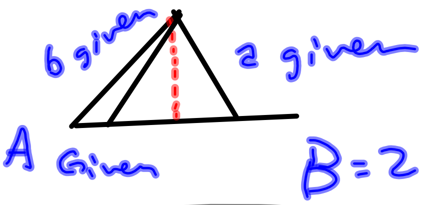
§3.1 Law of Sines

There're rules all laid out, but the trick is to draw the picture.

SSA when A is acute.



B=2

$\frac{h}{b} = \sin A \Longrightarrow h = b \sin A$

compare it to side a.

Need $h \le a$ to have a triangle.

SSA ambiguous

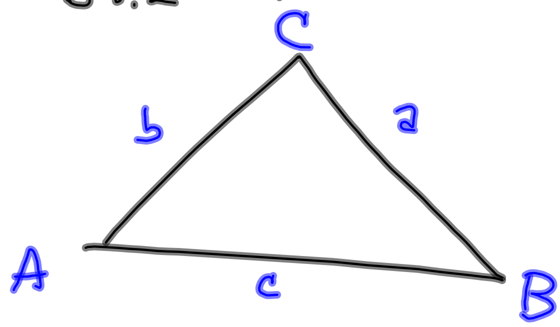
Really need to draw the picture.

what if $a > b$?

what if $a = h$? (and it's a right triangle.)

Then there's only one possibility

§3.2 Law of Cosines



up 'til now, we've handled
ASA
AAS
ASS

Law of Cosines helps us with
SSS and SAS

I'd keep Law of Sines & Cosines straight by always drawing the picture & seeing if Sines would work.

Law of Cosines

a,b,c given.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

In SSS case, you'd find A by solving this equation for $\cos A$ & then solving for A with $\cos^{-1}(\cos A)$

$$2bc\cos A = b^2 + c^2 - a^2$$

Book gives you:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

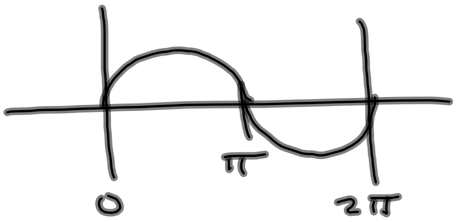
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Apr 6-8:25 AM

$x = 6\cos\theta \Longrightarrow$

$$\sqrt{36-x^2}$$
$$= \sqrt{36-(6\cos\theta)^2}$$
$$= \sqrt{36-6^2(\cos\theta)^2}$$
$$= \sqrt{36-36\cos^2\theta}$$
$$= \sqrt{36(1-\cos^2\theta)}$$
$$= \sqrt{36\sin^2\theta}$$
$$= \sqrt{36}\sqrt{\sin^2\theta}$$
$$= 6|\sin\theta|$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

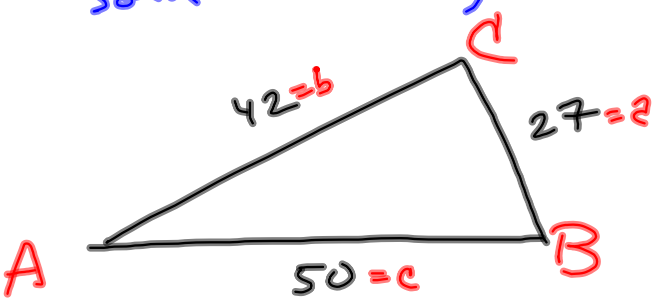


Bonus Related

If $0 \le \theta \le \pi$, then $6|\sin\theta| = 6\sin\theta$

" $\pi \le \theta < 2\pi$, " $6|\sin\theta| = -6\sin\theta$

Solve the triangle



$$a^2 = b^2 + c^2 - 2bc\cos A \implies$$

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{42^2+50^2-27^2}{2(42)(50)}$$

```
*42*50)
       .2464333333
(42²+50²-27²)/(2
*42*50)
        .8416666667
cos⁻¹(Ans)
        .5704340535
```

$$\cos A = .841\overline{6}$$

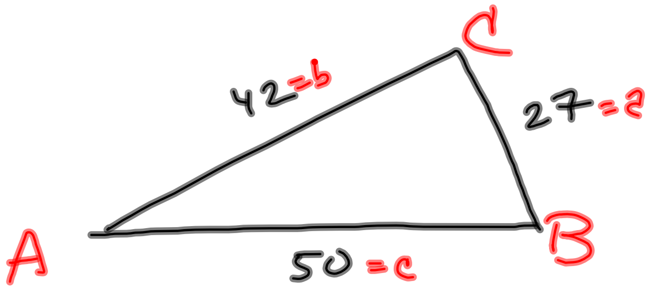
$$\implies A = \cos^{-1}(.841\overline{6}) \approx .5704340535 \text{ radians}$$

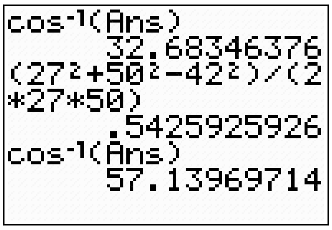
Better with degrees

$$\boxed{A \approx 32.6835^\circ}$$

```
cos⁻¹(Ans)
        .5704340535
(42²+50²-27²)/(2
*42*50)
        .8416666667
cos⁻¹(Ans)
        32.68346376
```

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$




```
cos⁻¹(Ans)
         32.68346376
(27²+50²-42²)/(2
*27*50)
         .5425925926
cos⁻¹(Ans)
         57.13969714
```

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{27^2+50^2-42^2}{2(27)(50)} \approx .5425925926$$

$$\Longrightarrow B \approx 57.13969714^\circ \approx 57.1397^\circ$$

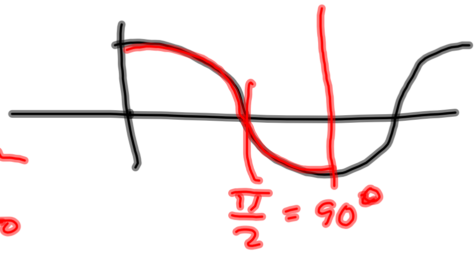
$$C \approx 180^\circ - 57.1397^\circ - 32.6835^\circ$$

$$= 90.1768^\circ$$ is obtuse.

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$ is negative. I'll betcha!

$$\approx -.0031$$

We restrict cosine's domain to $[0,\pi]$ or $[0^\circ, 90^\circ]$ to keep it 1-to-1, so that $\arccos(x) = \cos^{-1}(x)$ is a function



Cosine:

$D = [0,\pi]$

$R = [-1,1]$

cosine-inverse $\cos^{-1}(x)$ or $\arccos(x)$

$D = [-1,1]$

$R = [0,\pi]$