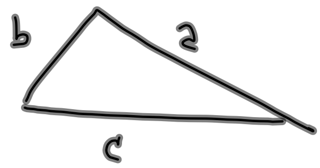
Today Finish §3.2 w/ Heron's formula

Begin §3.3 <u>Vectors</u>

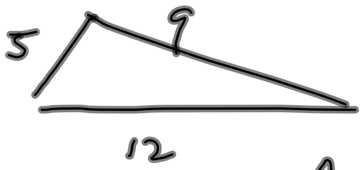


Heron says

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where $\frac{a+b+c}{2} = s$

$$\begin{array}{r} 32 \\ \times 13 \\ \hline 96 \\ 320 \\ \hline 416 \end{array}$$



$$s = \frac{5+9+12}{2} = \frac{26}{2} = 13$$

$$\text{Area} = \sqrt{13(13-5)(13-9)(13-12)}$$

$$= \sqrt{13(8)(4)(1)} = \sqrt{416}$$

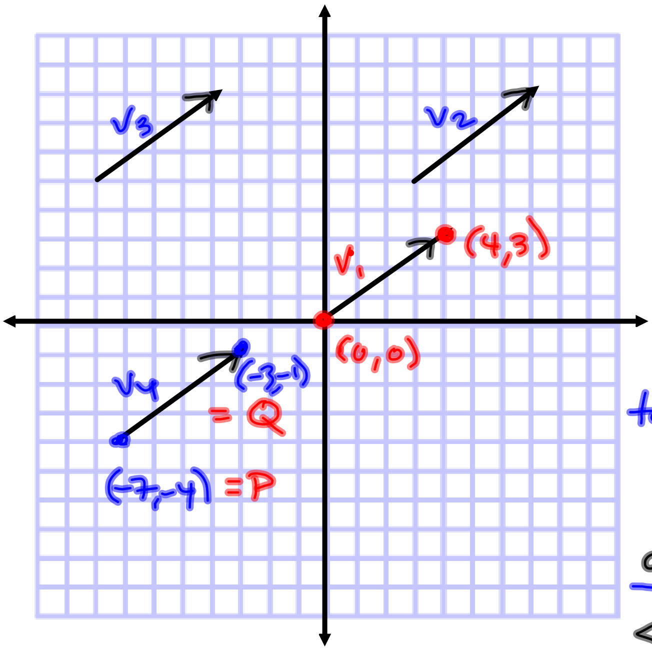
$$= 4\sqrt{26}.$$

2 | 416
2 | 208
2 | 104
2 | 52
2 | 26
13

§3.3 Vectors in the plane:

Directed line segments.

Equivalent vectors: same direction, same length



These are all the same vector.

$v_1 = \langle 4,3 \rangle$

v_1 is equivalent to ALL these others.

$\overrightarrow{PQ}$ is given in component form by

$\langle -3-(-7), -1-(-4) \rangle$

$= \langle 4,3 \rangle$

So all those vectors are represented as $\langle 4,3 \rangle$ in component form.

What's the length of a vector?

$$\|\bar{v}\| = \sqrt{v_1^2 + v_2^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$\bar{u} + \bar{v}$ is a way to impose some sort of "arithmetic" on vectors (points in the plane).

