§2.5 Finish

Need to create Homework for §2.4, 2.5

Identities we need to Know:

Double-angle

$\sin(2u) = 2\sin u \cos u$ → Know

$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan(2u) = \dfrac{\sin(2u)}{\cos(2u)}$

Power-Reducing

$\sin^2 u = \dfrac{1-\cos(2u)}{2}$

$\cos^2 u = \dfrac{1+\cos(2u)}{2}$

$\tan^2 u = \dfrac{\sin^2 u}{\cos^2 u}$

Product-to-Sum (Don't need to memorize)

$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$

$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$

4th is same deal.

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Solve: $x \in [0, 2\pi]$

$\sin(2x) + \cos x = 0$

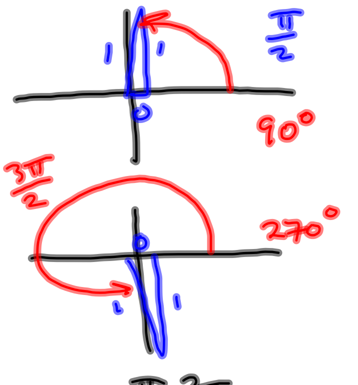
$2 \sin x \cos x + \cos x = 0$

$(\cos x)(2 \sin x + 1) = 0$

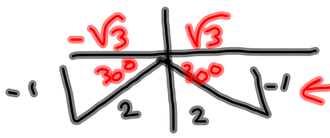
$\cos x = 0$ OR $2 \sin x + 1 = 0$

$2 \sin x = -1$

$\sin x = -\frac{1}{2}$



$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$



$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$180° + 30° = 210°$

$360° - 30° = 330°$

$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

#30 use double-angle

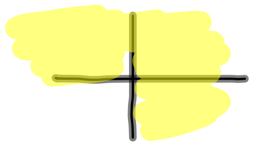
$6\sin x\cos x = 3\cdot\underline{2\sin x\cos x} = 3\sin(2x)$

#s 37-42

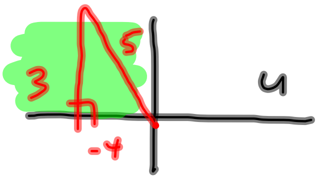
$\cos u = -\frac{4}{5}$ & $-\frac{\pi}{2} < u < \pi \Rightarrow -\pi < 2u < 2\pi$ No QIII involvement.

Find $\cos(2u)$, $\sin(2u)$, $\tan(2u)$

u



$\cos(u) = -4/5$ (QII or QIII)

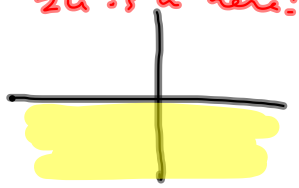


u Must be in QII

so $\frac{\pi}{2} < u < \pi$

& so $\pi < 2u < 2\pi$

2u is a here:



$\cos(2u) = \cos^2 u - \sin^2 u$

$= (-\frac{4}{5})^2 - (\frac{3}{5})^2$

$= \frac{16-9}{25} = \boxed{\frac{7}{25} = \cos(2u)}$

what does THIS say about where 2u lives?

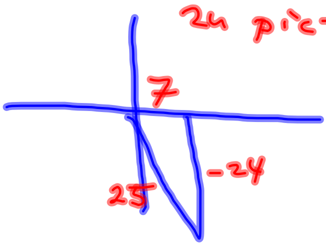
$\Rightarrow 2u \in$ QIV, because $\cos(2u) > 0$

$\sin(2u) = 2\sin u\cos u$

$= 2(\frac{3}{5})(-\frac{4}{5}) = \boxed{-\frac{24}{25} = \sin(2u)}$

$\Rightarrow \boxed{\tan(2u) = -\frac{24}{7}}$

2u picture.



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#s 43-52 Power-Reduce

$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

43 $\cos^4 x = \left(\cos^2 x\right)^2$

$= (\cos x)^4 = (\cos x)^{2\cdot 2} = \left((\cos x)^2\right)^2$

$(a+b)^2 = a^2+2ab+b^2$

$= \left(\frac{1+\cos(2x)}{2}\right)^2 = \frac{(1+\cos(2x))^2}{2^2} = \frac{1^2+2\cos(2x)+\cos^2(2x)}{4}$

$(1+\cos(2x))^2 = (1+\cos(2x))(1+\cos(2x))$

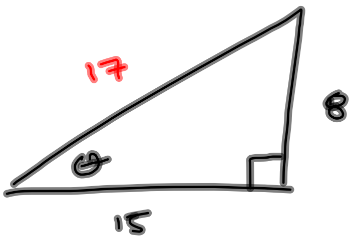
$= \frac{1}{4}\left[1+2\cos(2x)+\cos^2(2x)\right]$

$= \frac{1}{4}\left[1+2\cos(2x)+\frac{1+\cos(4x)}{2}\right]$

$= \frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{8} + \frac{1}{8}\cos(4x)$

$= \frac{1}{8}\cos(4x) + \frac{1}{2}\cos(2x) + \frac{3}{8}$

#s 53-58



$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\sin x = \pm\sqrt{\frac{1-\cos(2x)}{2}}$$

$$8^2 + 15^2$$
$$= 64 + 225$$
$$= 289$$

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

$$= \sqrt{\frac{1-\frac{15}{17}}{2}} = \sqrt{\frac{\frac{17-15}{17}}{2}} = \sqrt{\frac{\frac{2}{17}}{2}} = \boxed{\sqrt{\frac{1}{17}}}$$

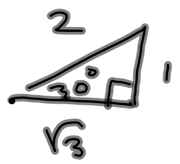
$$= \frac{\sqrt{1}}{\sqrt{17}} = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$

#s 59-66 Use $\frac{1}{2}$-angle to find sine, cosine, tangent of...

59 ... 75°

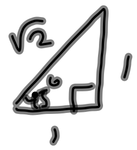
old way: $75^\circ = 30^\circ + 45^\circ$

$\sin(u+v) = \sin u \cos v + \sin v \cos u$



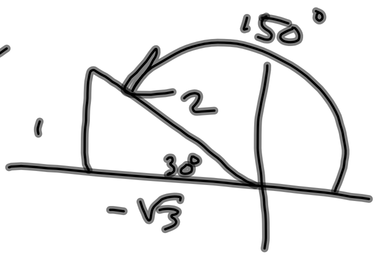
$= \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} = \boxed{\frac{1+\sqrt{3}}{2\sqrt{2}}}$



New way: $75^\circ \cdot 2 = 150^\circ$, i.e.,

$75^\circ = \frac{150^\circ}{2}$



$\sin\left(\frac{150^\circ}{2}\right) = \sqrt{\frac{1-\cos 150^\circ}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

Hmmmmmm looks different. is it?

$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{1+\sqrt{3}}{2\sqrt{2}} \overset{?}{=} \frac{\sqrt{2+\sqrt{3}}}{2}$

$= \frac{\sqrt{2}+\sqrt{3}\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2\cdot 2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

Come with this one figured out all the way

Counts as one homework.

Mar 19-8:51 AM