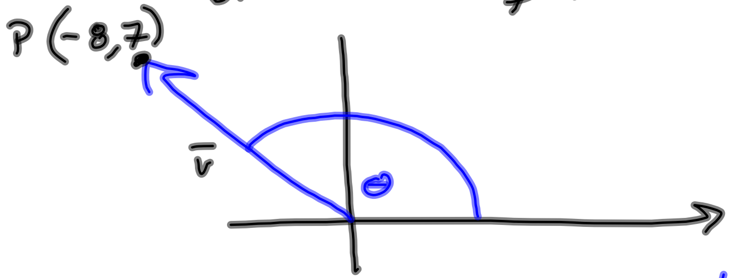
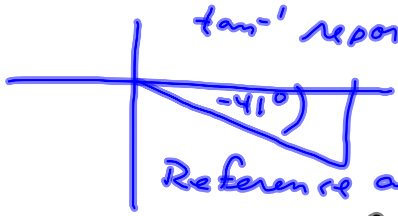
§3.3 More on vectors.

Direction angles



$\bar{v} = \langle -8, 7 \rangle$ is the position vector for P(-8,7)

tan⁻¹ reports this:



Reference angle is 41°

we find θ:

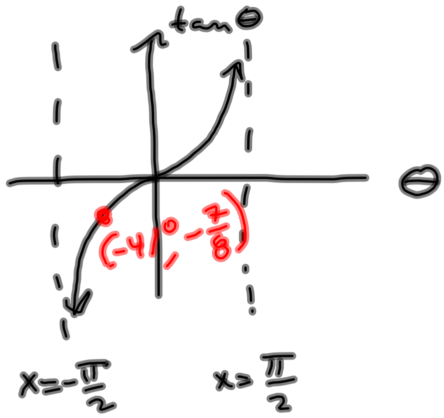
$$\tan\theta = \frac{7}{-8}$$

$$\tan^{-1}(\tan\theta) = \tan^{-1}\left(-\frac{7}{8}\right) \approx -41.19°$$

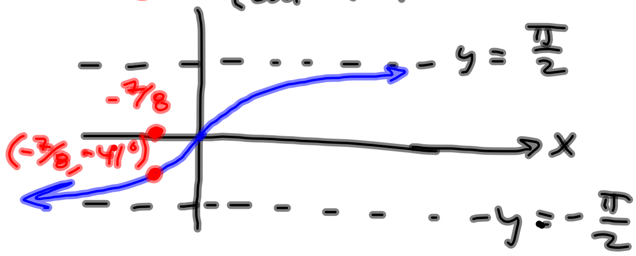
what's the range of $\tan^{-1}(x)$?

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = (-90°, 90°)$$

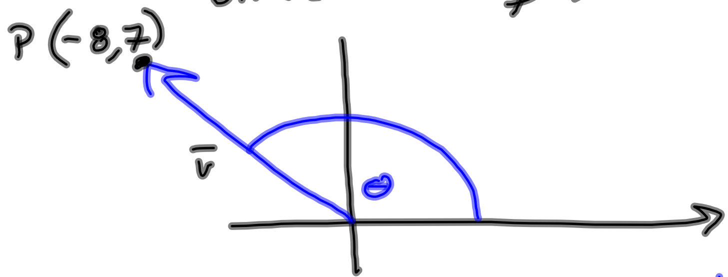
This comes from how we restrict the domain of $\tan\theta$ to make it 1-to-1, so that $\tan^{-1}(x)$ is function.



$\theta = \tan^{-1}(x)$

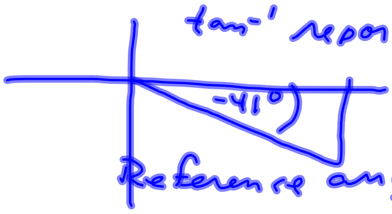


Direction angles



$\bar{v} = \langle -8, 7 \rangle$ is the position vector for $P(-8,7)$

tan^{-1} reports this:

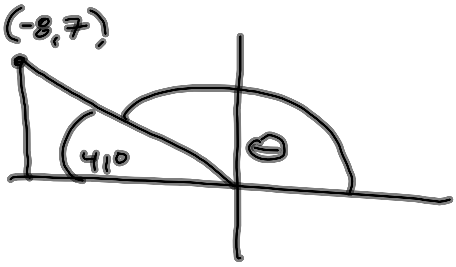


Reference angle is 41°

We find θ:

$$\tan\theta = \frac{7}{-8}$$

So the reference angle for θ is 41°.

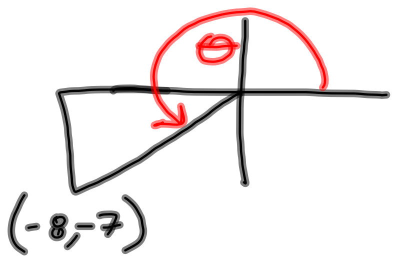


$\theta = ?$

$\approx 180° - 41.19°$

$= \boxed{138.81° \approx \theta}$

Used $\tan^{-1}\left(\frac{y}{x}\right)$ to find reference angle & then add/subtract to find θ.