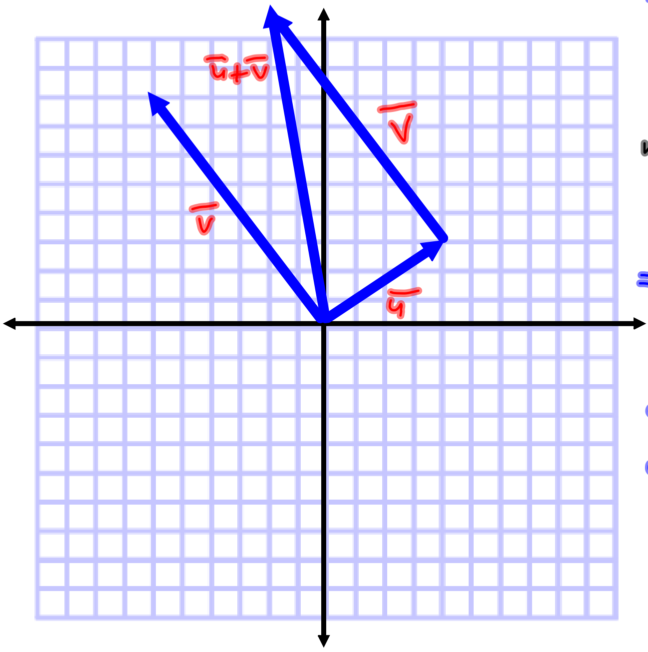
Apr 9-8:14 AM

$\bar{u} = \langle u_1, u_2 \rangle$ & $\bar{v} = \langle v_1, v_2 \rangle$, then

$\bar{u} + \bar{v} = \langle u_1 + v_1, u_2 + v_2 \rangle = \overline{w}$

$\bar{u} = \langle 4, 3 \rangle$ & $\bar{v} = \langle -6, 8 \rangle$

Then $\bar{u} + \bar{v} = \langle -2, 11 \rangle$



The visual is that $\bar{u} + \bar{v}$ is obtained by positioning $\bar{u}$ & $\bar{v}$ nose-to-tail

$\bar{u} - \bar{v}$

$= \langle 10, -5 \rangle$ is the vector from the end of $\bar{v}$ to the end of $\bar{u}$

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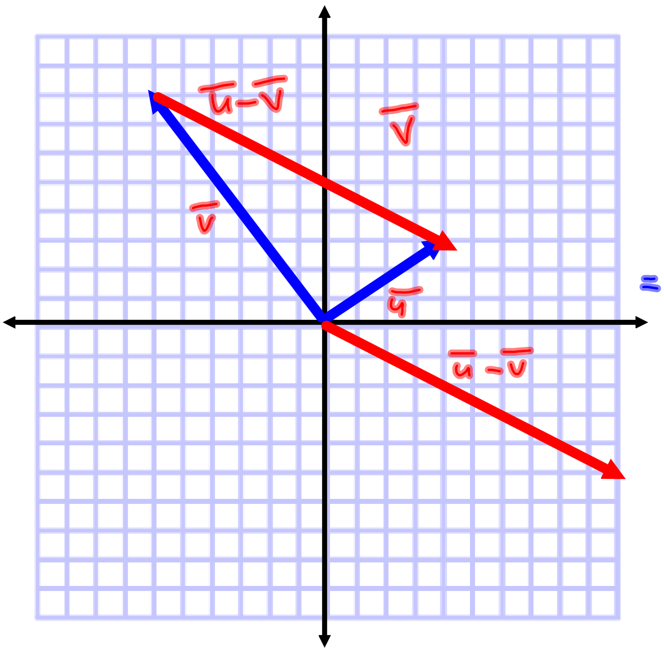
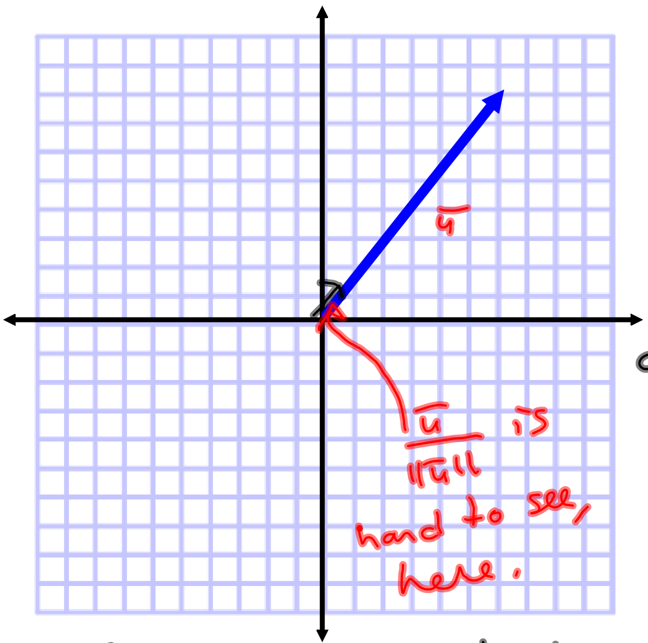


