§3.1 Oblique Triangles

'A' stands for angle.
'S' .. .. side.

Solving Triangles
Find all 3 angles
& all 3 sides.

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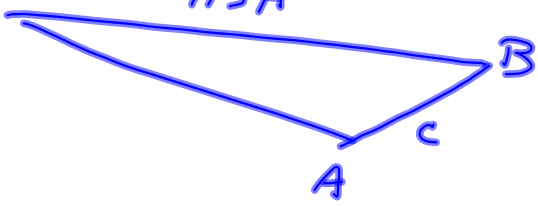
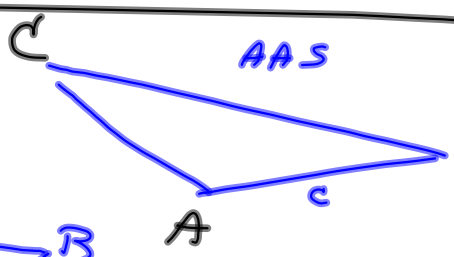
① AAS, ASA





These Kinds
can be solved, uniquely.

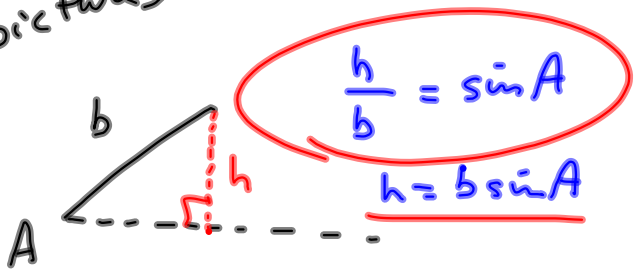
② SSA

See pg284

I wrote it out. When I got done, I realized I just need to draw pictures & find height = h

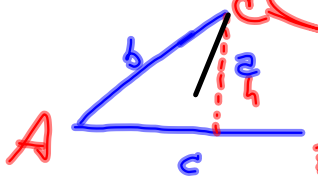
$h < a < b$

SSA isn't always solvable. Sometimes you get 2 solns. Sometimes 1. Sometimes None



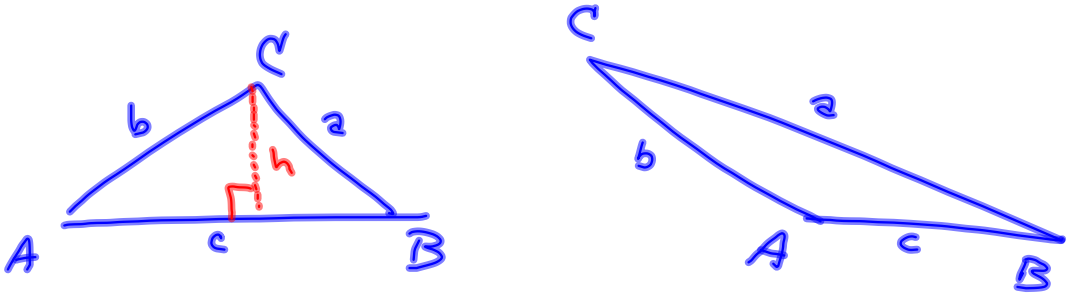
Suppose $a = h$. Then it's a right triangle!

Suppose $a < h$



No solution.
h is the shortest distance to the base.

Law of Sines



Law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Proof pg 331 sketch, where A is Acute.

$\sin A = \frac{h}{b}$ $\sin B = \frac{h}{a}$

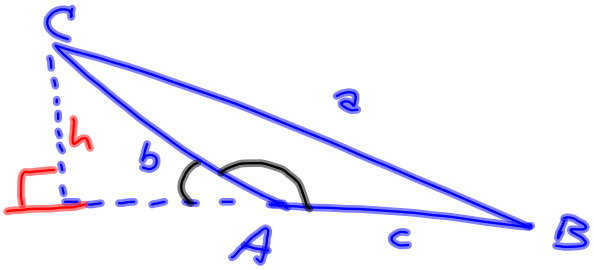
$\Rightarrow h = b \sin A$ $h = a \sin B$

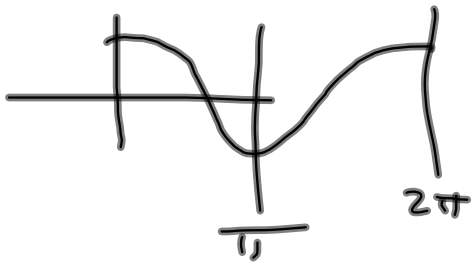
$h = h \Rightarrow$

$b \sin A = a \sin B \Rightarrow$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