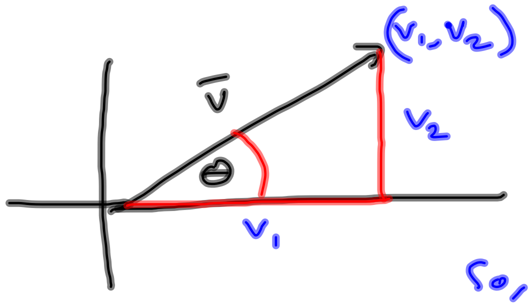
$\tan^{-1}\left(\frac{-7}{-8}\right) \approx 41.19°$

So 41.19° is reference angle

$\theta \approx 180° + 41.19°$

$= \boxed{221.19° \approx \theta}$

If $\bar{v}$ has direction angle θ,



$\bar{v} = \langle v_1, v_2 \rangle$

$\|\bar{v}\|$ = length of $\bar{v}$.

So, $\frac{v_2}{\|\bar{v}\|} = \sin\theta$

$\Longrightarrow v_2 = \|\bar{v}\| \sin\theta$

Likewise,

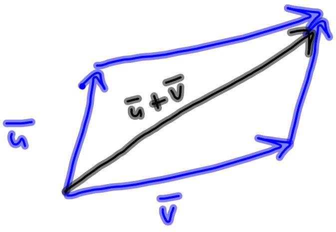
$v_1 = \|\bar{v}\| \cos\theta$

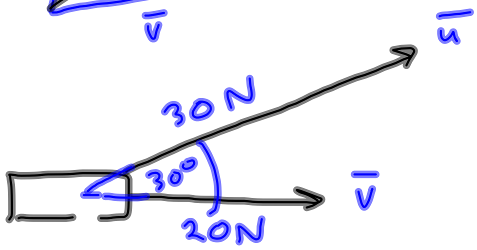
$\bar{v} = \langle v_1, v_2 \rangle$

$= \langle \|\bar{v}\|\cos\theta, \|\bar{v}\|\sin\theta \rangle$

$= \underbrace{\|\bar{v}\|}_{\text{length}} \underbrace{\langle \cos\theta, \sin\theta \rangle}_{\text{Direction of } \bar{v}}$ is another way to write $\bar{v}$.

Resultant of $\bar{v}$ & $\bar{u}$ is $\bar{u} + \bar{v}$ is the diagonal of the parallelogram.





$\bar{u} = 30 \langle \cos 30°, \sin 30° \rangle$
$= 30 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$
$= \langle 15\sqrt{3}, 15 \rangle$
$\bar{v} = 20 \langle \cos 0°, \sin 0° \rangle$
$= 20\langle 1, 0 \rangle = \langle 20, 0 \rangle$

what's the resultant force?
(Direction & magnitude)

$\bar{w} = \bar{u} + \bar{v} = \langle 15\sqrt{3}, 15 \rangle + \langle 20, 0 \rangle$
$= \langle 15\sqrt{3} + 20, 15 \rangle$
$\approx \langle 45.980, 15 \rangle$

$\begin{array}{r} 1.732 \\ \times\ 3 \\ \hline 5.196 \end{array}$

$5(5.196 + 4)$

$5(9.196)$

$\begin{array}{r} 9.196 \\ \times\ 5 \\ \hline 45.980 \end{array}$

Now, let's talk magnitude & direction

$\|\bar{w}\| \approx \sqrt{45.980^2 + 15^2}$

≈ 48.36 N

$\theta = ?$ $\tan^{-1}(\tan\theta) \approx \tan^{-1}\left(\frac{15}{45.98}\right)$

$\approx 18.06°$

§3.4 Dot product

Dot product is a kind of measure of how similar 2 vectors are.
How big is the shadow cast by one on the other?
"Projections"

$\bar{u} = \langle u_1, u_2 \rangle \longrightarrow \bar{u} \bullet \bar{v} = u_1 v_1 + u_2 v_2$

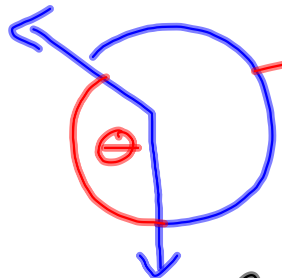
$\bar{v} = \langle v_1, v_2 \rangle$

NOTE: $\bar{u} \bullet \bar{u} = u_1 u_1 + u_2 u_2$
$= u_1^2 + u_2^2 = \|\bar{u}\|^2,$
since $\|\bar{u}\| = \sqrt{u_1^2 + u_2^2}$

$$\boxed{\cos\theta = \frac{\bar{u} \bullet \bar{v}}{\|\bar{u}\| \, \|\bar{v}\|}}$$

can be proved using the Law of Cosines. (See pg 334)

Assumption: $0 \le \theta < \pi$



we'll never measure θ this way.

Two vectors are orthogonal ($\theta = 90°$) if and only if their dot product is ZERO

$$\cos(90°) = 0 = \frac{\bar{u} \bullet \bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$

Must be zero.

Work a bunch of §3.3 odds & 3.4 odds