$\bar{u}-\bar{v}$

$= \langle 10, -5 \rangle$ is the vector from the end of $\bar{v}$ to the end of $\bar{u}$

Unit Vector: Same direction but unit length as $\bar{u}$: $\frac{1}{\|\bar{u}\|}\bar{u}$



$\bar{u} = \langle 6, 8 \rangle$

Then $\|u\| = \sqrt{6^2+8^2} = \sqrt{36+64}$
$= \sqrt{100}$
$= 10$

A unit vector in the direction of $\bar{u}$ is

$\frac{1}{10}\langle 6, 8\rangle = \langle \frac{3}{5}, \frac{4}{5} \rangle$

The main thing that's nice about unit vectors is, if $\bar{v}$ is a unit vector, then moving 20 units in the direction of $\bar{v}$ is achieved by $20\,\bar{v}$.

Standard unit vectors.

$$\bar{i} = \langle 1,0 \rangle$$
$$\bar{j} = \langle 0,1 \rangle$$

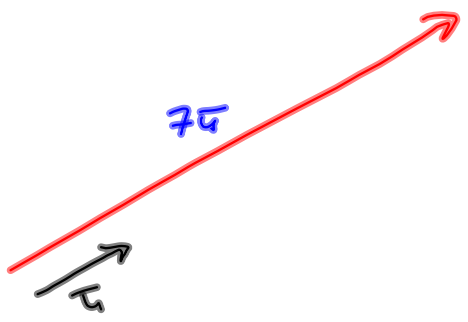
They're the standard because we can write ANY vector in the plane as a <u>linear combination</u> of $\bar{i}$ & $\bar{j}$ in a very nice way.

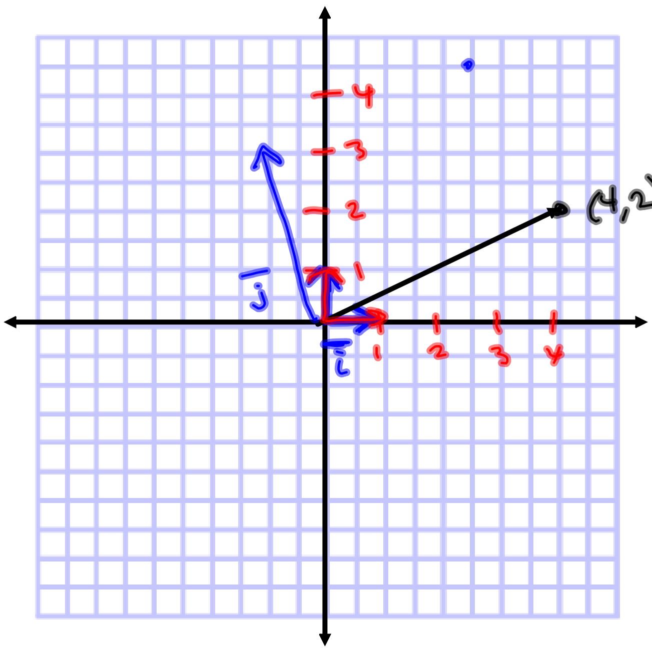
$$\bar{u} = \langle a,b \rangle = a\bar{i} + b\bar{j}$$
$$= a\langle 1,0 \rangle + b\langle 0,1 \rangle$$
$$= \langle a,0 \rangle + \langle 0,b \rangle$$

$$\bar{u} = \langle 4,3 \rangle = 4\bar{i} + 3\bar{j}$$

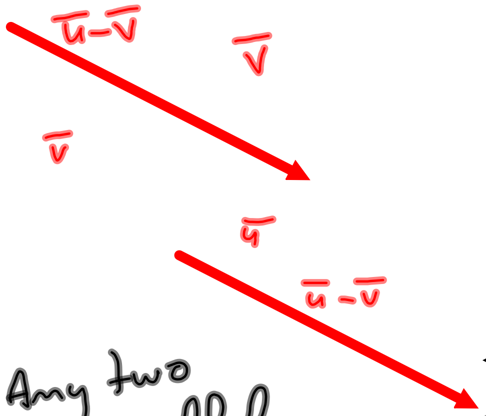
Linear combos are good for certain apps & calculations.