Apr 4-8:24 AM





Sketch of proof for A being obtuse

$$\frac{h}{b} = \sin(\pi - A) = \sin\pi\cos(-A) + \sin(-A)\cos\pi$$
$$= 0 \cdot \cos A - (\sin A)(-1)$$
$$= \sin A$$

$$\frac{h}{b} = \sin A \qquad \sin B = \frac{h}{a} \Rightarrow$$

$$h = b\sin A \qquad h = a\sin B$$

$$\Rightarrow \ldots \Rightarrow \boxed{\frac{\sin A}{a} = \frac{\sin B}{b}}$$

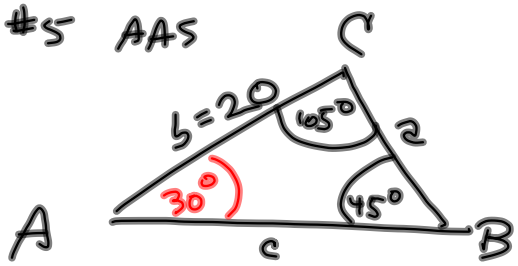
E1 AAS

E2 ASA

#s 5-24 solving triangles - mostly AAS, I think. They ALL have unique solns. AAS / ASA

#17 is SSA, but I bet it has a unique soln.

#5 AAS



$$\begin{array}{r} 105 \\ +\ 45 \\ \hline 150 \end{array} \quad \not\in \quad 180 - 150 = 30$$

$\boxed{A = 30^\circ}$

$$\frac{\sin B}{b} = \frac{\sin 45^\circ}{20} = \frac{\sin A}{a} = \frac{\sin 30^\circ}{a}$$

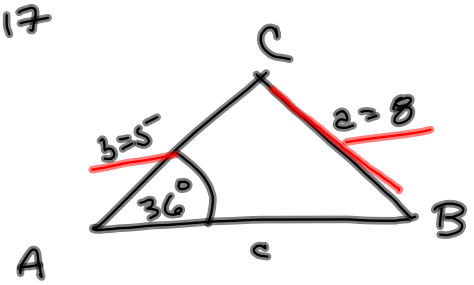
i.e. $\frac{\sin 45^\circ}{20} = \frac{\sin 30^\circ}{a}$ ← Find a.

Sure would be nice if 'a' started upstairs.

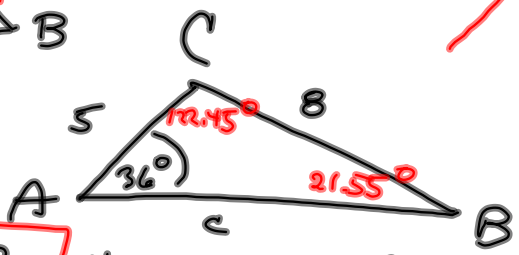
$$\frac{a}{\sin 30^\circ} = \frac{20}{\sin 45^\circ} \Rightarrow a = \frac{20 \sin 30^\circ}{\sin 45^\circ} \approx 14.14 \approx a$$

$$\frac{c}{\sin 105^\circ} = \frac{20}{\sin 45^\circ} \Rightarrow c = \frac{20 \sin 105^\circ}{\sin 45^\circ} \approx 27.32^\circ \approx C$$

#17



SSA



$\frac{\sin C}{c} = \frac{\sin A}{2} = \frac{\sin 36^\circ}{8}$ Has one soln

↳ 2 unknowns
No help.

$\frac{\sin B}{5} = \frac{\sin 36^\circ}{8}$

$\sin B = \frac{5\sin 36^\circ}{8}$

$\sin B \approx .3673657827$

$B \approx 21.55^\circ$

$B = \sin^{-1}\left(\frac{5\sin 36^\circ}{8}\right)$

$21.55 + 36$

$= 57.55$

180
$- 57.55$
$122.45^\circ \approx C$

$\frac{c}{\sin C} = \frac{2}{\sin A} = \frac{8}{\sin 36^\circ}$

$c \approx \frac{8\sin(122.45^\circ)}{\sin 36^\circ} \approx$

Next time:
3.1 Area of triangles.
3.2

Should be good for #s 1-34