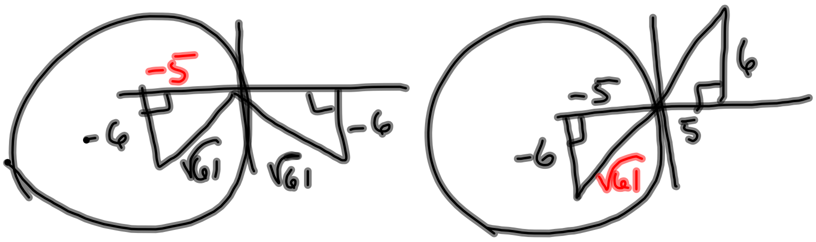
① $\sin x = -\frac{6}{\sqrt{61}}$ & $\tan x = \frac{6}{5}$



$\Longrightarrow \cos x = -\frac{5}{\sqrt{61}}$

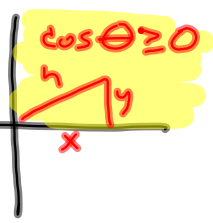
② $\sqrt{49 - x^2}$ & $x = 7\sin\theta \Longrightarrow$

$\sqrt{49 - (7\sin\theta)^2} = \sqrt{49 - 7^2\sin^2\theta} = \sqrt{49 - 49\sin^2\theta}$

$= \sqrt{49(1 - \sin^2\theta)} = 7\sqrt{1 - \sin^2\theta} = 7\sqrt{\cos^2\theta} = 7|\cos\theta|$

Knowing $0 \le \theta < \frac{\pi}{2}$

$\cos\theta > 0$



$7\cos\theta$

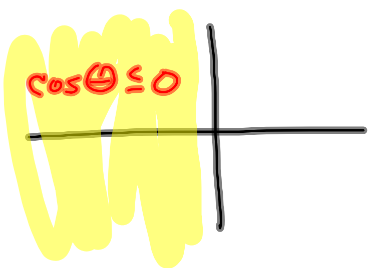
so $|\cos\theta| = \cos\theta$

Bonus: $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

so $|\cos\theta| = -\cos\theta$

so $|7\cos\theta|$

$= -7\cos\theta$



$=$

$$\boxed{\frac{2+3i}{5-7i}} \cdot \frac{5+7i}{5+7i} = \frac{10+14i+15i+21i^2}{5^2-(7i)^2} = \frac{10+29i-21}{25+49}$$

$$= \frac{-11+29i}{74} = \boxed{-\frac{11}{74}+\frac{29}{74}i} = a+bi$$

$a+bi$

$$\frac{4}{10} = \frac{2}{5}$$

$$(a+b)(a-b) = a^2-b^2$$

Show that $\frac{4}{10} = \frac{2}{5}$

$$\frac{4}{10} = \frac{\cancel{2}\cdot 2}{\cancel{2}\cdot 5} = \frac{2}{5}$$

$=$

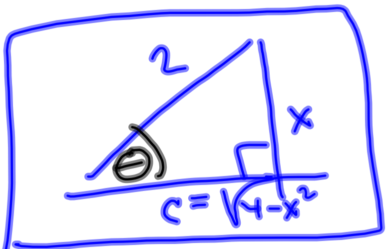
$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}}$$

$$\frac{\sqrt{(1-\cos\theta)^2}}{\sqrt{1-\cos^2\theta}} = \frac{|1-\cos\theta|}{\sqrt{\sin^2\theta}} = \frac{1-\cos\theta}{|\sin\theta|}$$

$$\frac{1-\cos\theta}{|\sin\theta|}$$

④ $\cot\left(\sin^{-1}\left(\frac{x}{2}\right)\right) = \frac{\sqrt{4-x^2}}{x}$

$\theta = \sin^{-1}\left(\frac{x}{2}\right)$



cotangent of the angle whose sine is $\frac{x}{2}$ is $\frac{\sqrt{4-x^2}}{x}$

$c^2 + x^2 = 2^2$

$2^2 - x^2 = c^2$

$4 - x^2 = c^2$

$\sqrt{4-x^2} = \sqrt{c^2}$

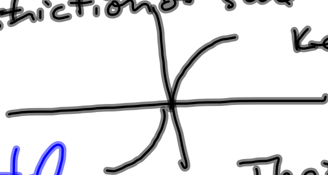
$\sqrt{4-x^2} = |c|$

Assume $c \geq 0$:

$c = \sqrt{4-x^2}$

$\sin^{-1}(x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Restriction of sine to keep it 1-to-1



That's what's needed to make $\sin^{-1}(x)$ a function

sin x must pass horizontal line test in order for $\sin^{-1}(x)$ to pass vertical line test

$2\sin^2 x - 7\sin x + 3 = 0$

$u = \sin x$

$2u^2 - 7u + 3 = 0$

$(2u - 1)(u - 3)$

$\vdots \qquad \vdots$

$u = \frac{1}{2}$ OR $u = 3$

$\sin x = \frac{1}{2}$ OR $\cancel{\sin x = 3}$

$a = 2, b = -7, c = 3$

$b^2 - 4ac = (-7)^2 - 4(2)(3)$

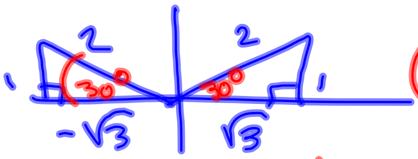
$= 49 - 24$

$= 25$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}$

$\frac{12}{4} = 3 \qquad \frac{2}{4} = \frac{1}{2}$



$(30^\circ)\left(\frac{\pi}{180^\circ}\right) = \boxed{\frac{\pi}{6} = x}$

$180^\circ - 30^\circ = (150^\circ)\left(\frac{\pi}{180^\circ}\right) = \boxed{\frac{5\pi}{6} = x}$

$\frac{5\pi}{6} + 2n\pi,\ n \in \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\frac{\pi}{6} + 2n\pi,\ n \in \mathbb{Z}$