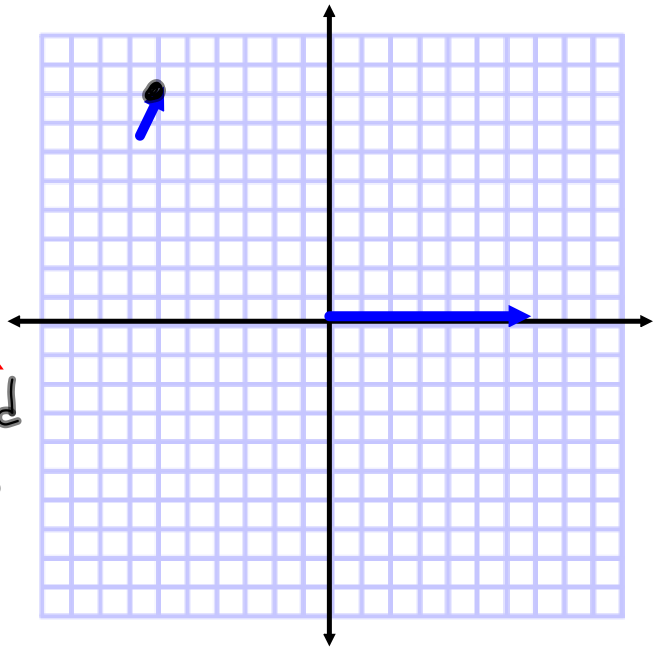
BTW: $7\bar{u} = 7\langle 4,3 \rangle = \langle 7(4), 7(3) \rangle$
$$= \langle 28, 21 \rangle$$



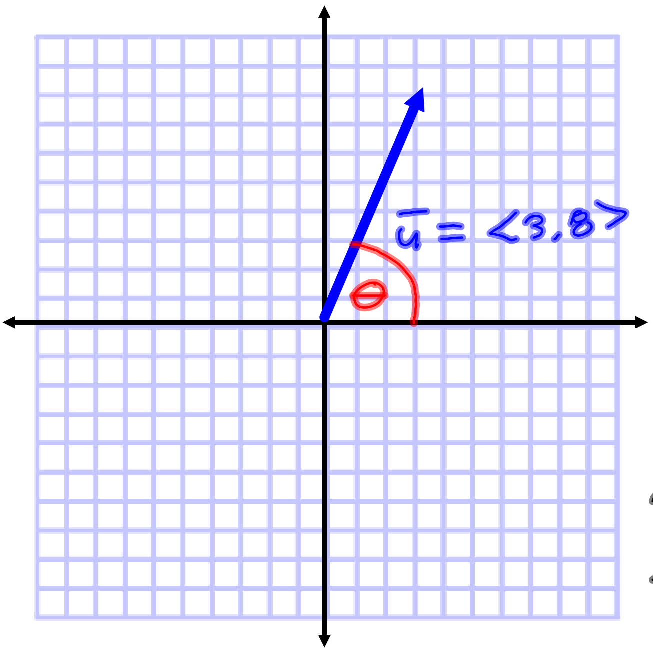


(4,2) corresponds to <4,2> $= 4\bar{i} + 2\bar{j}$





Any two non parallel vectors can be joined in a linear combo to reach <u>any</u> point in the plane.



Direction angles

$\tan\theta = \frac{y}{x} = \frac{8}{3}$

$\theta = \tan^{-1}\left(\frac{8}{3}\right)$ is fine for QI, but remember, the range of $\tan^{-1}(x)$ is $\theta = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$