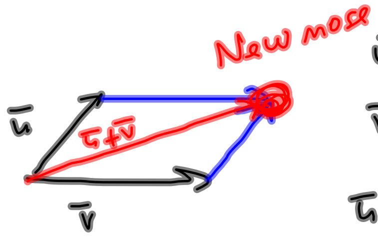
Resultant of two vectors:

Just add 'em



$\bar{u} = \langle 2,2 \rangle$ $\qquad \overline{V}$

$\bar{v} = \langle 3,0 \rangle$ $\qquad V$

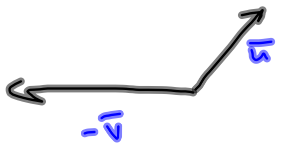
$\bar{u} + \bar{v} = \langle 5,2 \rangle$

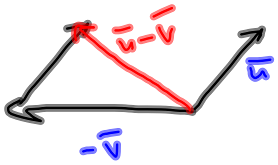
Think of it as:

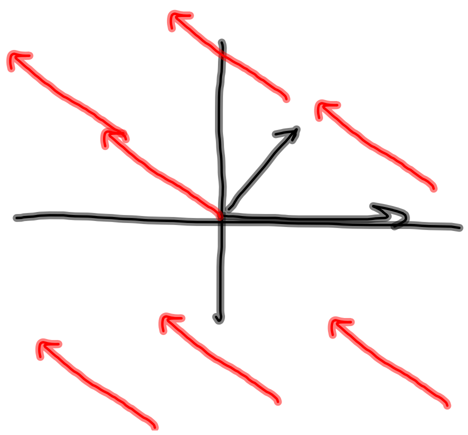
the diagonal of the parallelogram

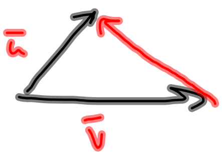
what you get when you position $\bar{u}$ & $\bar{v}$ start-to-tip Then draw a vector from the tail to the new nose

$\bar{u} - \bar{v} = \bar{u} + (-\bar{v})$









So, $\bar{u} - \bar{v}$ is like going from the tip of $\bar{v}$ to the tip of $\bar{u}$.

Dot Product

$\bar{u} = \langle u_1, u_2 \rangle$

$\bar{v} = \langle v_1, v_2 \rangle$

Then $\bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2$

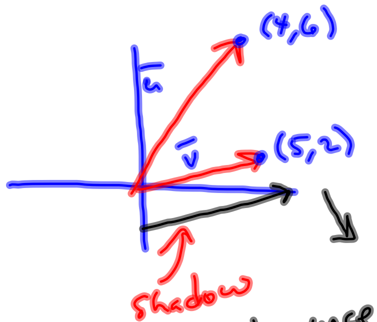
what's the magnitude of $\bar{u}$?

$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2}$

what's $\bar{u} \cdot \bar{u} = u_1 u_1 + u_2 u_2$

See the connection $= u_1^2 + u_2^2 = \|\bar{u}\|^2$

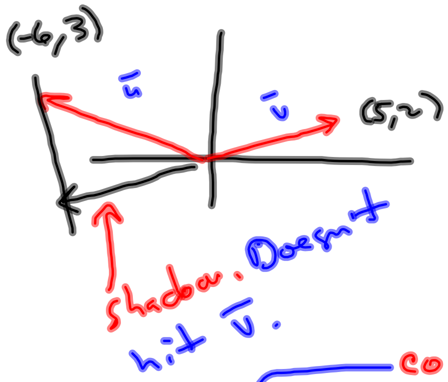
between $\cdot$ and length?



$\bar{u} \cdot \bar{v} = (4)(5) + (6)(2)$

$= 32$

A light source at right angles to $\bar{v}$. Look at the shadow cast by $\bar{u}$ on $\bar{v}$.



$\bar{u} \cdot \bar{v} = (-6)(5) + (3)(2)$
$= -30 + 6$
$= -24$

Direction angles:

1^{st} one, θ was acute.
cosine is positive $0 \le \theta < 90^\circ$

2^{nd} one, θ was obtuse
cosine is negative $90^\circ < \theta < 180^\circ$

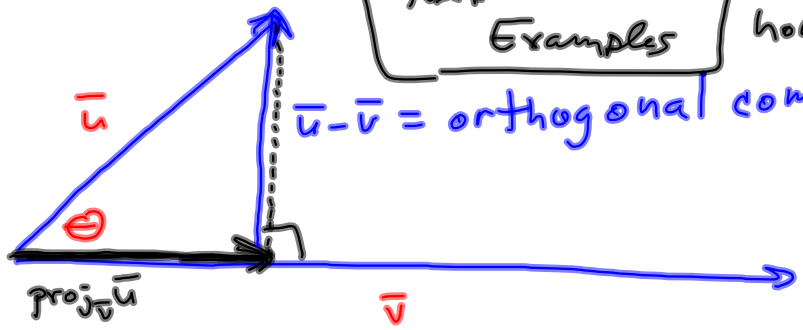
Shadow hits $\bar{v}$ $\quad \bar{u} \cdot \bar{v}$ positive

Shadow misses $\bar{v}$ $\quad \bar{u} \cdot \bar{v}$ negative

"Shadow" is the projection of $\bar{u}$ onto $\bar{v}$
$= \text{proj}_{\bar{v}} \bar{u}$

$\boxed{\text{Next time Examples}}$ Look @ homework

$\bar{u}-\bar{v}$ = orthogonal complement.



Assume θ is acute.

$$\|\text{proj}_{\bar{v}}\bar{u}\| = \|\bar{u}\| \cos\theta = \cancel{\|\bar{u}\|}\frac{|\bar{u}\cdot\bar{v}|}{\cancel{\|\bar{u}\|}\|\bar{v}\|}$$

cancel the $\|\bar{u}\|$'s

So there's the length.

Let's get a vector in the direction of $\bar{v}$ with that length.

Need length ✓

Need unit vector in direction of $\bar{v}$.

$$\frac{\bar{v}}{\|\bar{v}\|}$$

$\text{proj}_{\bar{v}}\bar{u} = \frac{|\bar{u}\cdot\bar{v}|}{\|\bar{v}\|}\cdot\frac{\bar{v}}{\|\bar{v}\|}$

$$\boxed{\text{proj}_{\bar{v}}\bar{u} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{V}}$$

If θ is acute, then $\bar{u}\cdot\bar{v}$ is positive.
If θ is obtuse, then $\bar{u}\cdot\bar{v}$ is negative.

$\bullet$ & $\cos\theta$ are linked

$\bar{u} - \text{proj}_{\bar{v}}\bar{u} = \bar{w} = \text{orth}_{\bar{v}}\bar{u}$ = orthogonal complement.

$\bar{w}$ is orthogonal to $\text{proj}_{\bar{v}}\bar{u}$