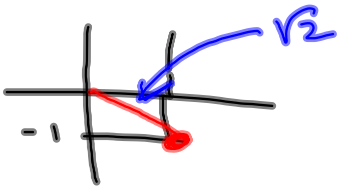
$\sqrt[3]{1}$



63 $(2+2i)(1-i) =$

$2(1+i)(1-i) = 2(1-i^2) = 2(1+1) = 4$





Book wants to see

$\left(2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)\left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right)$

$= 2\sqrt{2}\cdot\sqrt{2}\left(\cos 0 + i\sin 0\right)$

$= 4$