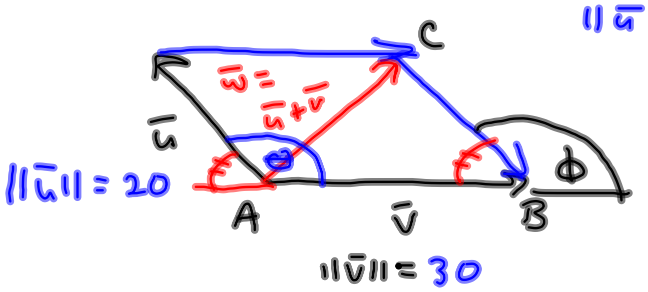
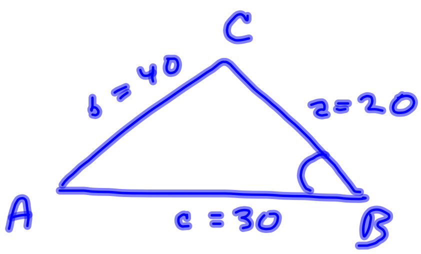
If you're given the magnitude of two vectors & the magnitude of the resultant (their sum), you can find the angle θ between them:

$\|\bar{u}+\bar{v}\| = \|\bar{w}\| = 40$

$\theta = \pi - B$ in radians.

or $180^\circ - B$ in degrees





SSS is Law of Cosines

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$40^2 = 20^2 + 30^2 - 2(20)(30)\cos B$$

$$1600 = 400 + 900 - 1200\cos B$$

$$1600 = 1300 - 1200\cos B$$

$$300 = -1200\cos B$$

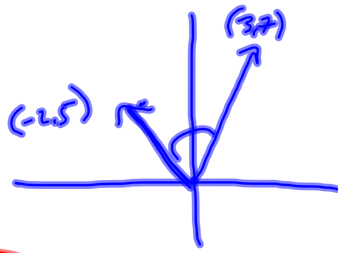
$$-\frac{1}{4} = \cos B$$

$$B = \arccos\left(-\tfrac{1}{4}\right) \approx 104.48^\circ$$

$$\Rightarrow \theta = 180^\circ - B \approx \boxed{75.52^\circ \approx \theta}$$

Courtesy Karla

$\bar{u} = \langle 2, 3 \rangle \quad \langle -12, 8 \rangle \qquad \langle 2, 3 \rangle$



$\bar{u} = \langle 3, 7 \rangle \quad \langle -2, 5 \rangle = \bar{v}$

$3\bar{i} + 7\bar{j} = 3\langle 1, 0 \rangle + 7\langle 0, 1 \rangle$

$= \langle 3, 0 \rangle + \langle 0, 7 \rangle = \langle 3, 7 \rangle$

$\sqrt{3^2 + 7^2} = \sqrt{9 + 49} = \sqrt{58}$

$\sqrt{2^2 + 5^2} = \sqrt{29}$

$$\cos\theta = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \|\bar{v}\|}$$

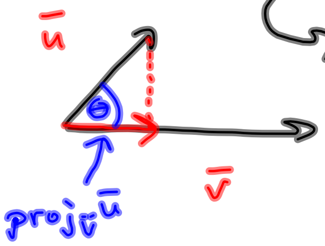
$$= \frac{(3)(-2) + (7)(5)}{\sqrt{58}\,\sqrt{29}}$$

$$= \frac{-6 + 35}{\sqrt{58 \cdot 29}} = \frac{29}{\sqrt{1682}}$$

$\approx .7071067812$

$\Longrightarrow \theta \approx 45^\circ$

$\bar{u} = \langle 7, 3 \rangle$, $\bar{v} = \langle -11, 2 \rangle$ Magnitude



$$\frac{\|proj_{\bar v}\bar u\|}{\|\bar u\|} = \cos\theta = \frac{|\bar u \cdot \bar v|}{\|\bar u\|\|\bar v\|}$$

$$proj_{\bar v}\bar u = \frac{\bar u \cdot \bar v}{\|\bar v\|} \cdot \frac{\bar v}{\|\bar v\|}$$

Magnitude times unit vector in the direction of $\bar v$.

$$proj_{\bar v}\bar u = \frac{\bar u \cdot \bar v}{\|\bar v\|^2} \cdot \bar v = \frac{-71}{125}\langle -11, 2\rangle$$

$\|\bar u\| = \sqrt{49+9} = \sqrt{58}$ $\left(\sqrt{121+4}\right)^2 = \sqrt{125}^2 = (5\sqrt{5})^2 = \|\bar v\|^2$

$\bar u \cdot \bar v = -77 + 6 = -71$

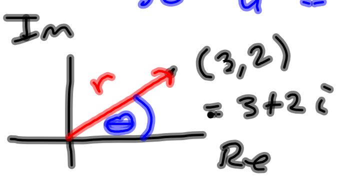
So $\bar{u} - \text{proj}_{\bar{v}} u = \text{orth}_{\bar{v}} \bar{u}$

$= \langle 7,3 \rangle - \frac{-71}{125} \langle -11, 2 \rangle$

$a+bi = r\cos\theta + i r \sin\theta$

$= \langle \frac{94}{125}, \frac{517}{125} \rangle$

So $\bar{u} = \underbrace{\frac{-71}{125} \langle -11, 2 \rangle}_{\text{parallel to } \bar{v}} + \overbrace{\underbrace{\langle \frac{94}{125}, \frac{517}{125} \rangle}_{\text{orthogonal to } \bar{v}.}}^{\bar{z}}$ AND



If so, then $\bar{z}$ is $\perp$ to $\bar{v}$

Check: $\bar{z} \cdot \bar{v} = \langle \frac{94}{125}, \frac{517}{125} \rangle \cdot \langle -11, 2 \rangle$

$= \frac{-1034}{125} + \frac{1034}{125} = 0$!

$$\frac{\bar{v}}{\|\bar{v}\|} = \frac{\langle v_1, v_2 \rangle}{\sqrt{v_1^2 + v_2^2}}$$

What's its length?

$$\left\| \frac{\bar{v}}{\|\bar{v}\|} \right\| = \frac{1}{\cancel{\|\bar{v}\|}} \cancel{\|\bar{v}\|} = 1$$

$\bar{v} = \langle 3, 4 \rangle$

$\|\bar{v}\| = \sqrt{3^2 + 4^2} = 5$

$$\frac{\bar{v}}{\|\bar{v}\|} = \frac{\bar{v}}{5} = \frac{1}{5}\bar{v}$$

$$\& \left\| \frac{\bar{v}}{\|\bar{v}\|} \right\| = \left\| \tfrac{1}{5}\bar{v} \right\|$$

$$= \left\| \langle \tfrac{1}{5}\cdot 3, \tfrac{1}{5}\cdot 4 \rangle \right\|$$

$$= \sqrt{\left(\tfrac{3}{5}\right)^2 + \left(\tfrac{4}{5}\right)^2} = \sqrt{\tfrac{9}{25} + \tfrac{16}{25}} = \sqrt{\tfrac{25}{25}} = 1$$